

Numerical Simulation of High-Speed Non-equilibrium Flow with Applied Magnetic Field

^{1,2}A.I. Ryakhovskiy <alexey.i.ryakhovskiy@mail.ioffe.ru>

^{1,2}A.A. Schmidt <alexander.schmidt@mail.ioffe.ru>

²V.I. Antonov <antonovvi@mail.ru >

¹Ioffe Institute

Polytechnicheskaya st., 26, Saint-Petersburg, 194021, Russian Federation

²Peter the Great St.Petersburg Polytechnic University

Polytechnicheskaya st., 29, Saint-Petersburg 195251, Russian Federation

Abstract. The paper describes the development and testing processes of a modification of an existing solver for hypersonic reacting flow within the OpenFOAM numerical simulation framework. The modification is suited to simulate the interaction between the flow and the constant applied magnetic field. The purpose of the development is to create a simulation tool for the study of the concept of magnetohydrodynamical flow control and its possible technological applications. Resulting application utilizes Navier-Stokes-Fourier system of equations supplemented with appropriate auxiliary models for the accurate assessment of process-specific additional terms. Solver testing has been carried out using the cases that highlight solvers capabilities to model MHD high-speed flow in different regimes.

Keywords: numerical simulation; fluid flow control; magnetohydrodynamics; aerodynamics; shock waves

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1. Introduction

In the recent years there has been a resurgence of interest towards plasma-assisted technology in high-speed flight. Prospective applications of this technology include aerobraking, communication blackout mitigation, aerodynamics heating reduction, wave drag and turbulence cancelation and scramjet engine intake control. The principal idea behind most of these concepts is magnetohydrodynamic (MHD) control

i.e. affecting the gas flow incoming towards an aircraft with an applied magnetic field. The potential of MHD flow control system has been studied and demonstrated on numerous occasions [1][2][3], however it's prospective effectiveness remains an open question. With the new challenges facing aerospace industry in the upcoming decade comes a need for more advanced numerical simulation tools.

Near space is the region of the atmosphere between controlled commercial airspace and the low earth orbit (LEO) [4]. This is the region where most of the aircraft with a potential to benefit from MHD flow control system operate. The difficulty of modeling the MHD interaction in hypersonic flow within the near space region is that the rarefaction of the gas can be high enough that the CFD model can become non-applicable in some cases, yet DSMC-type (Direct Simulation Monte Carlo) can be too computationally prohibitive. The issue has to be addressed in developing a comprehensive simulation tool for the high-speed magnetohydrodynamic flow.

There are several simulation packages for hypersonic reacting flow, such as DPLR, LAURA, LeMANS, US3D and others. All of them, however, can only be distributed to and used by United States citizens or US government contractors. The challenge posed by this restriction can be answered by developing an open-source code for the use in high-speed non-equilibrium flow research and multiphysics problems associated with it.

2. Development Basis

We are using OpenFOAM CFD toolbox as a framework for our development because of its' flexibility, robust class structure for numerical simulation and a broad community of contributions. In previous works there have been attempts to use standard toolbox solvers to simulate magnetic field effect on the supersonic flow around the object [5]. However, OpenFOAM's basic kit lacks a dedicated solver for high-speed reacting flow. Due to this fact we are basing our work on a solver, developed in the University of Strathclyde, Scotland [6] called *hy2Foam*. It combines the standard kit solvers *rhoCentralFoam* (density-based compressible flow solver based on central-upwind schemes of Kurganov-Tadmor) and *reactingFoam* (a solver for combustion with chemical reactions).

3. Mathematical Model

hy2Foam employs a Navier-Stokes-Fourier system of equations for reacting flow. To include a magnetic field interaction into the model the terms corresponding to Lorentz force and Joule heating have been added to the right hand sides of momentum and energy equation respectively. An important assumption needs to be made to allow for a reduction of the number of equations necessary to calculate the MHD terms. Magnetic Reynolds number for the studied flow regimes is generally low, which allows us to disregard self-induced magnetic field [7]. This means that the only magnetic field present is the applied one, which eliminated the need to solve the

Maxwell system of equations. The flux-divergence form of the resulting system is presented below.

