Fractal Analysis of Growing Cities and its Relationship with Health Centre Distribution

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Abstract. During recent years, many research group interested in the study of organizational networks from different fields, e.g. studies of behavior of cities growth following the fractal theory. Considering the fractal analysis as a strong tools to analyse the relation between grow of cities and its relation with health center distribution. The propose of this paper is to develop a methodology based on a fractal analysis framework of Chilean public health networks, the population growth of cities and the location of health centers. To produce empirical approach for the analysis of the locations of health centers and its effects. On the other hand the health has been related to other sciences, a clear example is the physics and the mathematics, where we found many studies related with the geometry to understood the nature of the multiple natural shapes, its developed was called fractal nature or mathematic fractal model, this approach consider the different scales. For this reason, we present in this paper some preliminary ideas and some initial results of the analysis of the complex and fractal behavior of urban expansion, and its relation to the location of public health centers through simple counters algorithms and spectral methods using ImaCal software. The study health patterns will become a useful tool for understanding human flow and behavior of health center, where using a traditional measurement as equipment has serious problems and limitations.

Keywords: Multi-Fractals; Urban growth; City hospitals; Health centres; Nonlinear process; Chile health centres

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1. Introduction

The objective of this work is to propose a methodology based on a multi-fractal analysis framework of Chilean health networks and the relationship between human flow, population growth of cities and he location of health centers, seeking to produce new ideas for the empirical study of the health situation and its scaling and geographic determinants using an open software application ImaCalc.

The establishment of inter organizational [1] relations has been one of the characteristics of the business of the last decades; these relations are constituted as relatively resistant flows and unions that occur between an organization and one or several organisations of its environment. These long-term relationships allow deeper understanding of the structures and how they may maintaining competitive advantages, as they offer the opportunity to work together in a shared knowledge or physical environment. The theoretical advantage is that each organization can concentrate on doing what it does best and hire independent companies to do the rest of its activities, but this simple approach does not take into consideration even scaling effects.

2. Networks in the Health Systems

The Pan American Health Organization (PAHO, 2010) [2] in response to the consequences of heavily fragmented and segmented health systems, has expressed the need to implement Integrated Health Services Networks in the health systems of Latin American and Caribbean countries.

The concept of integrated health services has been favored for decades in the health discourse, however, discrepancies in the scope of implementing real inter-organizational coordination and effective cooperation measures, have shown the difficulty to reverse obstacles, based on the local social, economic and political reality that often do not contribute to the sustainability of integrated systems.

3. Multi-Fractal Ideas and Analysis

Fractal objects are irregular in shape but their irregularity is similar across many scales [3], enabling them to be described mathematically and to be generated computationally. Following the theory of chaos and fractal and multifractal geometry theory, we apply these ideas using the advances of fluid visualization [4], to investigate urban human behaviour, urban growth and relationship with public health networks in Chile. The vision of complex natural things has changed in recent years

[5], because in science, the nonlinear and fractal evolution of plants, feathers, clouds, flowers, rocks, mountains, tapestries and many other things has helped their understanding [5,6,7]. We present some preliminary ideas and some initial results of the analysis of the complex; multifractal behaviour of urban, expansion and their relationship with the location of health centers public through simple box-counting algorithms and spectral methods (Digiflow, ImaCalc, Matlab).These are open distribution, free, or limited academic license programs.

Here we consider multifractals as a statistic distribution that yields useful information even if the underlying structure does not show a complete self-similar or self-affine behaviour, but generalised spaces may help. A clear example of how fractal mathematics benefits health is to occupy it in various structures of the human body, such as in nervous networks, in blood vessels, in lung alveolar structure and even in the distribution of health centers. Chaos and non-linearity, fractal geometry, emergency and catastrophe theory, dynamic systems, and network theory constitute new methods that represent significant potential in the production of scientific knowledge and technological development in the area of health [8, 9].

For a long time, health has been related to other sciences, clear examples are chemistry, physics and mathematics, and now, from a human point of view: topology, when studying geometry multiple natural forms appear triangles, squares, hexagons, etc., but in practice irregular forms of fractal or multi-scale nature dominate. Within the methodology to study the flow location or distribution of health centers in a complex linked network, it is important to consider that the mathematical theory of fractal growth behavior, depends on of the diversity of parameters used in the description *{Xi},* their time derivative *{dXi/dt},* acceleration, spatial gradients etc.; with different viewpoints and considerations in a generalized system of coordinates, from a dimensional point of view, the flux of property *Xi* is the derivative in time per unit area [10,11].

