

## INTERNATIONAL CONFERENCE

# FLUXES AND STRUCTURES IN FLUIDS



**St.-Petersburg  
June 25-28, 2013**

Conference Venue  
Russian State Hydrometeorological University



98, Malookhtinsky pr., St.-Petersburg, 195196

## CALL FOR PAPERS

### FIRST ANNOUNCEMENT

The conference program includes invited lectures, oral and poster presentations

## CALL FOR PAPERS

Conference language is English.  
Authors submit in electronic form the following:  
- *Completed Registration form*;

- *Abstract* (no more than 3 pages prepared in MS WORD, framed by 16x24 cm and spaced by 12 pt. **TITLE OF THE PAPER**, authors' names, postal and electronic addresses are displayed in bold and italic according to the template presented at the conference website;

Abstract book will be handed at the registration desk. The Programme will contain general lectures (30 min, including questions), contributed papers (15 min) and presentations (3 min oral + showing the poster 1x2 m)

The lecture rooms are equipped with blackboards, NTSC VCR and PC video projection units.

## KEY DATES

Submission of the registration form and abstracts **March 01, 2013;**

Notification to authors **April 01, 2013.**

## REGISTRATION FEE

Regular participant fee is **250 USD.**

Student conference fee is **50 USD.**

Fee can be paid in cash at the conference desk or by money transfer order. Requisites are available on request.

## HOUSING INFORMATION

**ACCOMMODATION** of participants is available in more than 500 hotels, hostels and guest houses in Saint-Petersburg (site [booking.com](http://booking.com) and others) and at the University hostel (number of beds is limited).

## CONFERENCE TOPICS

- Dynamic processes in the Ocean and the Atmosphere: from global to microscales;**
- Waves, vortices, coherent structures, turbulence: transport and accumulation of substances on Interfaces;**
- Impact of physical, chemical and biological effects on formation of spatial structures in fluids;**
- Analytical, numerical and laboratory modelling of environmental systems and processes;**
- Conventional and novel Instruments;**
- Environmental and technological applications;**
- Special young scientists session.**

Papers on other related topics are equally welcome.  
More than one abstract may be submitted.



Space view on Crimea Peninsula

## REGISTRATION

Registration will take place in the main building of Russian State Hydrometeorological University starting from **June 24, 2013**

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E-mail: [chakin@ipmnet.ru](mailto:chakin@ipmnet.ru), [yulidch@gmail.com](mailto:yulidch@gmail.com)

Conference site: [www.ipmnet.ru](http://www.ipmnet.ru)

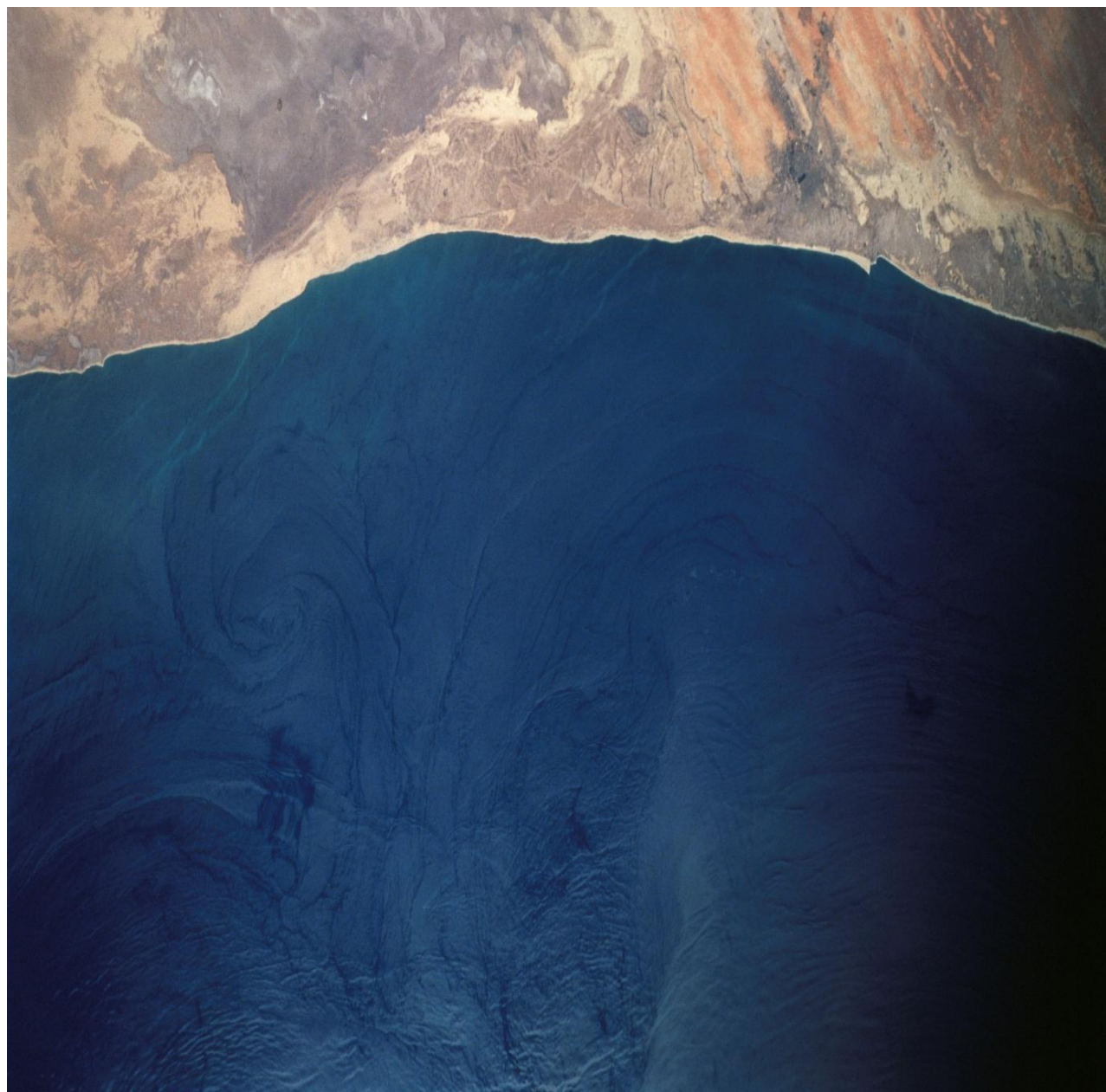
CONFERENCE CONTACTS

“Все течет, все изменяется...”  
«Все течет и движется, и ничего  
не пребывает в неизменности”  
Гераклит (544-483 years. BC.)

# ДИФФЕРЕНЦИАЛЬНАЯ МЕХАНИКА ЖИДКОСТЕЙ – ИНТЕГРИРОВАННОЕ АНАЛИТИЧЕСКОЕ, ЧИСЛЕННОЕ И ЛАБОРАТОРНОЕ МОДЕЛИРОВАНИЕ

Ю.Д. Чашечкин  
chakin@ipmnet.ru





Guinea Gulf: Filament structure of seashore currents (space view)







The best NASA Photo  
"Plankton around the Gotland in Baltic Sea"



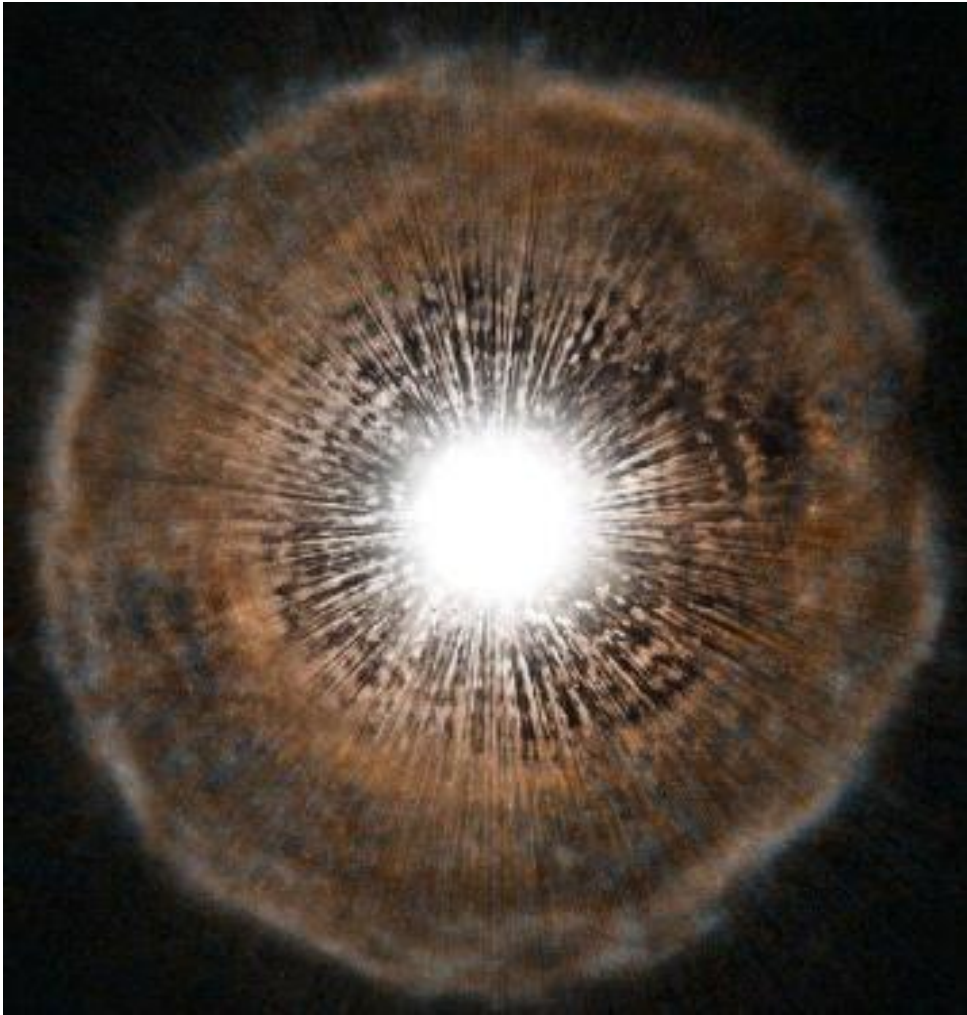
Photo: NASA's image of Gotland which has been compared to Van Gogh's 'Starry Night';