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial (\mathcal{F}_{inv} + \mathcal{F}_{visc})}{\partial x_i} = \mathbf{w};$$

$$\mathbf{u} = \begin{pmatrix} \rho \\ \rho_s \\ \rho u \\ \rho v \\ \rho w \\ \varepsilon_{ve,m} \\ \varepsilon \end{pmatrix}, \mathcal{F}_{inv} = \begin{pmatrix} \rho u_i \\ \rho_s u_i \\ \rho u_i u + \delta_{i1} p \\ \rho u_i v + \delta_{i2} p \\ \rho u_i w + \delta_{i3} p \\ \varepsilon_{ve,m} u_i \\ (\varepsilon + p) u_i \end{pmatrix}, \mathcal{F}_{visc} = \begin{pmatrix} 0 \\ -J_s \\ \tau_{i1} \\ \tau_{i2} \\ \tau_{i3} \\ \varphi_{v,visc} \\ \varphi_{visc} \end{pmatrix}, \mathbf{w} = \begin{pmatrix} 0 \\ \dot{\omega}_s \\ f_{i1} \\ f_{i2} \\ f_{i3} \\ \dot{\omega}_{v,m} \\ Q_m \end{pmatrix}.$$

\mathcal{F}_{inv} and \mathcal{F}_{visc} correspond to inviscid and viscous components of the flux. A more detailed explanation of each term can be found in the original paper.

The magnetic terms are calculated using generalized Ohm's law. We need several of additional models for different associated physical phenomena to complete model. Among those are Landau-Teller model for trans-vibrational relaxation and Millikan-White model for V-T relaxation times with Park's correction. Thermal diffusivities of different species are calculated using Eucken's formula. Mixture quantities are recovered using Gupta's mixing rule. Species' viscosity follows Blottner's formula. Diffusion and heat conduction are modeled by Fick's and Fourier's law respectively. Total pressure is recovered from partial pressures by Dalton's law [6]. Chemical reactions and their rates are taken from the paper by Park [7].

A. Conductivity models

Electrical conductivity of the flow plays a crucial role in estimating the intensity of its interaction with magnetic field. An electric conductivity model interface has been added to the *hy2Foam*, so that different conductivity model can be implemented and tested for use in high-speed flow problems. Five different electric conductivity models have so far been implemented alongside a dummy constant conductivity model. Two of them are semi-analytical Spitzer and Harm [8] and Chapman and Cowling [9] models. Another two models, developed originally by Bush [10] and Raizer [11] are semi-empirical. The fifth model was taken from the paper by Bityurin and Bocharov [1] and is based on Chapman-Enskog theory of multicomponent gas mixture. This last model is the most advanced and utilizes the gas mixture data that is available from solving Navier-Stokes-Fourier equations.

Table 1. Electric conductivity models

Boltzmann	Spitzer-Harm	Chapman-Cowling	Bush	Raizer
$\frac{e^2 n_e \tau_e}{m_e}$	$\frac{c_{1,s} T^{\frac{3}{2}}}{\ln \left(c_{2,s} \frac{T^{\frac{3}{2}}}{\sqrt{n_e}} \right)}$	$c \frac{\alpha}{Q \sqrt{T}}$	$\sigma_0 \left(\frac{T}{T_0} \right)^n$	$c_{1,R} e^{-\frac{T_1}{T}}$

4. Test Results

The developed code has been tested on a case from the book by Surjikov [12]. The case presents a hypersonic flow of molecular nitrogen around a cylinder with a 0.015 meters radius. Freestream conditions are presented in the table below.

Table 2. Surjikov case freestream conditions

Quantity	Value	Measurement Units
Pressure	1300	Pa
Temperature	3030	K
Density	0.0014	kg/m ³
Velocity	10300	m/s

To test the capabilities of our MHD modification we have added a magnetic field to be generated by a coil located inside the cylinder. The distribution of magnetic flux intensity generated by it is shown in a Fig. 1.