4. Study of the Central Chile Area (CCA) Health Structure

Recent research has focused on the management of health networks, which, because of their establishment, growth and development, have formed relationships that are relatively resistant, with quite stable flows and unions that occur between an organization and one or more organizations of its neighborhood, which determine the characteristics of their success, business efficiency and their management [12].

The fundaments of this analysis are to consider the network as objects of health/disease propose a fractal structure of network on the base of the fractal unit, and in each point, to be double or increase the size in the same way. In figure 1, we represent the network of health as groups of four elements as cultural diversity, social development, economic development and the environment [9]. Figure 2 represents the evolution since 2010 and projection of the urban growth of Chillan.

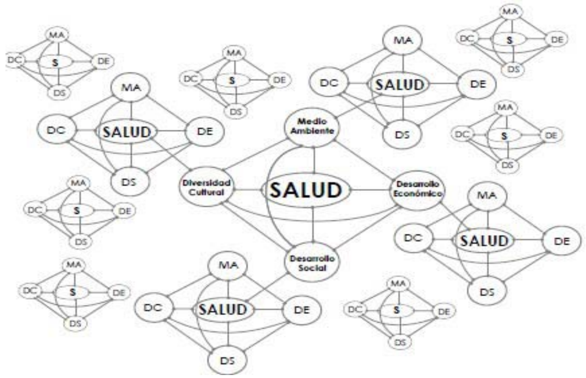

Fig. 1. Health as Fractal Network [9]

4.1 Urban Growth Behavior in Central Zone Chile

The accumulation, now seen from its growth projections, yields quite important figures, almost comparable to the projections the growth of the entire region (Institute Nacional de Statistical - INE 2002). According to projections, without substantial changes in future growth, growth will occur almost entirely in the most attractive cities, therefore, in the central area.

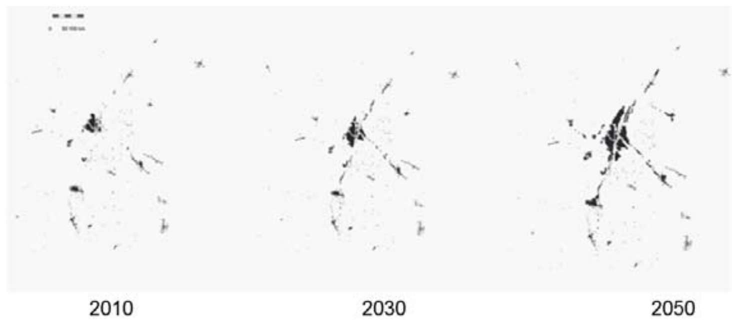

Fig.2. Urban growth – projections Chillan-Chile

There is a great amount of small towns, villages and small villages dispersed in most of the territory, with population voids at greater distances of the central area. These entities are close to large arable areas, natural areas, or simply abandoned areas, which could be characterized as the edges of the supra-region. This, in turn, shows a high rural percentage in the territory and consequently it may be described as a ring of unprotected areas, for example, in terms of planning, transport, infrastructures, etc.

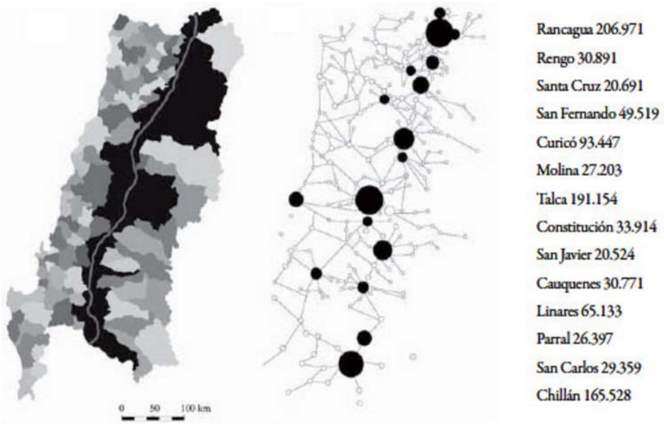

Fig.3. Geographical information and population of the study region, the size of cities is proportional to the number of inhabitants

The urban population of this region is concentrated mainly along the centre of Chile, in a few major cities, which are administrative provincial or regional capitals, including the province of Ñuble. The rest of the population is dispersed throughout the regional territory in innumerable villages, hamlets and even smaller centres. Figure 3 shows the visual information of the population distribution, while figure 4, and also presents the main human flows, both associated to the city structure, the transport infrastructures and the local conditions.

