

# ISPRAS OPEN 2016

## 3D flow modeling of Herschel-Bulkley fluid on the slope in OpenFOAM

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# Slope flows

## Examples of slope flows:

- snow avalanche
- landslide
- mudflow
- rockfall



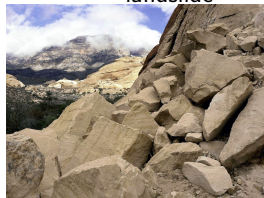
landslide



snow avalanche



mudflow



rockfall

## Snow avalanche

### Snow avalanche arise if:

- slope angle more than  $15^\circ$
- depth of snow cover 30 - 40 cm and more



Many studies of avalanches have been done, avalanche department is daily monitoring the situation in the mountains. Despite all these measures people still die in avalanches. The latest incident is artificial release of avalanche in the city of Kirovsk in Khibiny mountains. It happened at 10 pm 18 February 2016. The avalanche became disastrous, three people died. The railway and car road were blocked, windows were broken in three nearby houses.



## Software



OpenFOAM (Open source Field Operation And Manipulation) is a C++ toolbox for the development of customized numerical solvers, and pre-/post-processing utilities for the solution of continuum mechanics problems, including computational fluid dynamics (CFD). We use interFoam solver which realizes two-phase algorithm. The feature of the problem is a transient flow of two fluids separated by a sharp interface, or free surface. To find the shape of the free surface has been used a methods which do not define the interface as a sharp boundary (interface-capturing methods). The computation is performed on a fixed grid, which extends beyond the free surface. The shape of the free surface is determined by computing the fraction of each near-interface cell that is partially filled. This can be achieved by solving a transport equation for the fraction of the cell occupied by the liquid phase (the Volume-of-Fluid or VOF scheme).

## Mathematical model

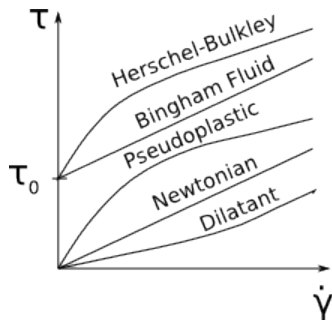
Avalanche is considered as a turbulent two-phase flow — snow and air. We will take the Herschel-Bulkley fluid as model for snow and Newtonian fluid as a model for air.

Newtonian model:

$$\tau = \nu \dot{\gamma}$$

Herschel-Bulkley model:

$$\tau = \tau_0 + k \dot{\gamma}^n$$



The shear rate dependency of shear stress in simple shear flow for different rheological models.

# Momentum equation and continuity equation.

We use the Reynolds averaged equations.

Continuity equation:

$$\frac{\partial(\rho\bar{u}_i)}{\partial x_i} = 0,$$

Momentum equations:

$$\frac{\partial(\rho\bar{u}_i)}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho\bar{u}_i\bar{u}_j + \overline{\rho u'_i u'_j} \right) = \rho g_i - \frac{\partial \bar{p}}{\partial x_j} + \frac{\partial \bar{\tau}_{ij}}{\partial x_j},$$

where  $\bar{\tau}_{ij}$  - components of the averaged molecular stress tensor,  
 $-\overline{\rho u'_i u'_j}$  - Reynolds stresses.

## Rheological model

$$\bar{\tau}_{ij} = 2\mu(|\dot{\bar{\gamma}}|)\bar{e}_{ij},$$

$$|\dot{\bar{\gamma}}| = \sqrt{2I_2(\dot{\bar{e}}_{ij})},$$

where  $\mu$  is defined as

$$\mu = \rho\nu,$$

$$\nu = (1 - \alpha)\nu_{air} + \alpha\nu_{snow},$$

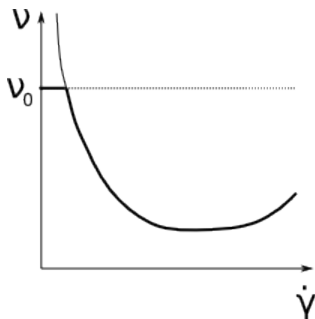
$$\rho = (1 - \alpha)\rho_{air} + \alpha\rho_{snow}.$$

$\alpha$  - is a volume fraction of snow.

$$\nu_{air} = \text{const},$$

$$\nu_{snow} = \min\left(\nu_0, \frac{\tau_0 + k|\dot{\bar{\gamma}}|^n}{|\dot{\bar{\gamma}}| \rho_{snow}}\right),$$

$$\rho_{air} = \text{const}, \quad \rho_{snow} = \text{const}.$$



The shear rate dependency of effective viscosity.

## Turbulence model

We use  $K - \varepsilon$  turbulence model for Reynolds stresses.

$$-\overline{\rho u'_i u'_j} = 2\mu_t \bar{e}_{ij} - \frac{2}{3}K\delta_{ij}$$

Turbulent viscosity:

$$\mu_t = \rho C_\mu \frac{K^2}{\varepsilon}.$$

The equation for the turbulent kinetic energy  $K$ :

$$\frac{\partial(\rho K)}{\partial t} + \frac{\partial(\rho \bar{u}_j K)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \mu \frac{\partial K}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\sigma_k} \frac{\partial K}{\partial x_j} \right) + P_K - \rho \varepsilon,$$

The equation for the dissipation  $\varepsilon$ :

$$\frac{\partial(\rho \varepsilon)}{\partial t} + \frac{\partial(\rho u_j \varepsilon)}{\partial x_j} = C_{\varepsilon 1} P_K \frac{\varepsilon}{K} - \rho C_{\varepsilon 2} \frac{\varepsilon^2}{K} + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_j} \right).$$

## The equation for snow volume fraction

The equation of snow volume fraction  $\alpha$ :

$$\frac{\partial \alpha}{\partial t} + \frac{\partial(\alpha u_j)}{\partial x_j} + \frac{\partial u_j^r \alpha(1 - \alpha)}{\partial x_j} = 0.$$

$\vec{u}_r$  — additional velocity field introduced to compress the interface.