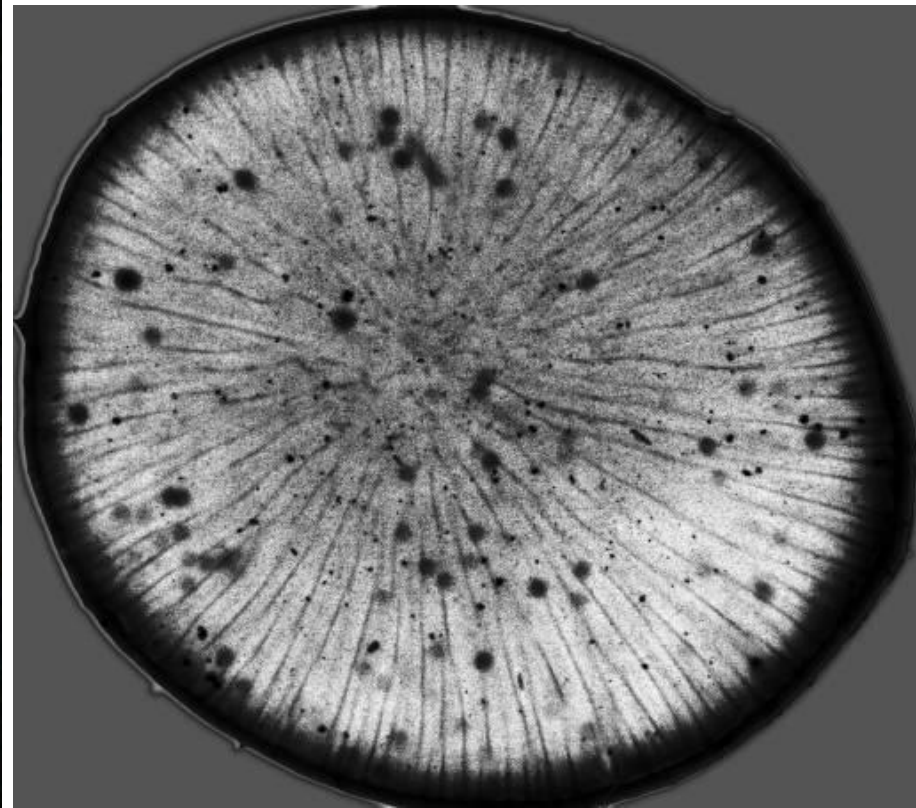
III Международная конференция «Облачные вычисления. Образование. Исследования. Разработка» Москва, РАН, 6-7 декабря 2012 г.  
Расширенный семинар сообщества лаборатории механики сплошной среды.



## Hubble Sees Red Giant U Camelopardalis Blow a Bubble

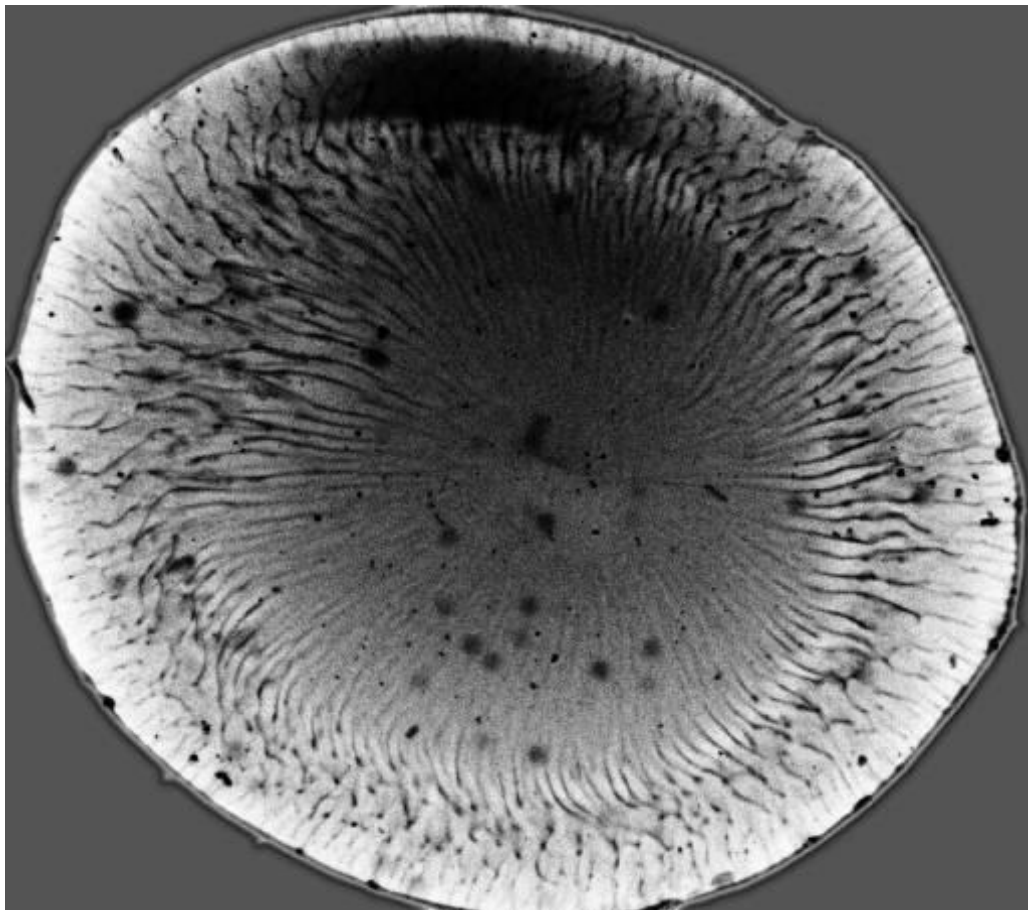


## Radial structure in drying drop of nanoparticles suspension

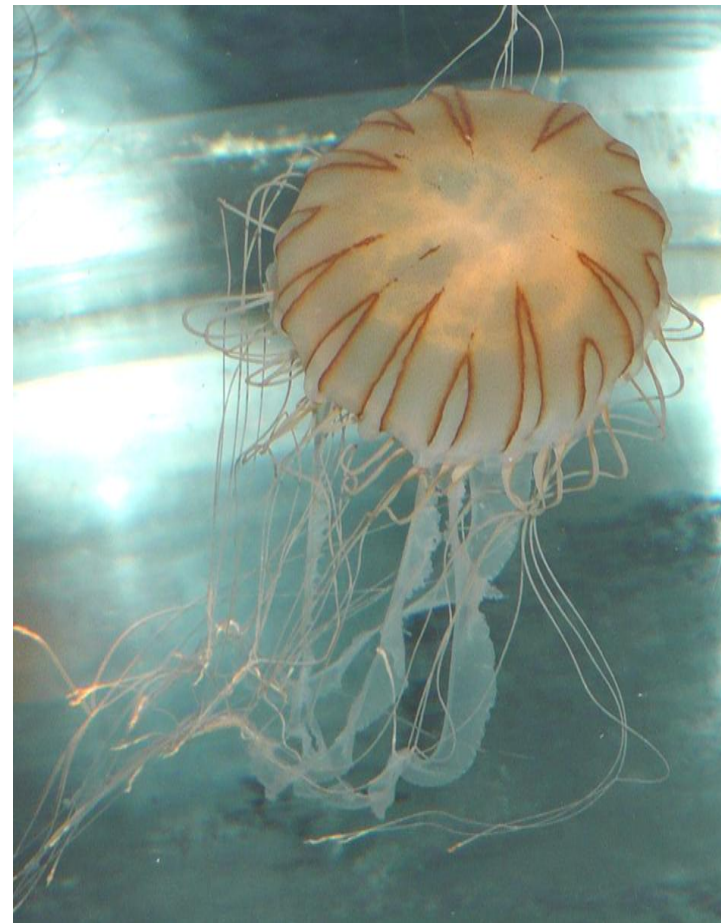


Drying drop of quartz nanoparticles  $s$  in vodka  $D = 0.63$  cm

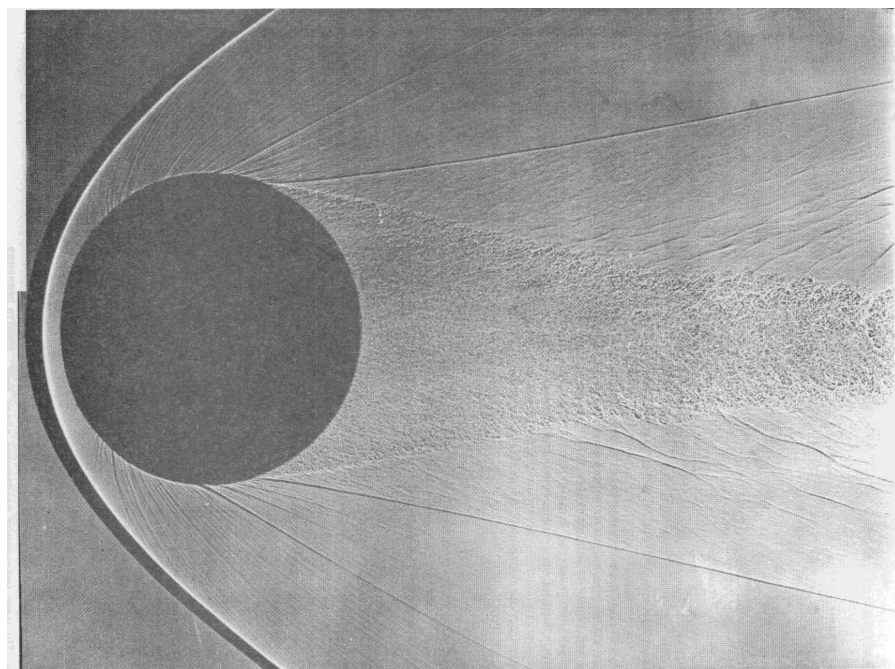




Radial structure in drying drop of  
quartz nanoparticles in vodka  $D = 0.63$  cm