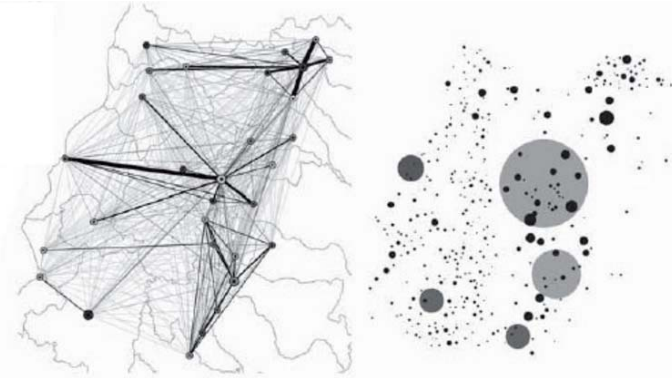

Fig.4. Human flow, population – location near the study area

4.2. The Integrated Networks of Health Services in Central Chile

A network of organizations that provide, or arrange to provide, equitable and comprehensive health services to a defined population, and which is accountable for its clinical and economic results and for the health status of the population at large.

We combine here the population structure and the health infrastructures of the area.

Fig.5. Hospital network in the province of Ñuble

The micro-regional capabilities and their integrations, connect areas of the same hierarchy, that is, connecting micro-regional areas with others, in theory, a series of longitudinal corridors are configured. As a result of the geographic conditions and real physical possibilities that the territory offers, alternative areas are developed to the central corridor, which finally acquires the scale of the entire macro-region. Figure 5 represents the main hospital distribution in the CCA.

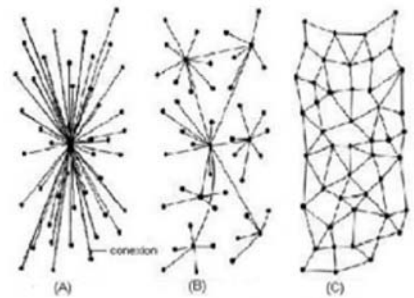

Fig. 6. (A) Centralized, (B) De-centralized y (C) Distributed. The sciences of complexity and medical innovation, Institute of Physics of the Center for Interdisciplinary Research in Sciences and Humanities, National Autonomous University of Mexico, Grama Editora, S.A., 2006

Fractal geometry studies and classifies several possible distributions both in fixed or in dynamical coordinates, for example figure 6 shows three different network (or flux) possibilities of the same geographical data (UNAM), so complex systems allow a wide range of possibilities. From a topological point of view in figure 7, we show how statistically random, but simple fractal geometry, and the evolution of the monofractal or (on-off) black-white structure is described with *Do= 1.89*. This is the Sierpinski Carpet in a random set, to show the structure and the classification of complex geometry and how it is possible to determine in this case a single fractal dimension.

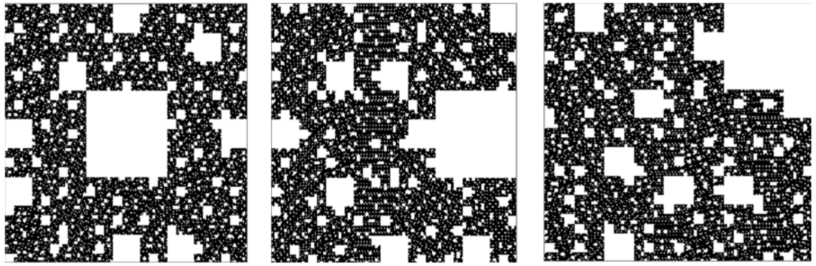

Fig.7. Three types of Random Sierpinski Carpets with five size levels of recurrence. They represent possible levels of Health Centers in an urban area. In spite of their different Networks, the spatial Fractal dimension D = log8/log 3 = 1.8928 is the same but in a statistical sense [13, 14]