The snow volume fraction in a computational cell:

$$\alpha = \begin{cases} 1 & \text{snow} \\ 0 < \alpha < 1 & \text{on the interface} \\ 0 & \text{air} \end{cases}$$

## Input parameters for the model

Air properties:

$$\rho_{air} = 1 \text{ kg/m}^3$$

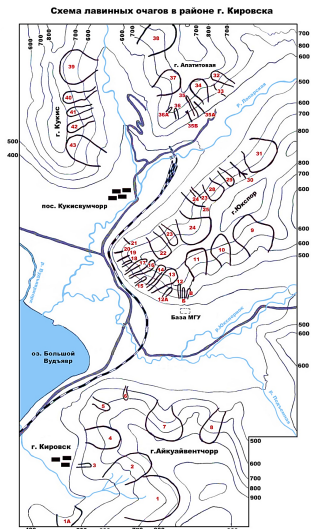
$$\nu_{air} = 1.48 \cdot 10^{-5} \text{ m}^2/\text{s}.$$

Snow properties such as  $\rho_{snow}$ ,  $k$ ,  $n$ ,  $\tau_0$ ,  $\nu_0$ , were set during the calibration of the model.

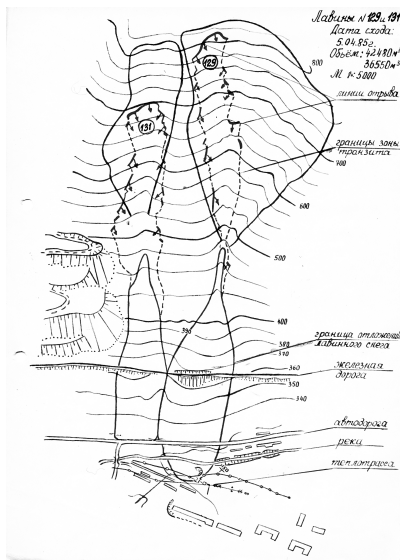
The turbulence model contains five constants, which are defined as follows:

$$C_\mu = 0.09; C_{\varepsilon 1} = 1.44; C_{\varepsilon 2} = 1.92; \sigma_K = 1.0; \sigma_\varepsilon = 1.3.$$

# Geographic maps



Scheme of avalanche sites nearby Kirovsk city.



Avalanches №129, №131



## Data about an avalanche №129 in site №22

Apatit company provided avalanche passport №129. It includes the avalanche description.

**We take the following parameters from the avalanche passport:**

- maximum thickness of the fracture line - 1.5 m
- medium thickness of the fracture line - 0.8 m
- maximum density avalanche deposits -  $320 \text{ kg/m}^3$
- medium density avalanche deposits -  $290 \text{ kg/m}^3$
- minimum density avalanche deposits -  $260 \text{ kg/m}^3$
- snow conditions - dry
- the maximum thickness of the avalanche deposits - 4 m
- the medium thickness of the avalanche deposits - 0.72 m

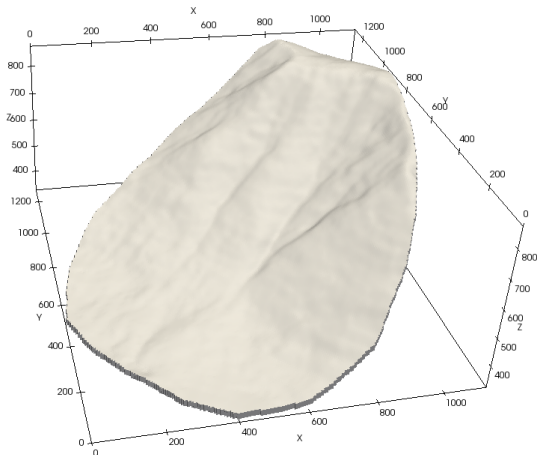
Lomonosov Moscow State University, Faculty of Geography Research Laboratory of Snow Avalanches and Debris Flows provided a digital raster map ASCII GRID format 22nd avalanche site, including avalanche occurrence area and the avalanche deposit area of the 129th avalanche.

# Slope



Photo of the avalanche site №22.

# Computational domain.

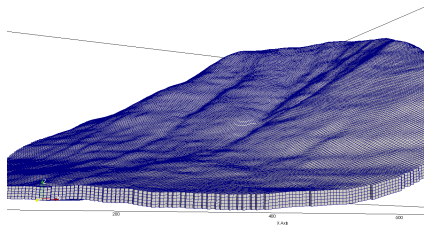
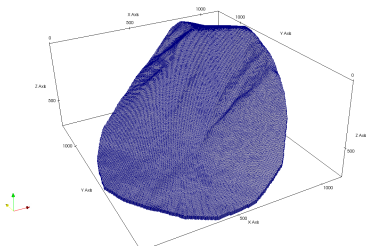


Computational domain is a 3D domain 20 meters up from the slope surface.