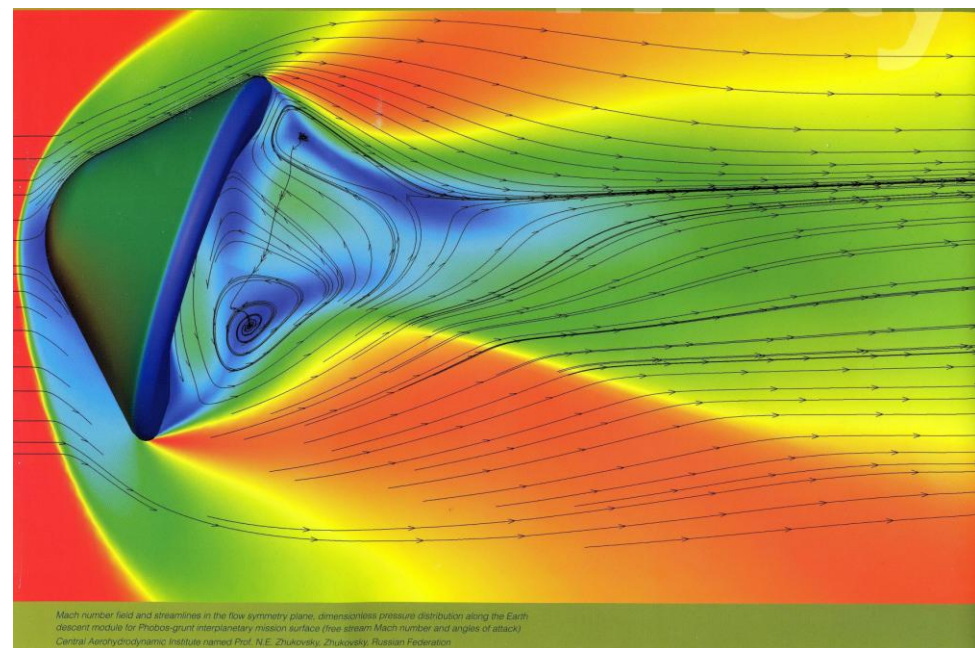


Медуза “Японская морская  
крапива” в аквариуме Бостона  
20 ноября 2012 г.



Теневая картина обтекания сферы  $d = 8 \text{ см}$   
 $M = 2.5$

Герасимов С.И., Файков Ю.И. Теневое фотографирование  
в расходящемся пучке света. Саров, ФГУП «РФЯЦ\_ВНИИЭФ». 2010. 344 с.



Расчет картины течения около спускаемого аппарата

Begel House, Publishers



**Введение.** Тонкая структура течений в природных и промышленных условиях;  
Течения, индуцированные диффузией в природе и лаборатории  
Расчет, визуализация и измерения присоединенных внутренних волн  
Перенос маркеров в вихревых течениях  
**Общая теория течений жидкости**

Тождественность преобразований: сохранение симметрий и условие совместности;  
**Течения с малой диссипацией:** классификация структурных компонент;

**Заключение**

Макроскопические проявления действия атомно-молекулярных сил  
(всплески и звуки при падении капель в жидкость)

**Окончание:**

концепции «движение» и «течение» - идентификация и измерения



Smoke from Electric Power plant chimney in Yushno-Sakhalinsk on September 19, 2011





Smoke from Heat Power plant chimney in Yushno-Sakhalinsk on September 19, 2011



Smoke from Heat Power plant chimney in Yushno-Sakhalinsk on September 19, 2011





Smoke from two industrial chimneys in Yushno-Sakhalinsk on September 19, 2011.



The SeaWiFS image provides a good view of the Sea of Azov.

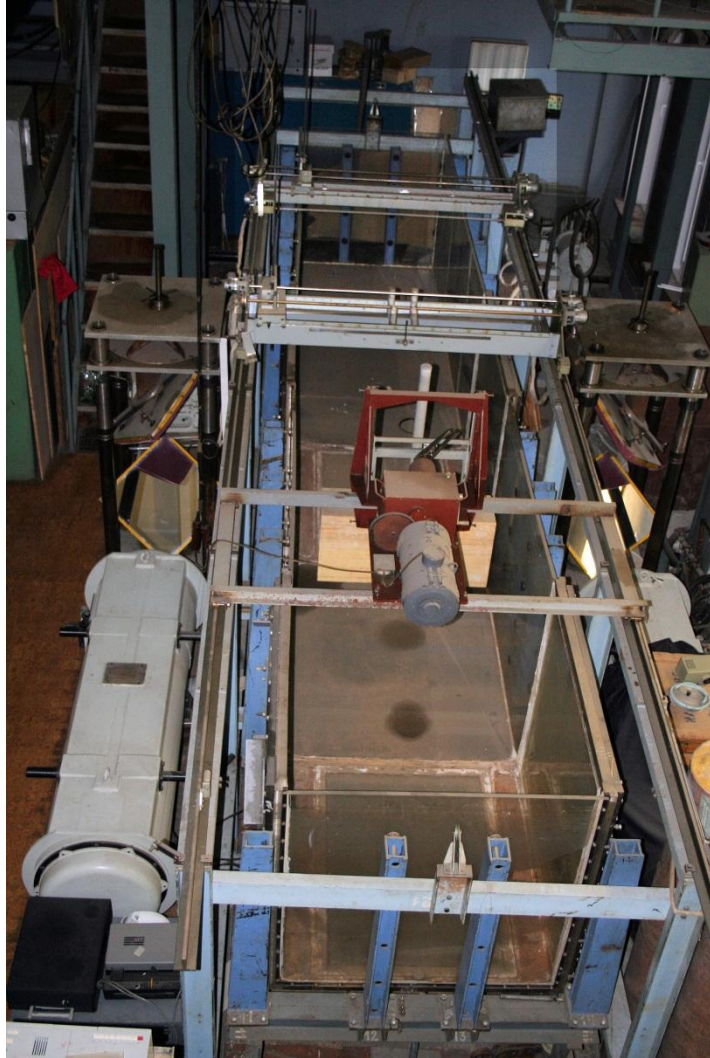
[http://eoimages.gsfc.nasa.gov/images/imagerecords/54000/54054/S2000104100904\\_md.jpg](http://eoimages.gsfc.nasa.gov/images/imagerecords/54000/54054/S2000104100904_md.jpg)