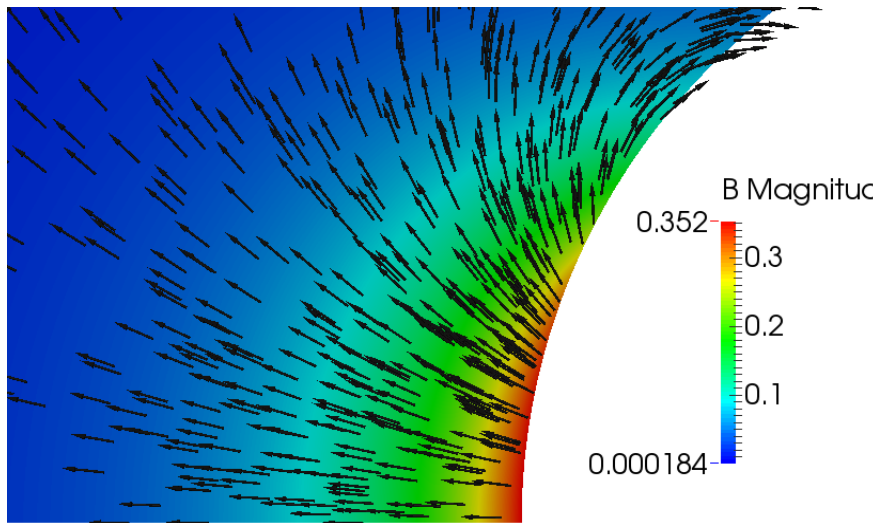


Fig. 1. Geometry of the case 1 and the distribution of magnetic flux density

Investigating this case with our MHD hypersonic solver serves 2 purposes. First, we can validate *hy2Foam* and its' auxiliary libraries on a well-known problem for which there are experimental data to use for comparison. Second, we can see how the modifications that we have added perform, when MHD interaction is introduced to the established case. Additionally, the numerical solution of the case with magnetic field interaction can give us insights on the potential of MHD control in the considered conditions.

The results for non-MHD case show good agreement with both experimental and numerical results from [10]. Apparent discrepancies can be attributed to the difference in choice of boundary conditions for temperature on the object (constant value in the source, zero gradient in our calculations) and using a different model for chemical reactions and their rates. The solution obtained for the case with magnetic interaction shows that without assisted ionization the effect of the applied magnetic field is negligible, albeit visible. Figures below show a clear increase of the shock standoff distance, the magnitude of which differs depending on model used to calculate electrical conductivity.

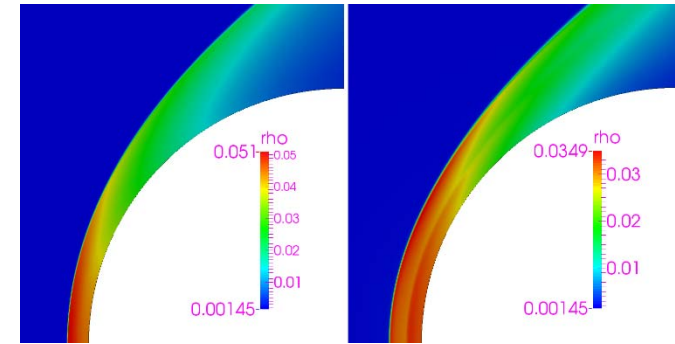


Fig. 2. Bow shocks compared for cases 1 without (left) and with (right) MHD interaction.

The difference in the effect for different conductivity models can be better illustrated using the case with freestream conditions characteristic for atmospheric entry.

Table 3. Re-entry case freestream conditions

Quantity	Value	Measurement Units
Pressure	33	Pa
Temperature	200	K
Density	0.001	kg/m ³
Velocity	5411	m/s

The simulation results indicate that for these conditions the size and the parameter distribution within stagnation layer will vary significantly depending on the used electric conductivity model. The shock standoff distance can differ by as much as 2.5 times.