## Computation domain.

Calculations were performed using the grid of the parallelepiped cells with a linear size 5m. Computation domain consists of:

- 206 630 points
- 532 887 faces
- 447 345 internal faces
- 163 372 cells

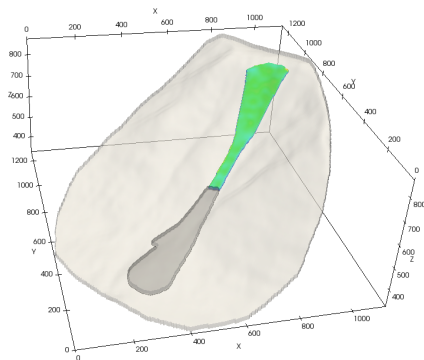


# Boundary conditions

- No-slip condition on the lower boundary of the computation domain (slope surface).
- Standard passing condition on sides and upper boundaries. The value of pressure equal to atmospheric pressure on these boundaries.

## Initial condition

At  $t = 0$ :  $\vec{U} = 0$ ,  $\varepsilon = 0$ ,  $K = 0$ ,  $\mu_t = 0$  everywhere. Snow depth in starting area is 1.5 m. The marker function  $\alpha$  equals 1 there, and 0 otherwise.



Avalanche starting area (green), avalanche deposit area (dark grey)

## Solution SLAE

The equation for the snow volume fraction  $\alpha$ , turbulent kinetic energy  $K$ , dissipation  $\varepsilon$ , and the momentum equation are solved by one of the relaxation techniques - symmetric Gauss-Seidel method.

The continuity equation is solved by the conjugate gradient method with preconditioner. The incomplete Cholesky decomposition was used as a preconditioner.

# Hardware specifications

Calculations made on the high-performance computing server NIISI RAS, consisting of:

- 2 x Xeon E5-2670v1 (Sandy Bridge) 8 cores 2.6 GHz (16 threads)  
4xchannel DDR3 memory controller
- MEM 256 GB DDR3-1333 ECC
- OS Ubuntu 15.10 64-bit



## Results

During the calculations the values of  $u_i(x, y, z, t)$ ,  $\alpha(x, y, z, t)$ ,  $\rho(x, y, z, t)$  were obtained, as well as the border of the avalanche flow, which was determined by the condition  $\alpha = 0.1$ .

Various calculations were performed, where the parameters  $\rho$ ,  $\nu_0$ ,  $\tau_0$ ,  $k$ ,  $n$  of the snow varied in the following ranges:

$\rho$ [kg/m <sup>3</sup> ]	$\nu_0$ [m <sup>2</sup> /s]	$\tau_0$ [m <sup>2</sup> /s <sup>2</sup> ]	$k$ [m <sup>2</sup> s <sup><math>n-1</math></sup> /s]	$n$
200	1e2	1	1e-5	2
300	1e5	1e-1	1e-4	5e-1
	1e-2	1e1	1e-6	

Thus, 108 cases were counted. The total time of all runs was about 36 hours. Parameter  $\tau_0$  is necessary to calculate the avalanche run-out distance. Parameter  $\nu_0$  is also important.

## Results

First, it was found that the taken values of the parameter  $k$  gave too large values of avalanche velocity. So additional study were performed using the values of  $k$  in the range  $1 - 1e2 \text{ m}^2\text{s}^{n-2}$ .

Finally, the following set of parameters

$$\tau_0 = 10 \text{ m}^2/\text{s}^2$$

$$\nu_0 = 10^5 \text{ m}^2/\text{s}$$

$$\rho = 200 \text{ kg}/\text{m}^3$$

$$n = 0.5$$

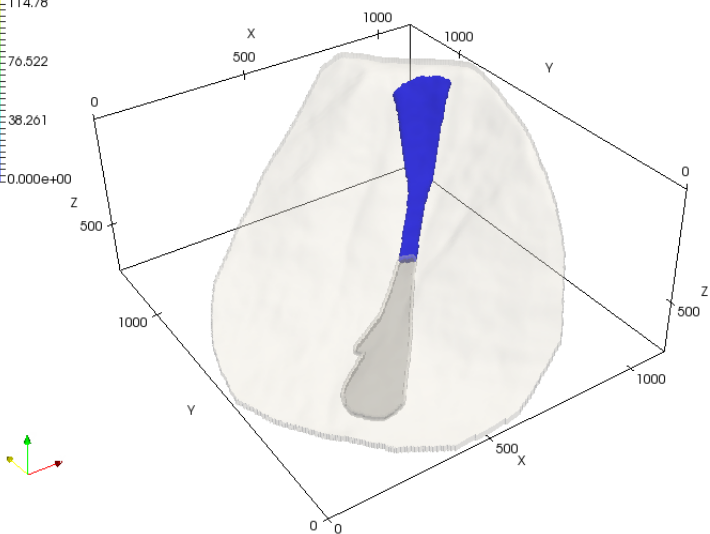
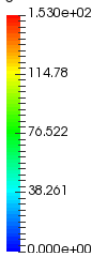
$$k = 5 \text{ m}^2\text{s}^{n-1}/\text{s}$$

was obtained. The results of calculations with these values were close to real avalanche data.