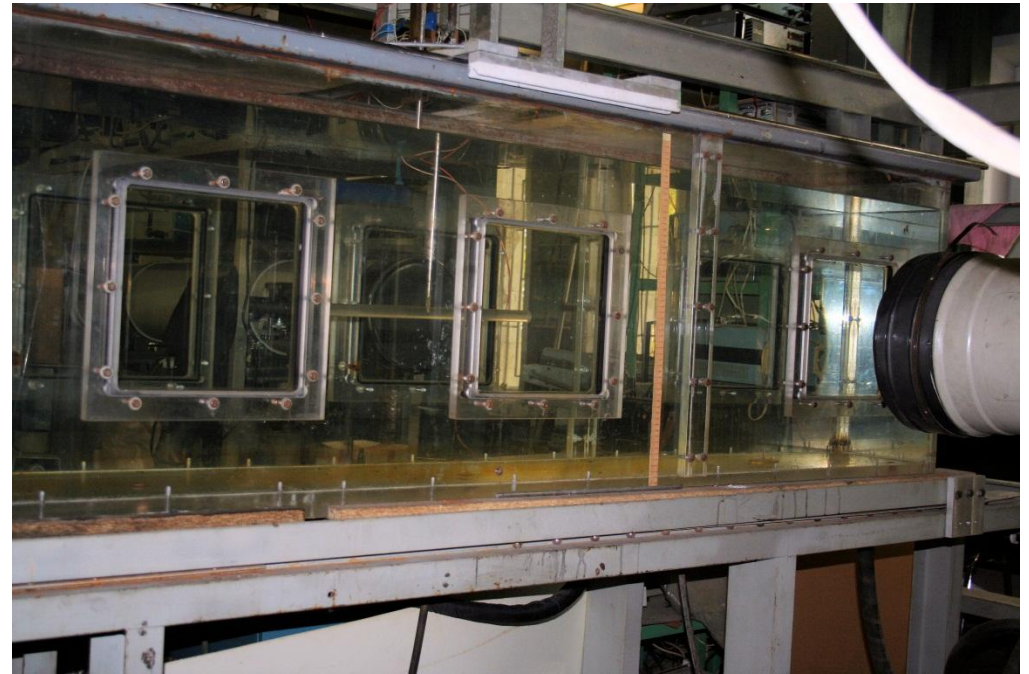




## Unique facilities of the Laboratory of Fluid Mechanics IPMech RAS



Stratified tank  $7 \times 1.2 \times 1.2$  м



$2.4 \times 0.4 \times 0.6$  м

Stratified tanks

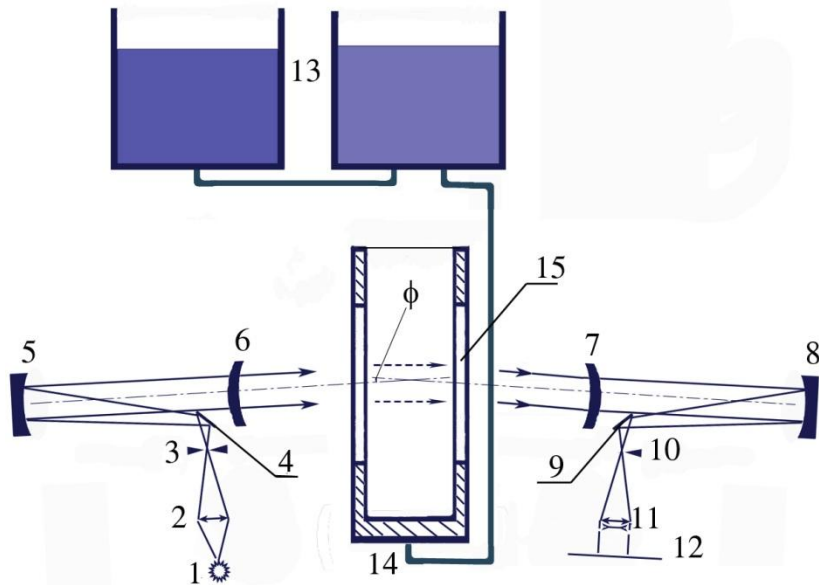
J.Lighthill, D.Mowbray, T.N.Stevenson UK, 1967

A.McEwan, Australia, 1970

D.Peters, W. Merzkirch, Germany (1976)