Results show that using both Spitzer-Harm and Raizer model can greatly exaggerate the MHD interaction effect. Bush model, on the other hand, considerably underestimates it compared to the other models. The remaining models provide similar results and can both be used reliably for this type of conditions.

Our third case serves to illustrate one of the applications of the concept. It is a model of a scramjet with a design Mach number of 10. The freestream Mach number, however, is much lower, which results in a "shock spillage". This phenomenon is known to adversely affect the flight characteristics of the aircraft. The proposed solution is to use the magnetic field to preserve the position of oblique shock during the changes of flight regime. Similarly to our first test case we need to artificially increase the flow's electric conductivity to achieve the necessary intensity of the

effect. For this purpose, we have used constant conductivity model. The location of inductor that generates the field is shown in the bottom picture of Fig. 6.

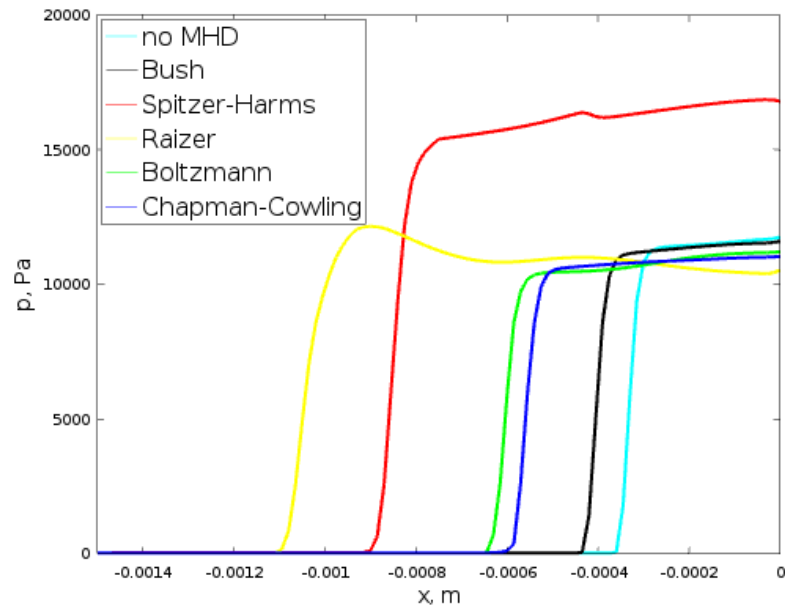


Fig. 3. Pressure distribution on the stagnation line, calculated using different conductivity models

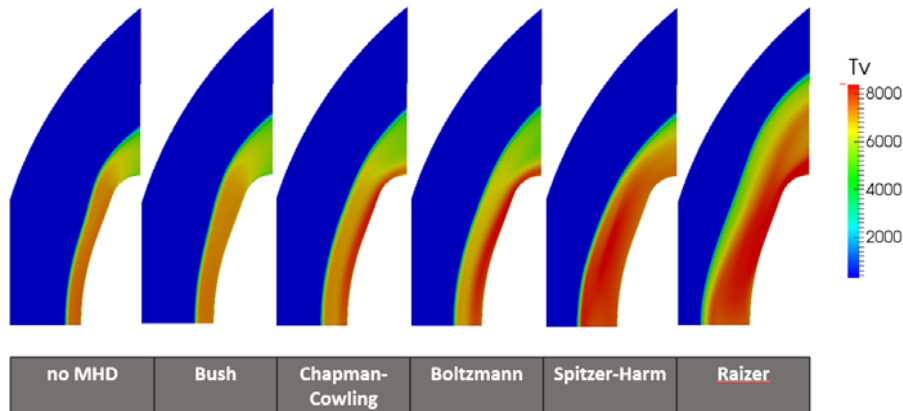


Fig. 4. Vibrational temperature distribution calculated using different electric conductivity models.