# Variations of flow velocity and avalanche contour with time

U Magnitude

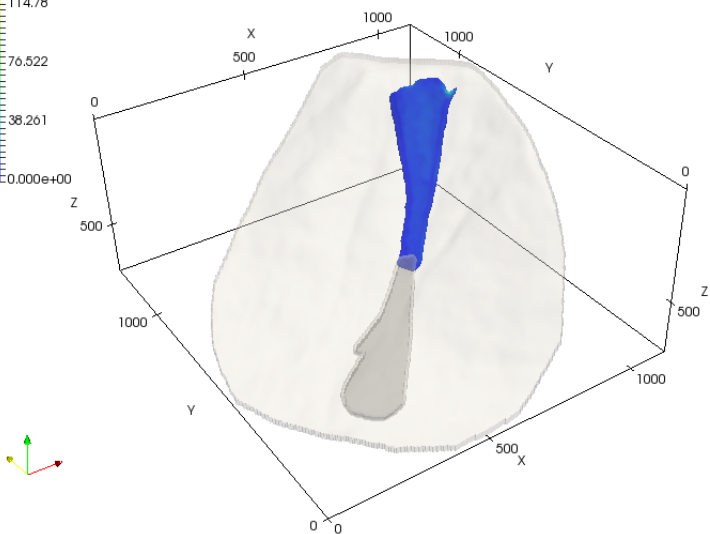
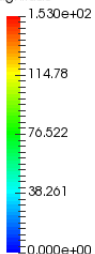
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# Variations of flow velocity and avalanche contour with time

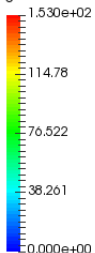
U Magnitude

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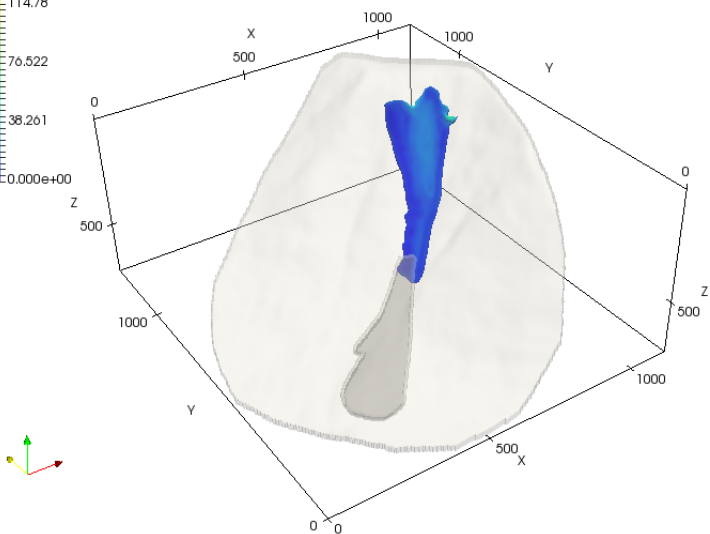


# Variations of flow velocity and avalanche contour with time

U Magnitude



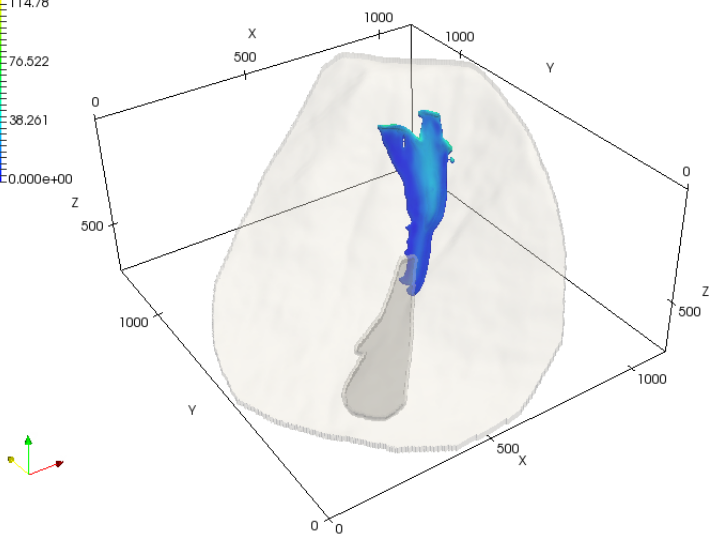
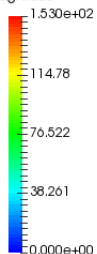
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# Variations of flow velocity and avalanche contour with time