## Refraction and dispersion of a white light in a stratified liquids

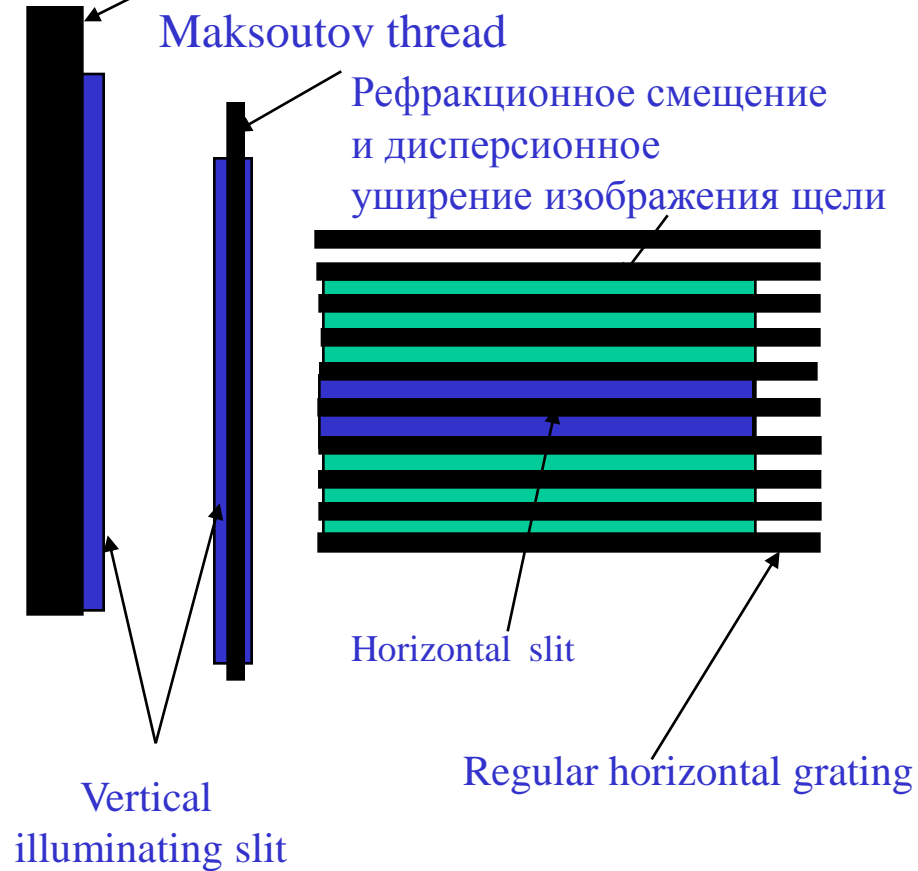


Optic scheme of stands for stratified flows studies

Foucault knife

Maksoutov thread

Рефракционное смещение и дисперсионное уширение изображения щели

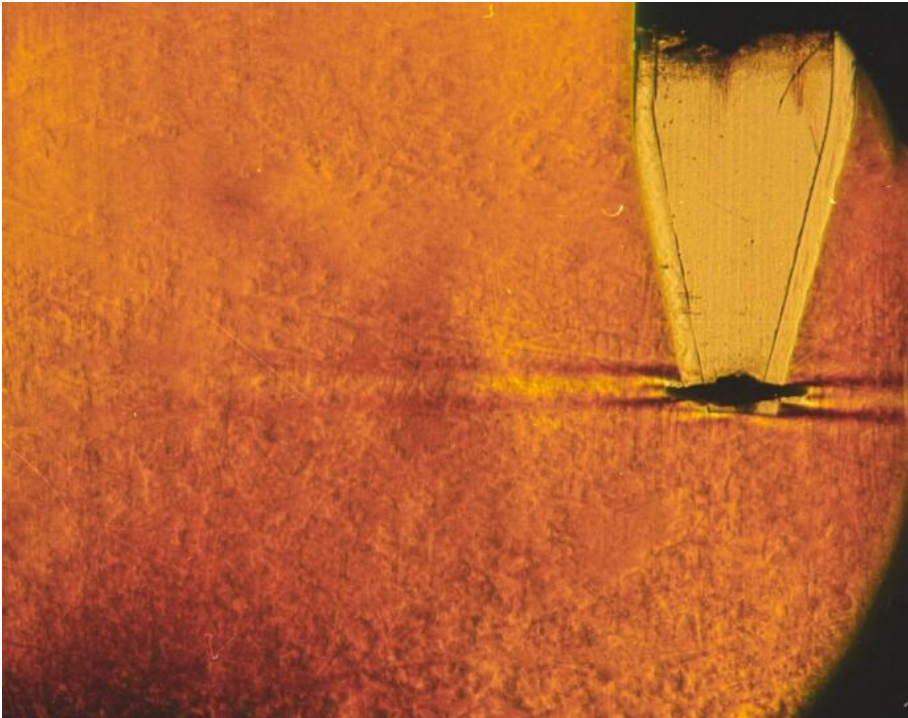


Horizontal slit

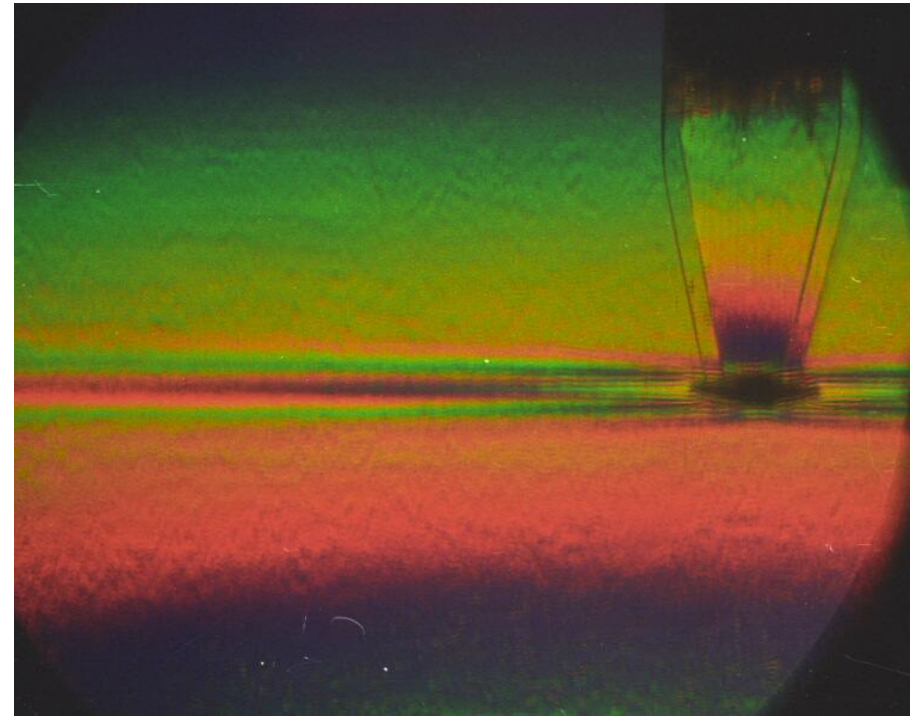
Regular horizontal grating

Vertical illuminating slit

## Diffusion induced flow on a motionless strip in a fluid at rest



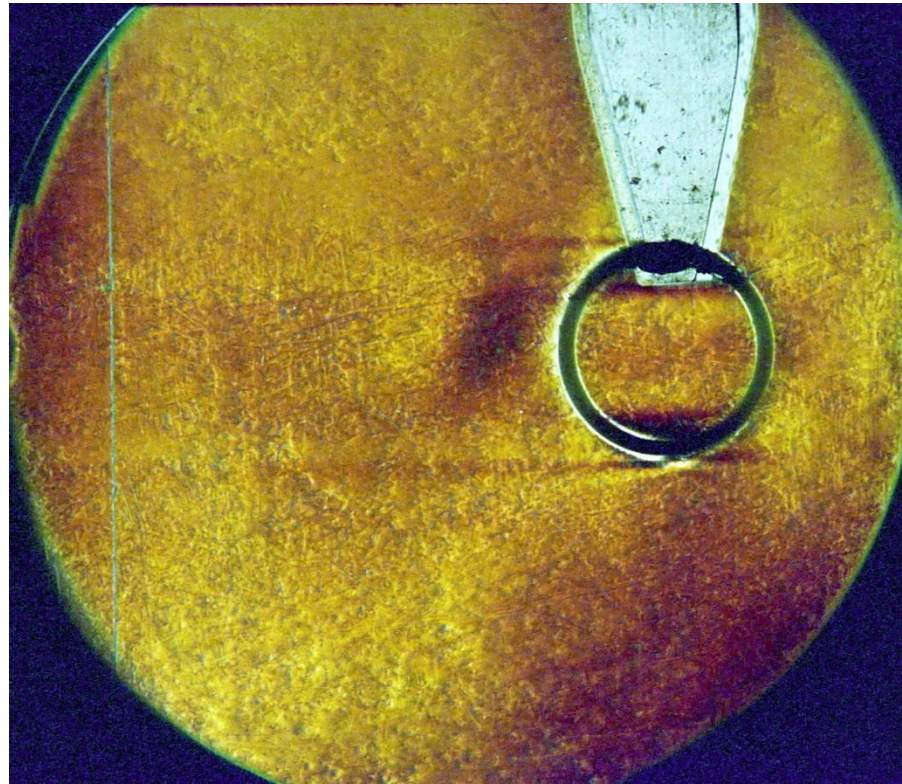
**Foucault knife**



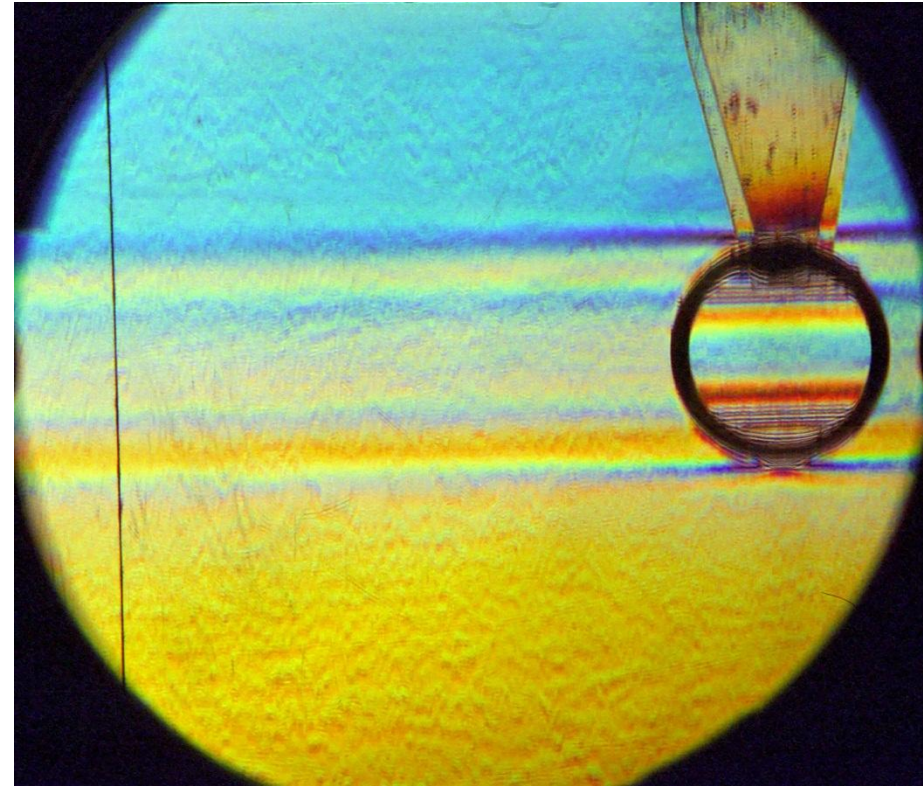
**Natural rainbow schlieren method**



The diffusion induced flow on motionless cylindrical tube inside continuously stratified fluid at rest (48 hours after submerging the body)



Convention Schlieren image



Rainbow colour schlieren image

$$T_b = 10.5 \text{ s} \quad D = 5 \text{ cm}$$

## Diffusion induced flows on a strip of limited length or on a wedge

$$\operatorname{div} \mathbf{v} = 0; \quad \rho = \rho_{00} \left( \exp(-z/\Lambda) + s \right),$$

$$\Lambda = \left| d \ln \rho / dz \right|^{-1},$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho_{00}} \nabla P + \nu \Delta \mathbf{v} - s \mathbf{g};$$

$$\rho = \rho_{00} \left( 1 - \frac{z}{\Lambda} + s \right),$$

$$\frac{\partial s}{\partial t} + \mathbf{v} \cdot \nabla s = \kappa_s \Delta s + \frac{v_z}{\Lambda};$$

$$N_b = \sqrt{\frac{g}{\rho} \frac{d\rho}{dz}},$$

$$T_b = \frac{2\pi}{N_b}$$

Length of the strip  $L_\xi$        $\delta_N^{(v)} = \sqrt{\frac{\nu}{N}}$        $\delta_N^{(\kappa_s)} = \sqrt{\frac{\kappa_s}{N}}$        $U_N = \sqrt{\nu N}$

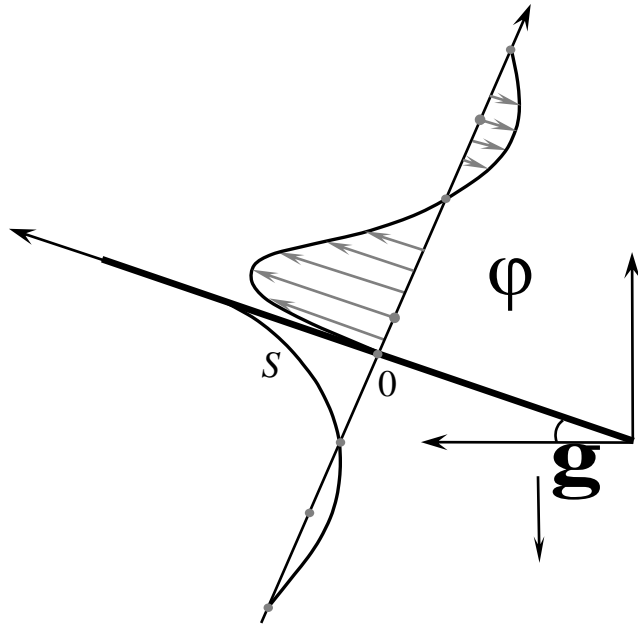
No-slip and no-flux boundary conditions

$$v_x|_\Sigma = v_y|_\Sigma = v_z|_\Sigma = 0, \quad \left[ \frac{\partial S}{\partial n} \right]_\Sigma = 0 \quad \text{or} \quad \left[ \frac{\partial s}{\partial n} \right]_\Sigma = \frac{1}{\Lambda} \frac{\partial z}{\partial n}$$

The first completely solved 2D problem: flow structures, dynamics, forces, momentum, transport of substances....– analytic, numeric, laboratory experiment



## Diffusion induced flows on a strip of limited length or on a wedge



L. Prandtl, O. Phillips, C. Wunsch  
Stationary solutions

Universal inverse length scale

$$\gamma = \left( N^2 \sin^2 \varphi / 4\nu\kappa_s \right)^{1/4}$$

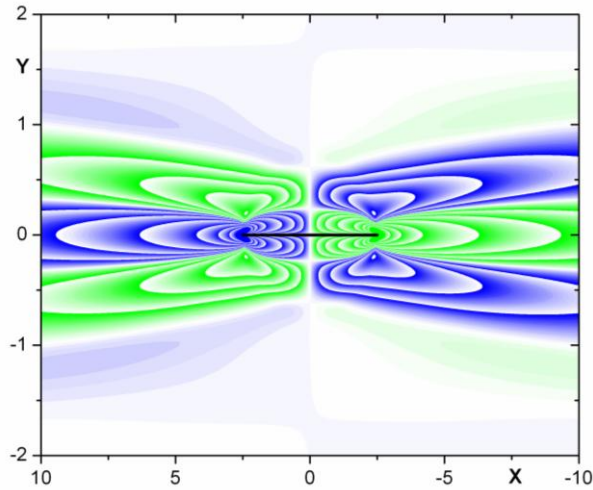
$$v(\zeta) = 2\kappa_s \gamma \operatorname{ctg} \varphi \exp(-\gamma\zeta) \sin(\gamma\zeta)$$

Asymptotic solutions for a short time approximation

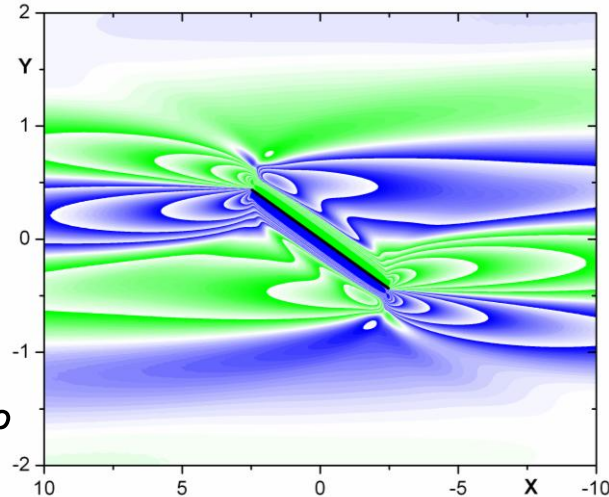
$$S = -2 \frac{\sqrt{\kappa_s t}}{\Lambda} \operatorname{i} \operatorname{erfc} \left( \frac{\zeta}{2\sqrt{\kappa_s t}} \right) \cos \varphi \quad \text{Two different length scale}$$

$$v_\xi = \frac{N^2}{2} \frac{(4\kappa_s t)^{3/2}}{\nu - \kappa_s} \left[ \operatorname{i}^3 \operatorname{erfc} \left( \frac{\zeta}{2\sqrt{\nu t}} \right) - \operatorname{i}^3 \operatorname{erfc} \left( \frac{\zeta}{2\sqrt{\kappa_s t}} \right) \right] \sin 2\varphi$$