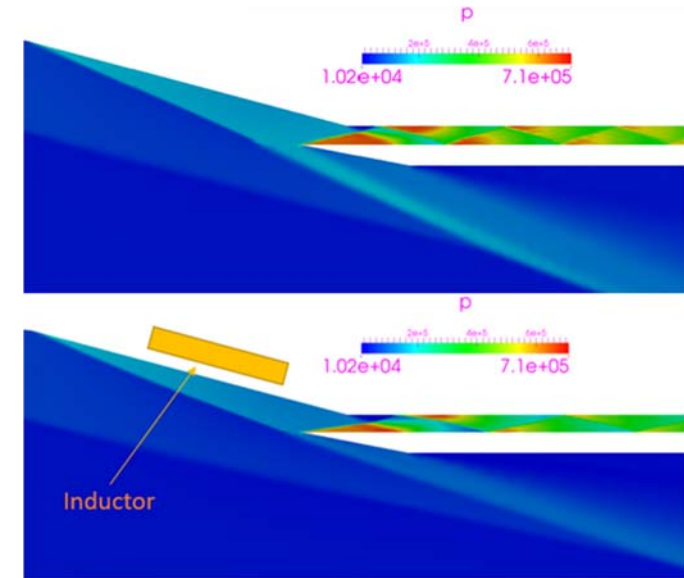


Fig. 5. Pressure distribution for scramjet case with and without MHD control effect

5. Conclusion

We have demonstrated that even at this stage of development the code can be used as a simulation tool by the researchers studying the interaction between the applied magnetic field and high-speed weakly ionized gas flow. Being developed within OpenFOAM framework the solver and its auxiliary libraries can be easily modified and supplemented by anyone familiar with OpenFOAM structure, like it had been done by the authors of this paper with the solver's "parent" code hy2Foam.

Table 4. Scramjet case freestream conditions

Quantity	Value	Measurement Units
Pressure	26436.3	Pa
Temperature	223.150	K
Density	0.412707	kg/m ³
Velocity	1800	m/s

The results of simulations provide an example of how the electric conductivity models can be tested for use in modeling of MHD hypersonic flow control with the developed software. They also showcase the potential effect of MHD flow control system on the

shockwave configuration and the applications this effect might have in hypersonic aircraft design.

Further development of the code is going to include additional models for the electric conductivity flow and the processes related to the assisted ionization of the flow. It may also be necessary to develop a hybrid Direct Simulation Monte-Carlo – CFD code to adequately simulate MHD control of the flow around atmospheric entry vehicles, which is another potential application of this technology.

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Численное исследование высокоскоростного неравновесного течения с приложенным магнитным полем

^{1,2}А.И. Ряховский <alexey.i.ryakhovskiy@mail.ioffe.ru>

^{1,2}А.А. Шмидт <alexander.schmidt@mail.ioffe.ru>

²В.И. Антонов <antonovvi@mail.ru>

¹ФТИ им. А.Ф. Иоффе

Политехническая ул., 26, Санкт-Петербург, 194021, Российская Федерация

*²Санкт-Петербургский Политехнический Университет Петра Великого
Политехническая ул., 29, Санкт-Петербург, 195251, Российская Федерация*

Аннотация. Статья описывает разработку и тестирование модификации решателя для гиперзвукового реагирующего течения в среде численного моделирования OpenFOAM. Модификация создается для моделирования взаимодействия между течением и приложенным постоянным магнитным полем. Цель разработки – создать численный инструментарий для исследования концепции магнитогидродинамического управления потоком и его возможных применений. Создаваемое приложение использует математическую модель на основе уравнений Навье-Стокса, дополненных необходимыми вспомогательными моделями для описания сопряженных процессов. Тестирования решателя проводилось на задачах, демонстрирующих основные возможности созданного приложения в моделировании высокоскоростных МГД-течений различных режимов. Тестовые примеры представляют собой задачи обтекания двумерных плоско и цилиндрически симметричных тел, имеющих форму, характерную для аппаратов, для которых известны потенциальные способы применения МГД управления. Исследовалось влияние выбора модели электропроводности на результаты численного моделирования. Сравнение результатов показало зависимость важности выбора модели электропроводности от разреженности рассматриваемого газового потока.

Ключевые слова: численное моделирование; управление потоком; магнитогидродинамика; неравновесные течения; ударные волны

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