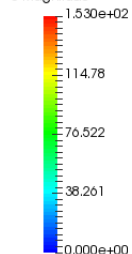
U Magnitude

Time: 15.000000

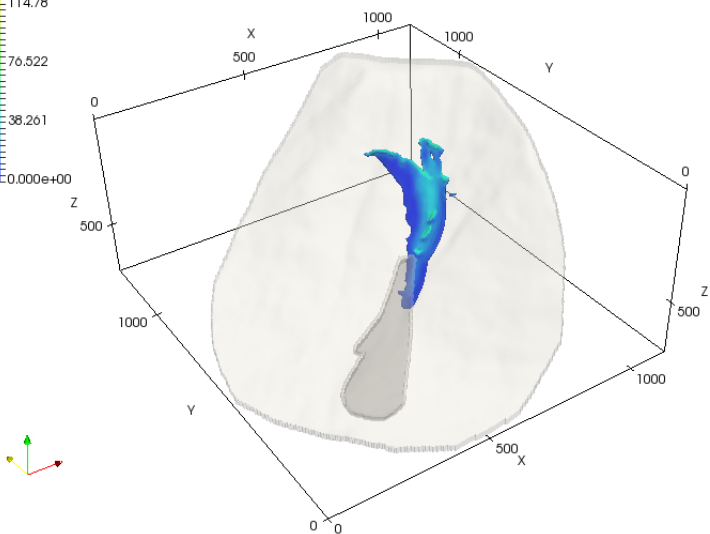


# Variations of flow velocity and avalanche contour with time

U Magnitude



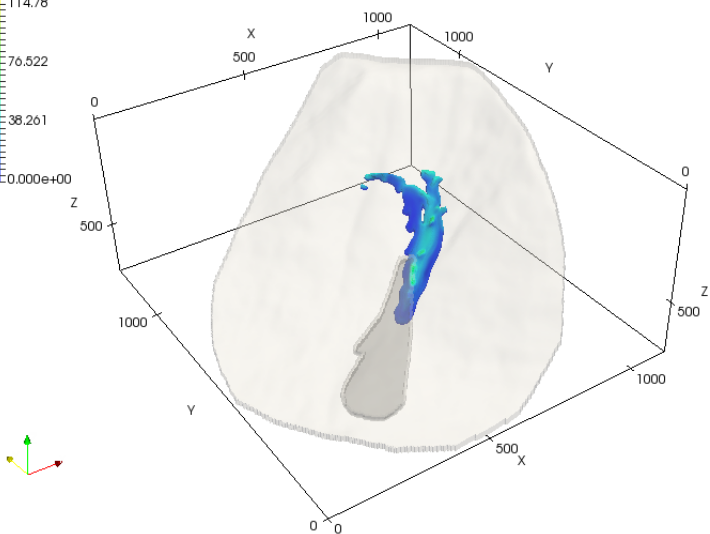
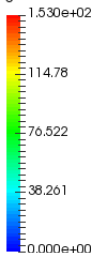
Time: 20.000000



# Variations of flow velocity and avalanche contour with time

U Magnitude

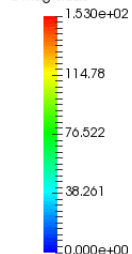
Time: 25.000000



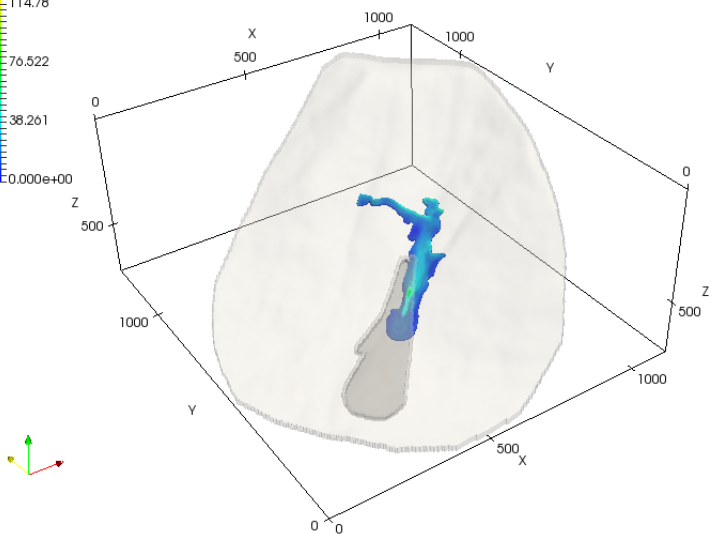


# Variations of flow velocity and avalanche contour with time

U Magnitude

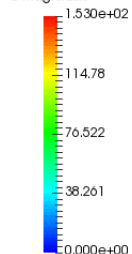


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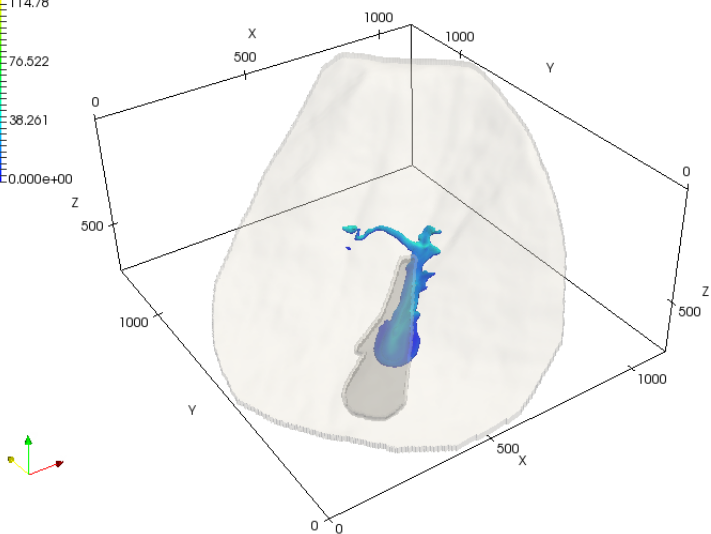


