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1.

2e

()

).

NP-

(

).

2.

T -

: $Opt(T,C)$, $A(T,C) -$
()

A.

h_{max} .

$$R_A = \sup_{T,C} \left\{ A(T,C) / Opt(T,C) \right\}$$

$$R_A^\infty = \lim_{k \rightarrow \infty} \sup_{T,C} \left\{ A(T,C) / Opt(T,C) \mid Opt(T,C) \geq k \right\}$$

algorithms).

(on-line

$$\text{last}(R) = \max \{ k : w_k \geq w(R) \}$$

M_1, \dots, M_m

$$R \in M_i \Leftrightarrow \text{last}(R) = i$$

$$S(R) = h(r) \cdot w(R) - \frac{Q}{R}$$

$S(Q)$

$\frac{Q}{T}$

$$S_i(T) = S(\{R \in T \mid R \in M_i\})$$

$A(T)$

$$y = (y_1, \dots, y_m), \quad y_k = \frac{A_k}{T}$$

k .

$$d_i = \sum_{j=1}^i \tilde{w}_j$$

$$D_i(T) = \sum_{j=1}^i S_j(T)$$

$$h(T) = \max_i \frac{D_i(T)}{d_i}$$

$D_i(T)$

$h(T)$

1.

$$Opt(T, C) \geq h(T)$$

1.

[3]

A

$$R_A^\infty \geq e$$

3.

« »

$$r \in (0, 1)$$

$$r^{k+1} < h(R) \leq r^k$$

2e.

(one-dimensional bin-packing).
(First Fit),

$$Shelf(r).$$

3.1.

A_r .

T

$$A_r(T) = y = (y_1, \dots, y_k)$$

R

:

$$h = h(T + |R|).$$

$$k = \max_{w(R) \leq w_i} i \quad \frac{y_i}{w_i} \leq eh$$

k

k,

Shelf(r).

2.

A_r

T.

3.

A_r

$$R_{A_r}^\infty \leq 2e/r$$

2.

S -

h_{max}

Shelf(r),

$$Shelf(r) \leq \frac{2}{r} \frac{S}{w} + \frac{h_{max}}{r(1-r)} + 1$$

3.

$$A_r(T, C) \leq \frac{2e}{r} Opt(T, C) + \left(\frac{1}{r(1-r)} + 1 \right) \frac{h_{max}}{\min}$$

$$\frac{S}{w_j} \leq eh(T) \leq e \cdot Opt(T, C)$$

$$A_r(T, C) \leq \frac{2}{r} \frac{S}{w_j} + \left(\frac{1}{r(1-r)} + 1 \right) \frac{h_{\max}}{w_j} \leq \frac{2e}{r} \cdot Opt(T, C) + \left(\frac{1}{r(1-r)} + 1 \right) \frac{h_{\max}}{\min}$$

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On-line algorithm for scheduling parallel tasks on a group of related clusters

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Abstract. A classical problem of scheduling the set of tasks optimizing load balancing for a set of given processors was considered in theory in 1966. Graham's algorithm sending each task to a least loaded machine (processor) was the first example of approximate algorithm with guaranteed constant approximation quality. But all tasks and processors (machines) in Graham's example were simple (only one processor was sufficient for each task). On the other hand Graham's algorithm was an on-line one that is each task was sent to processor without knowing future tasks. With wide use of computational clusters very actual became the problem of scheduling parallel tasks on the set of parallel clusters in the class of on-line algorithms.. This problem has very close relation to a two dimensional packing problem, so-called multiple strip packing problem. Few years ago the author proposed an on-line algorithm for this problem and proved that for arbitrary set of parallel tasks (rectangles) the obtained schedule (packing) is within 2e of the optimal. In this paper the author generalized his on-line algorithm for the case of related parallel machines when all clusters have different speeds of processors in different clusters. An on-line algorithm for scheduling parallel tasks on a group of clusters with different speeds of processors is presented and it was proved that its approximation ratio is 2e for any set of parallel tasks. This result is non-trivial because even the case of one-processor related machines was intensively investigated and some algorithms with constant approximation ratio were presented.

Keywords: parallel computations, load balancing, scheduling parallel tasks on related clusters

References

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