## Diffusion induced flows on motionless strip in continuously stratified fluid at rest

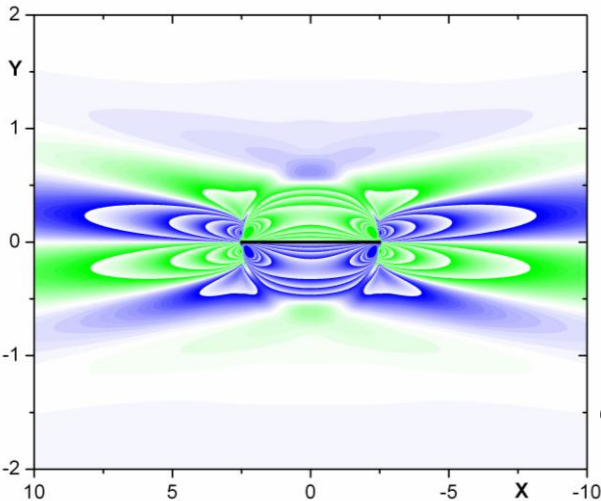


$\varphi = 0^\circ$

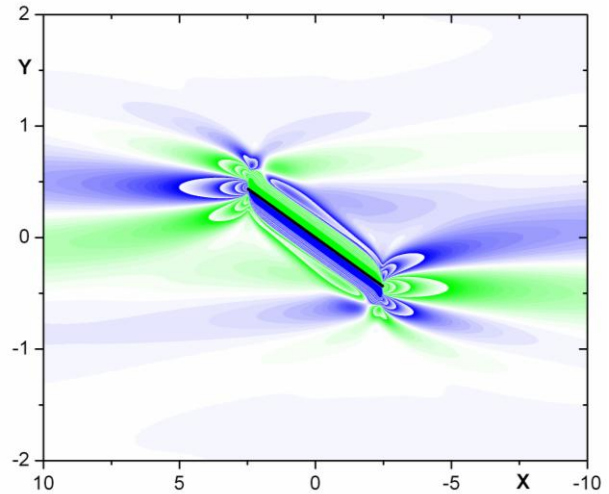


$\varphi = 10^\circ$

Fields of horizontal velocity components



$\varphi = 0^\circ$



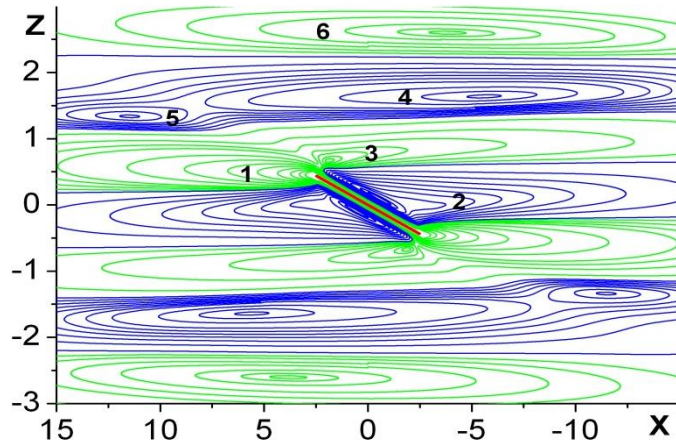
$\varphi = 10^\circ$

Fields of vertical velocity components

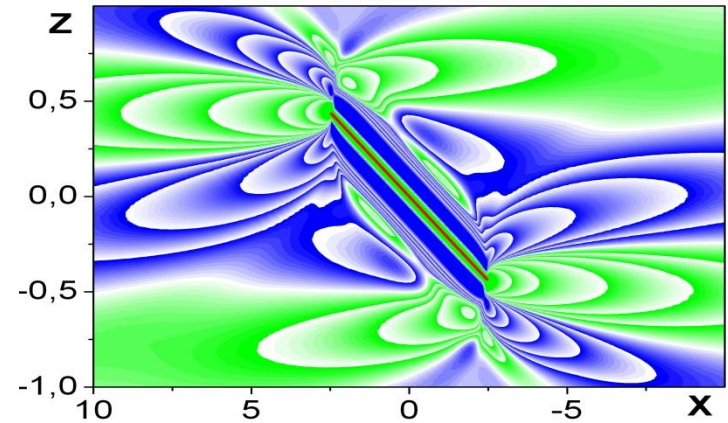
$$L = 5 \text{ cm}, N = 1.26 \text{ 1/c}$$



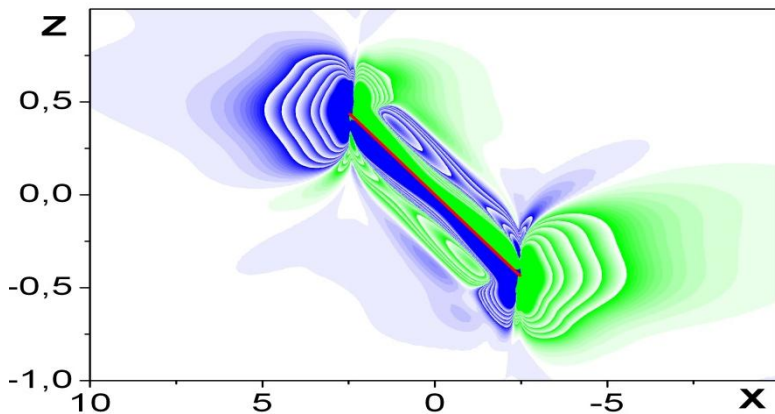
- Flow patterns of diffusion induced flows on a strip of limited length



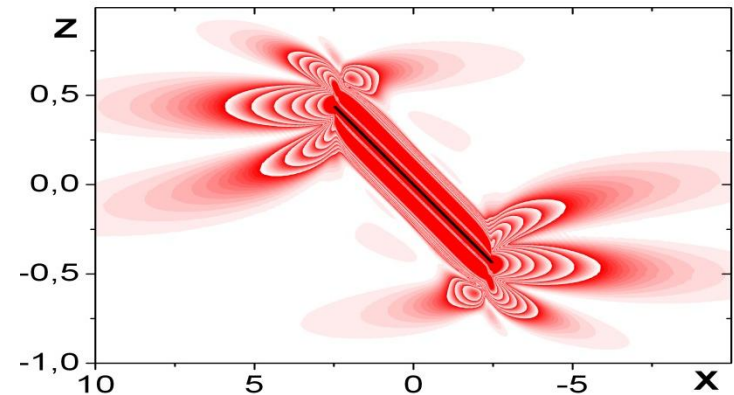
Stream lines



Vorticity  $\mathbf{\Omega} = \text{rot } \mathbf{v}$



Rate of baroclinic vorticity generation  $\dot{\mathbf{\Omega}} = \nabla p \times \nabla \frac{1}{\rho}$



Rate of mechanical energy dissipation

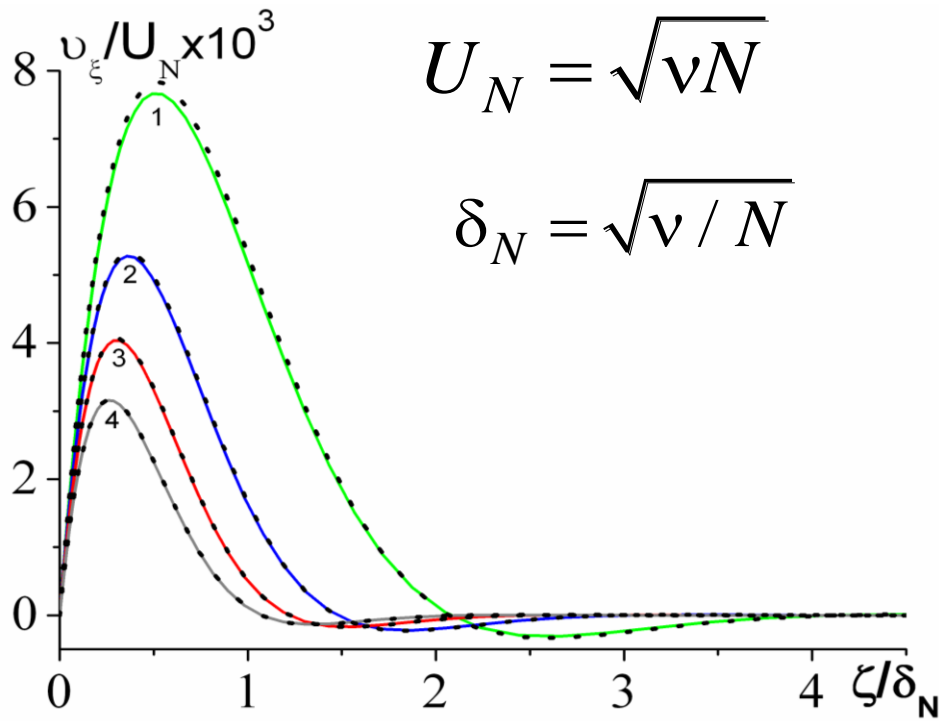
$L = 5 \text{ cm}$      $N = 1.256 \text{ c}^{-1}$      $T_b = 5 \text{ c}$