# Variations of flow velocity and avalanche contour with time

U Magnitude

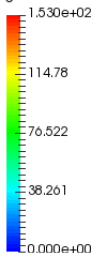


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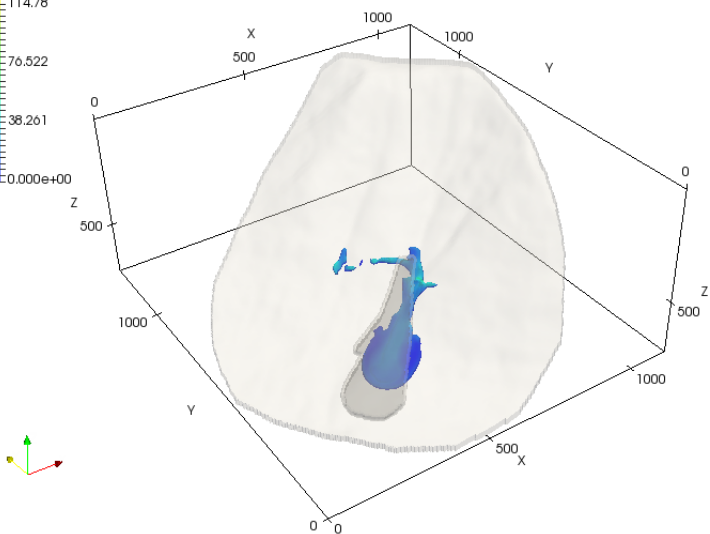


# Variations of flow velocity and avalanche contour with time

U Magnitude

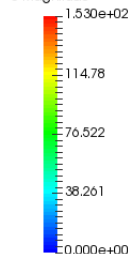


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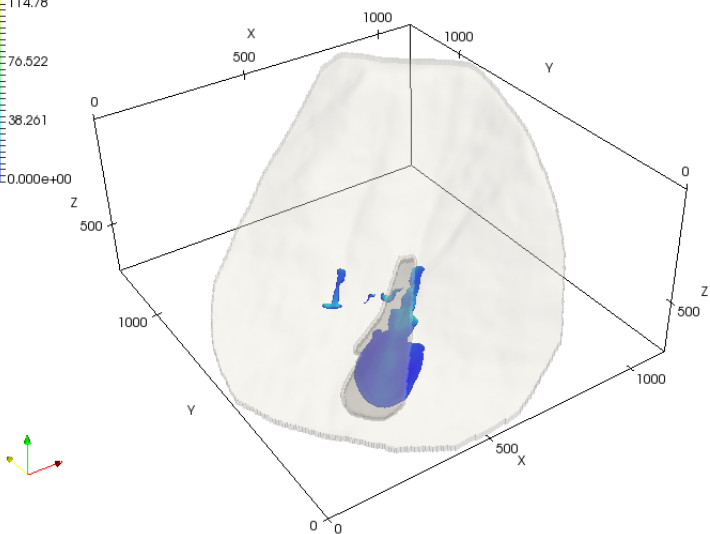


# Variations of flow velocity and avalanche contour with time

U Magnitude

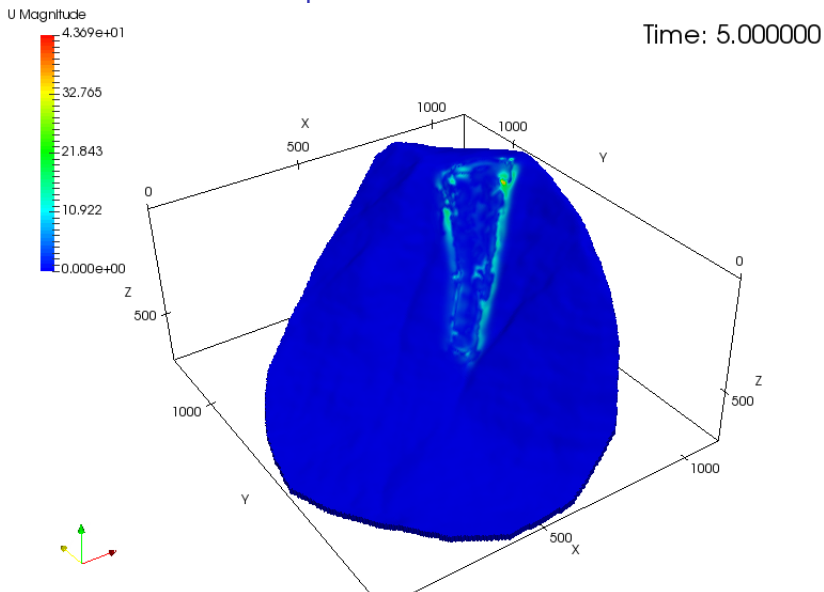


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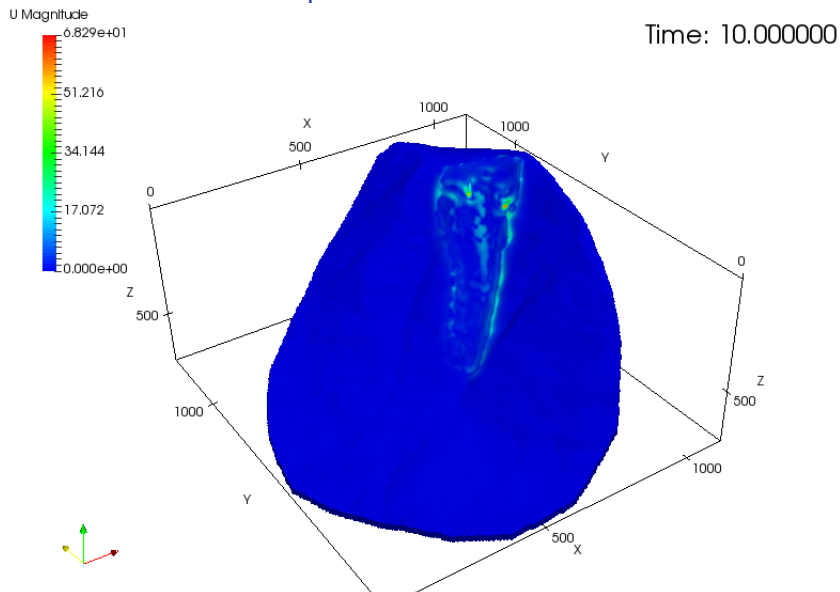