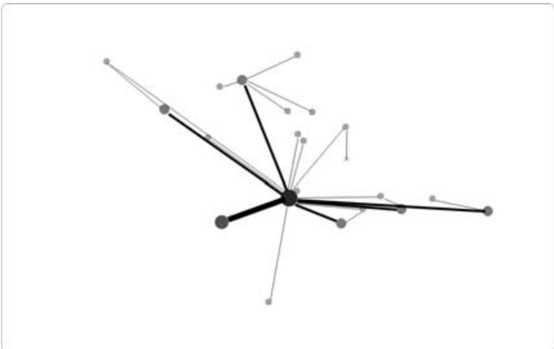

Fig.8. Geographical composition of human flows between the main hospitals in the province of Ñuble. Calculation of hospital levels, high-level complexity (8), medium level complexity (4) and low complexity (1).

Programs coupled to complex non-linear behaviour have been used since R. Thom (1988) [15,17,18], using concepts such as evolution Entropy, but advanced image processing for fluid mechanics to Flux-Force ideas is recent. As an example, the thickness and intensity of the marked roads are proportional to the transit not unlike a blood vessel or a Thermo-Magnetic Flux. In order to show the complexity of the network relations and interactions, we use the Fractal Dimension as a function of the traffic intensity *i,* we can thereby define the fractal dimension *D (i)* also as a function of the scale *e* of the image. The fractal dimension usually is calculated following this equation:

$$D(i) = -\mathrm{Log}(N(i)) / \mathrm{Log}(e, i),$$

where *N(i)* is the number of boxes of size e, needed to cover the image contour of intensity *i.* The algorithm used by ImaCalc 1.5 [16, 18] (available in free access from: https://www.academia.edu/420566/ImaCalc_Executable_Program), this program is oriented to calculate the fractal dimension of images using the Box-Counting algorithm. It also includes simple tools for processing and analysis of images operates dividing the 2D surface into smaller and smaller square boxes and counting the number of them that have values close to the level under study, for different iterations, *n*). For each box of size *l/n* it is then decided if the convoluted line, is intersecting that box. The slope of *N* versus the size of the box *e* in a log-log plot, within experimental limits, gives the fractal dimension for a single intensity or for the condition of small boxes: *Lim e -> 0* can be measured by counting the number N of boxes needed to cover the Flux Map under investigation for decreasing box sizes and estimating the limit of the slope.

There are several methods for implementing multifractal analysis; the moment method uses mainly three functions: The mass group of similar transport is conditional to many specific health related aspects, such as size, speciality distribution, etc. this is similar to traffic intensities described in [11, 17]. This method of box-counting in ImaCalc software is used to detect the self-similar behaviour. This analysis can be made for the Scalar marker, for a vector, such as a Vorticity or a generalised variable. In a monofractal object, the number *N* of features of a certain size *e* varies as

$$N(e) = c(e) \exp\{-Do\},$$

where the fractal dimension *Do* at the limit of small size exponent function $\tau(q)$ the coarse Holden exponent, α and the multifractal spectrum, $f(\alpha)$ as seen in figure 8. We have only used a grey scale between 0 to 255, but with more data, it is possible to improve the histogram of the analysed images.

A complex network could be decomposed spatially in terms of infinitely many intertwined sets of fractal dimensions. If that is the case, one fractal dimension cannot characterise all the complexity and several fractal dimensions will be estimate depending on the position and other relevant parameters, such as the patient type of ailment or condition.

Applying box counting “up-scaling” partitioning process we can obtain the partition function relating *m* as the mass of the measure, *q,* the mass exponent, *e* the length size of the box and *N* is the number of boxes in which the statistical moment of the measure, defined as a group of non-overlapping boxes of the same size partitioning the area studied. *Dq (i)* are related to $f(\alpha)$ as multifractal measures are the concept of generalized dimensions Dq. This set of descriptor corresponds to the scaling exponents for the moment of the measure.