$\varphi = 10^\circ$      $\tau = 120$

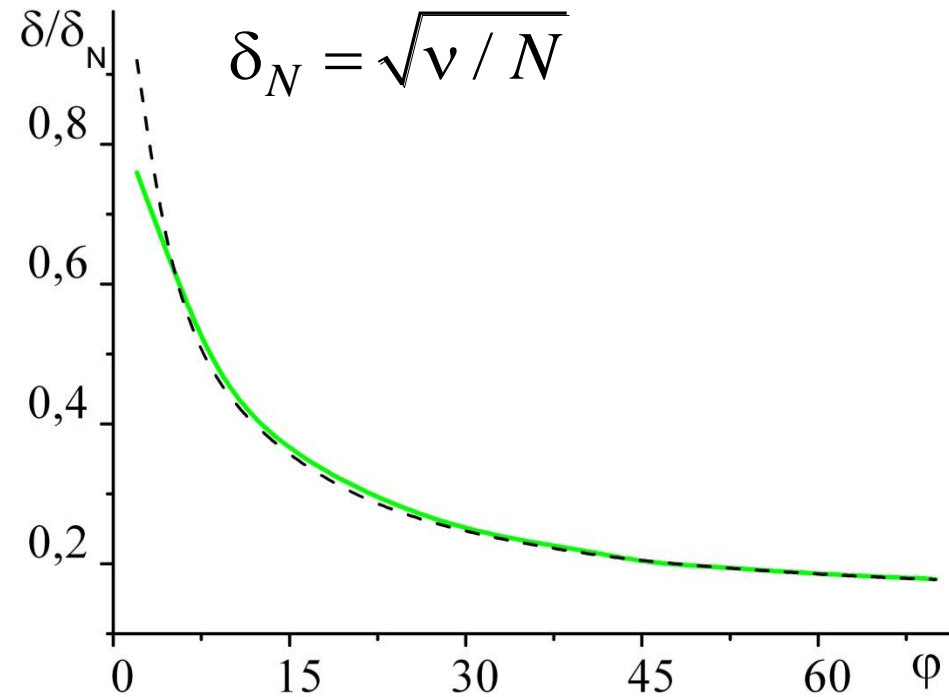
## Steady diffusion induced flows in the center of a strip of limited length (numerical and stationary solutions)

$\varphi = 5^\circ; 10^\circ; 20^\circ; 45^\circ$

$L = 10 \text{ cm}, N = 1.3 \text{ 1/c}$



Profile of longitudinal velocity component at the strip center



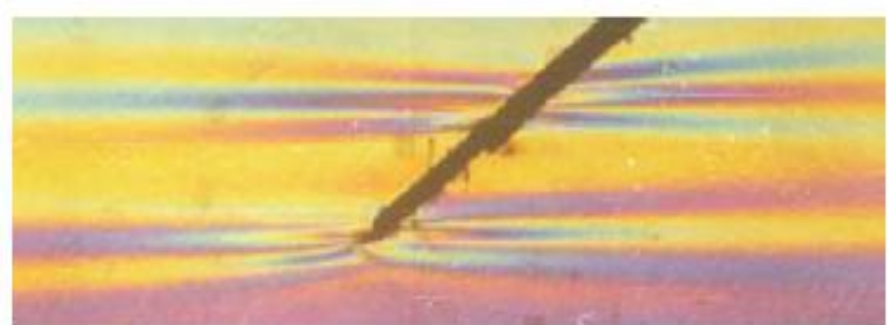
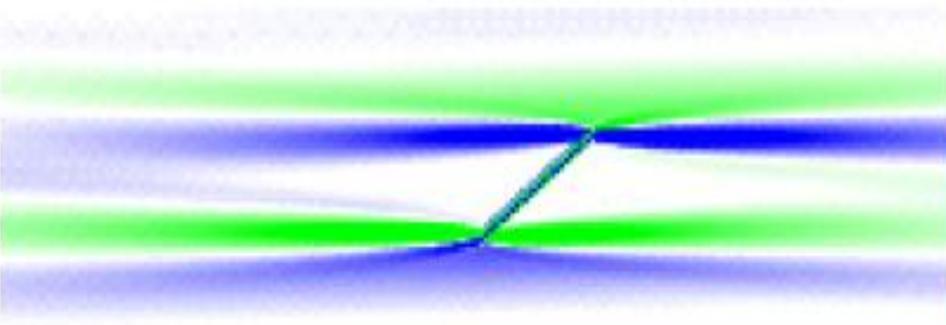
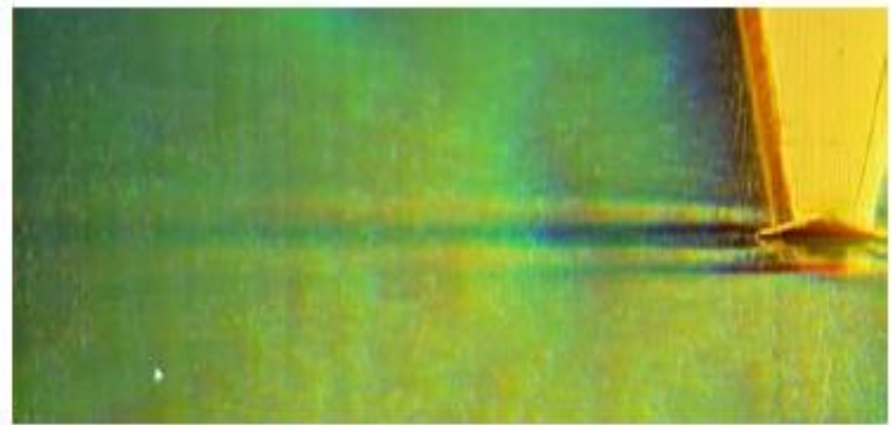
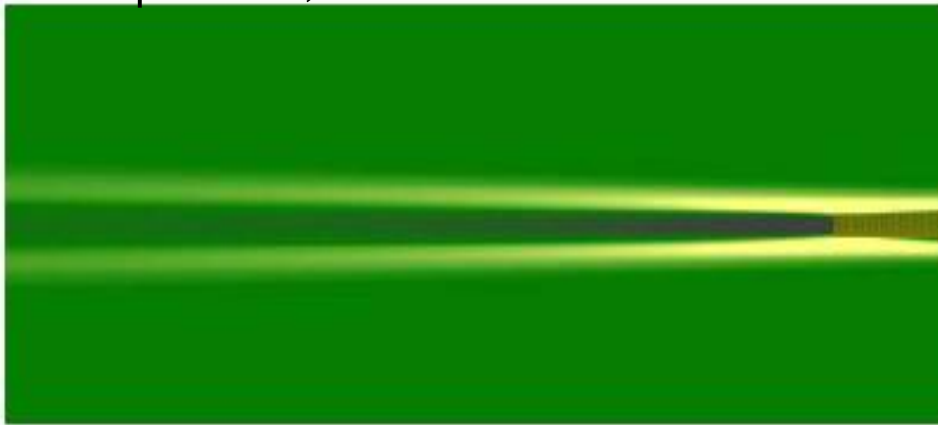
Thickness of the main jet for different slope at the strip center

Dash lines are stationary solution



Visualization results of numeric and laboratory experiments on diffusion induced flows on a sloping strip

$\varphi = 0^\circ$ ,



$T_b = 7.6$  с,  $L = 7.5$  см

$\varphi = 40^\circ$

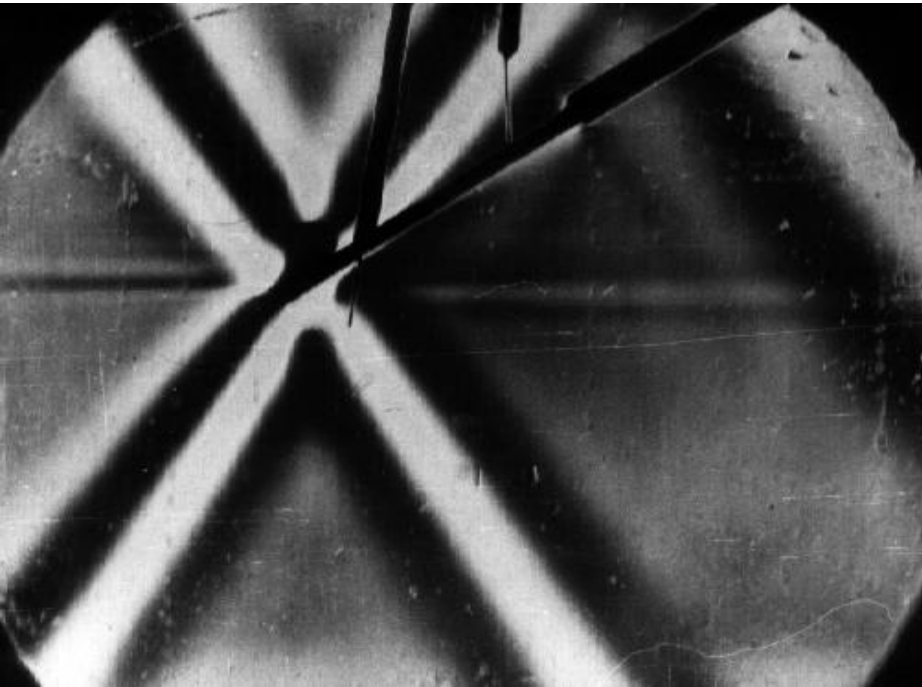


Дым из труб теплотанций в долине Южно-Сахалинска 19 сентября 2011 г.