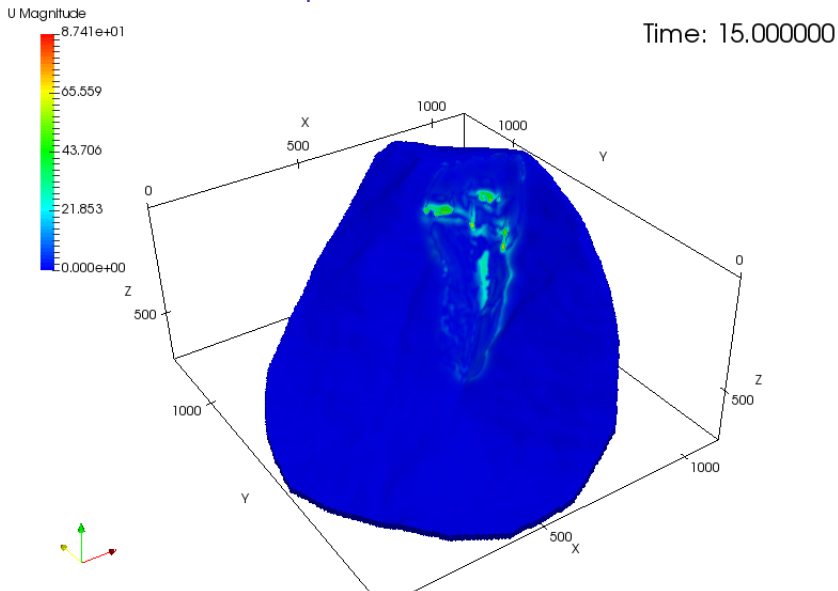
# The speed of the layer at the upper boundary of the computational domain



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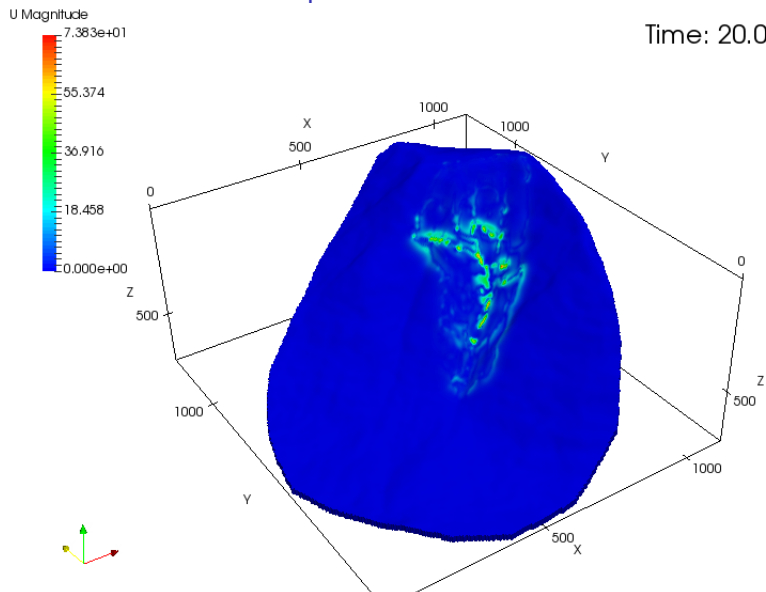
# The speed of the layer at the upper boundary of the computational domain