Fractal Dimension as a function of the growing population map of province de Ñuble as intensity *i* of, we can thereby define the fractal dimension *D (i)* also as a function of the scale e of the image. This dimension is usually calculated using: *D (i) = - Log*

(N(i)) / Log (e, i), Where *N(i)* is the number of boxes of size *e,* needed to cover the image contour of intensity *i.*

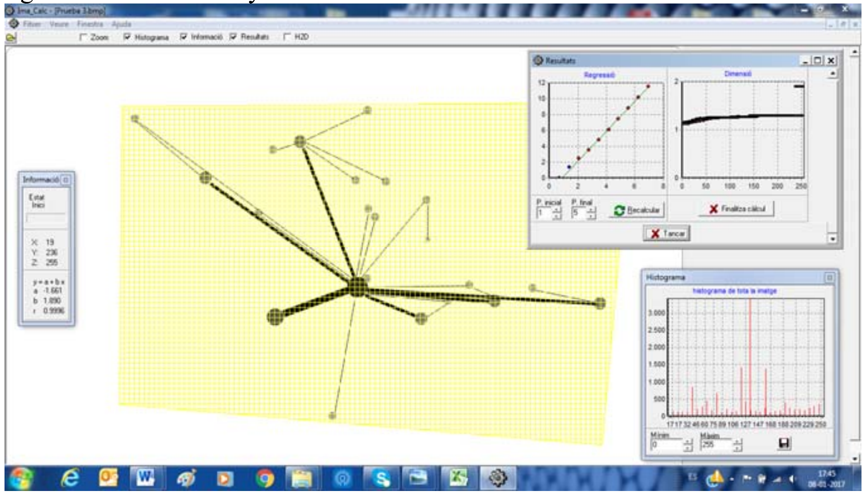

Fig.9. Visualization of ImaCalc applied on the human flow in the province of Ñuble

The algorithm used by ImaCalc operates dividing the 2D surface into smaller and smaller square boxes and counting the number of them that have values close to the level under study, for different iterations [14]. The Sierpinski carpet is used as a heavily urbanized city simple model, with fractal dimension D_{max}= log 8 / log 3 = 1.892789, but using multifractal generation at different intensities, the differences between the spectra of the city structure and that of the sanitary network may be quantified as D_{max} *(i,e)*- D_{health} *(i,e)*.For the multifractal spectra shown in the bottom right corner of figure 8 the urban flow in the province of Ñuble shows a variation of the fractal dimension between D_{max} (i) = 1.15 and 1.25.

This follows the methodology of [17, 19], where use of program ImaCalc is explained further.

5. Conclusions

In this case, the methodology carried out can become a useful tool to understand the human behaviour of the flow approaching the inlet where the traditional measuring equipment has serious problems and limitations.

An important achievement in human studies is the development of new techniques for the measurement and prediction of complex network relations and interactions. The technological advances in “Big Data” digital processing networks and the advances in image processing techniques give the researchers an enormous potential to measure and study the behavior of human population flows, in this context, it is possible to use these non-linear network techniques to study health patterns and patient fluxes related to the scale of the hospitality or road network. The results

presented in figure 9, corresponding to the general patient mobility could be upgraded to include the different medical specialties as well as different Indexes associated to the health practice.

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Фрактальный анализ растущих городов и его взаимосвязь с распределением центров здоровья

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Аннотация. В последние годы многие научные коллективы исследуют поведение организованных сообществ в различных областях, например, изучают поведение растущих городов в соответствии с теорией фракталов. Теория фракталов является мощным инструментом для анализа взаимосвязи между ростом городов и распределением сети центров здравоохранения для населения. Целью данной работы является разработка методологии, основанной на теории фракталов, для изучения сообщества общественных организаций в области здравоохранения в государстве Чили, а также для изучения роста населения и эффективного расположения медицинских центров для обслуживания населения. Авторами статьи предложен эмпирический подход для анализа ситуации в области здравоохранении и различных эффектов приложения данного подхода. Известно, что фрактальная теория применима к задачам физики и математики. Имеется много примеров, в которых изучается геометрия объектов для обеспечения понимания многообразных форм в природе. В связи с этим, мы представляем в этой работе ряд идей и свои первые результаты анализа фрактального поведения процесса урбанизации в городах, влияния урбанизации на расположение центров здравоохранения с использованием простейших счетных алгоримтов и спектральных методов с использованием программного обеспечения ImaCal. Результаты

исследования позволят лучше понимать разные аспекты миграции населения в связи с размещением центров здоровья людей.

Ключевые слова: сообщества; рост городов; центры здравоохранения; население; теория фракталов; урбанизация; нелинейные процессы.

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