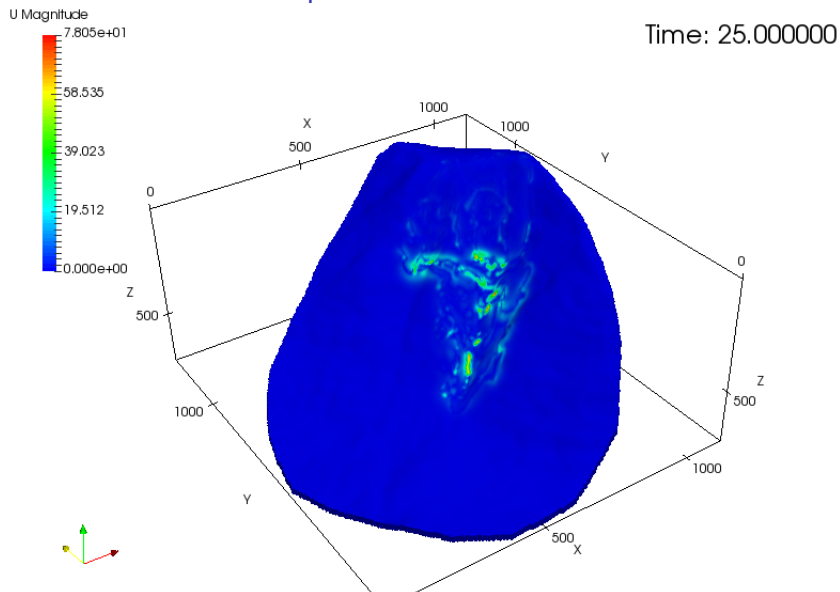


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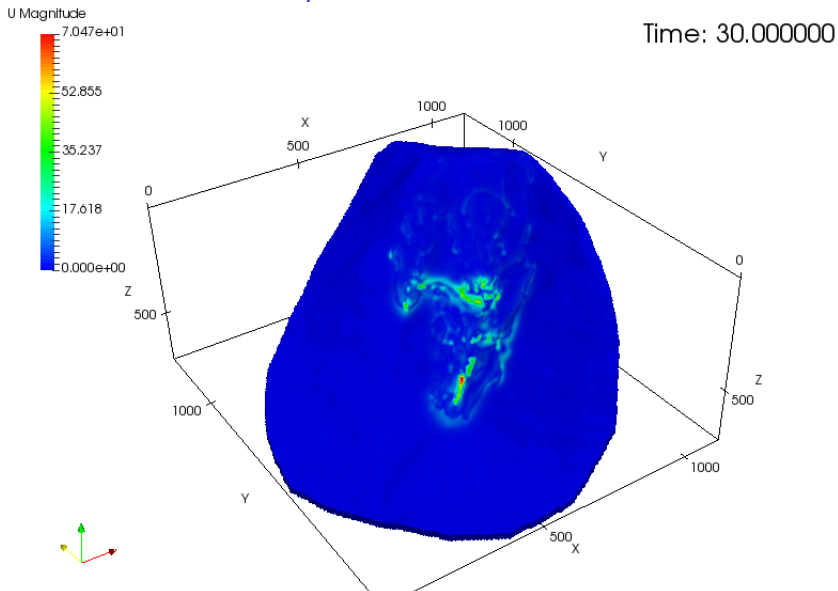
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# The speed of the layer at the upper boundary of the computational domain

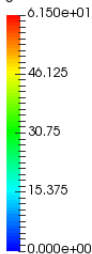


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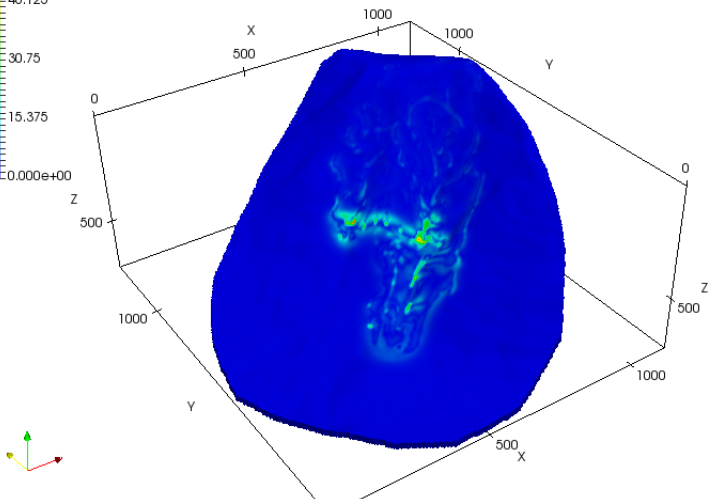


# The speed of the layer at the upper boundary of the computational domain

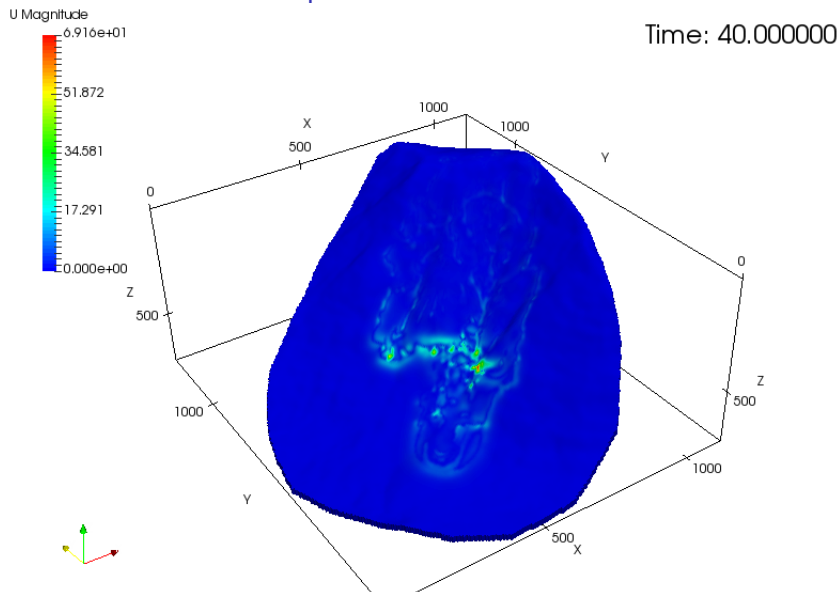
U Magnitude



Time: 35.000000



# The speed of the layer at the upper boundary of the computational domain





## Conclusions

- 3D numerical modeling of real avalanche in Khibiny mountains was performed using OpenFOAM.
- Both dense flow and snow-powder layer were simulated.
- The calibration of model was done for Khibiny mountains.
- Calculated shape of avalanche deposits area is close to that for real event.
- The calculated average flow velocity 44.8 m/s also corresponds to typical value for Khibiny's avalanches.
- In future we plan to do similar calculations for avalanches in other regions, make more detailed study, try different software and compare with results obtained by OpenFOAM.

# Acknowledgements

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for the avalanche data and discussion of the results



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