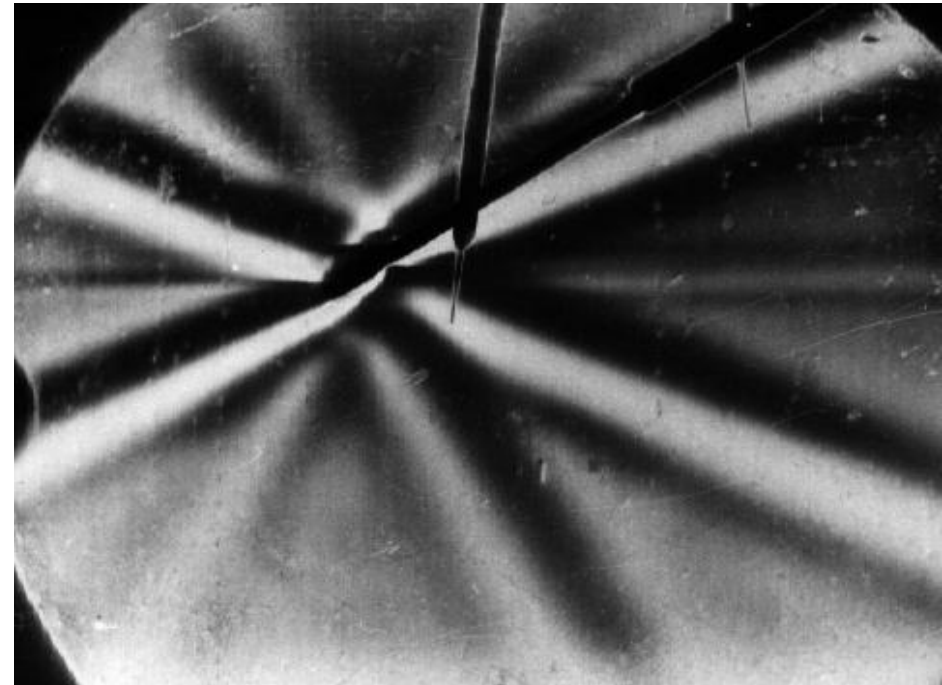


## Unimodal periodic 2D internal wave beams (schlieren images)

$$\omega = 1.01 \text{ rad s}^{-1}$$



$$\omega = 0.53; \text{ rad s}^{-1}.$$



$$N = 1.14 \text{ rad s}^{-1}; \quad L_x = 1 \text{ cm}$$

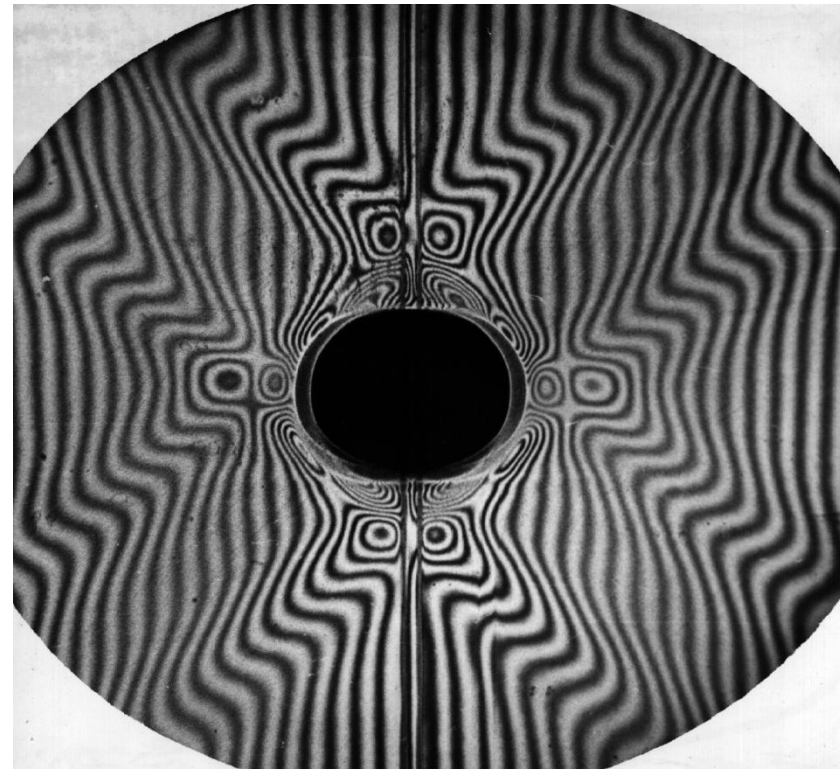
$$L_v = \sqrt[3]{g v} / N$$

Bimodal periodic 2D internal wave beams  
(schlieren and interferometric images)

$$L_x = 6 \text{ cm}$$

$$N = 1.14 \text{ rad s}^{-1}$$

$$L_v = \sqrt[3]{g\nu/N}$$



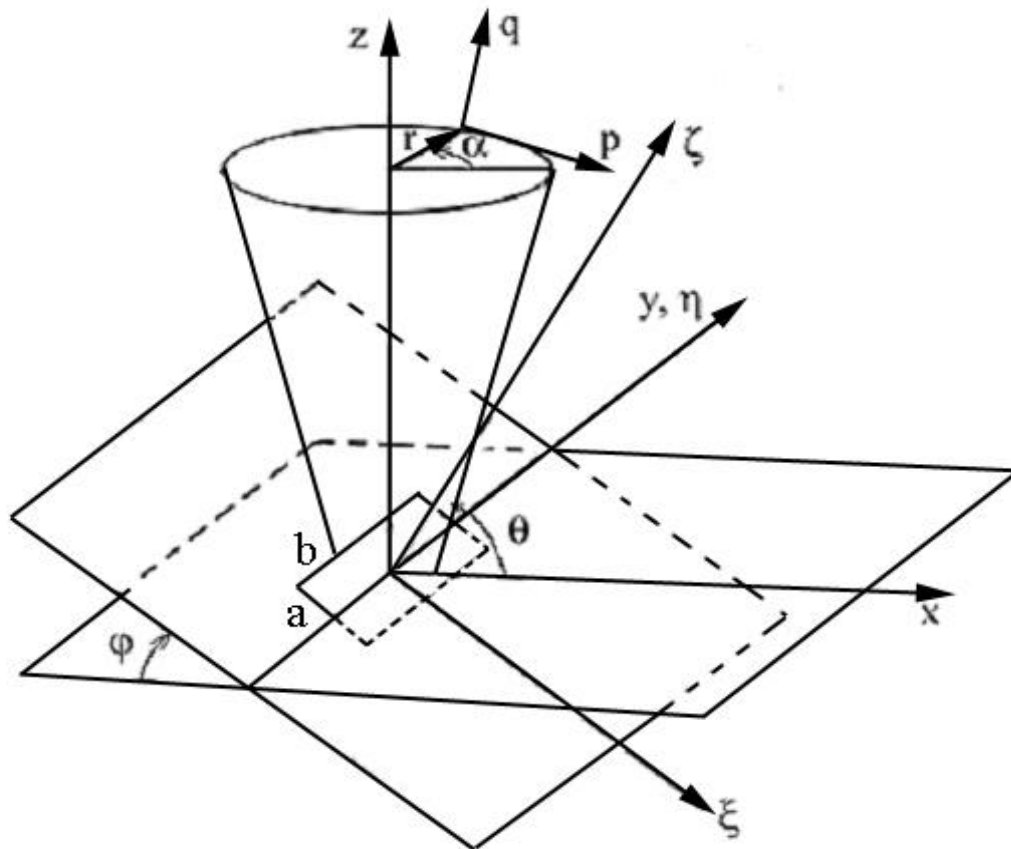
Critical angle

$$\theta = \varphi$$

$$d = 4 \text{ cm}$$



## Generalization of Stokes Problem – calculation of all roots of dispersion relations and calculation of all integrals – construction of the complete solutions



Normally and Sliding oscillating source

Toroidal-poloidal decomposition

Coordinate frames:

Laboratory (Descartes and cylindrical),

Local (attached to the plane),

Concomitant (oriented along wave cone)

Variables - Salinity, Velocity:

$$\mathbf{v} = \nabla \times \mathbf{e}_z \Psi + \nabla \times (\nabla \times \mathbf{e}_z \Phi)$$

## Generation of internal waves by a strip or rectangular oscillating on a plane

$$\operatorname{div} \mathbf{v} = 0; \quad \rho = \rho_{00} \left( \exp(-z/\Lambda) + s \right)$$

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho_{00}} \nabla P + \mathbf{v} \Delta \mathbf{v} - s \mathbf{g};$$

$$\frac{\partial s}{\partial t} = \kappa_s \Delta s + \frac{v_z}{\Lambda};$$

$$\rho = \rho_{00} \left( 1 - \frac{z}{\Lambda} + s \right), \quad \Lambda = |d \ln \rho / dz|^{-1}, \quad N_b = \sqrt{\frac{g}{\rho} \frac{d\rho}{dz}}, \quad T_b = \frac{2\pi}{N_b}$$

No-slip and no-flux boundary conditions

$$v_x|_{\Sigma} = v_y|_{\Sigma} = v_z|_{\Sigma} = 0, \quad \left[ \frac{\partial S}{\partial n} \right]_{\Sigma} = 0 \quad \text{or} \quad \left[ \frac{\partial s}{\partial n} \right]_{\Sigma} = \frac{1}{\Lambda} \frac{\partial z}{\partial n}$$



## Toroidal-poloidal decomposition

$$\operatorname{div} \mathbf{v} = 0 \quad \mathbf{v} = \nabla \times \mathbf{e}_z \Psi + \nabla \times (\nabla \times \mathbf{e}_z \Phi)$$

Governing equations set for auxiliary functions

$$\left( \left( \frac{\partial}{\partial t} - D\Delta \right) \left( \frac{\partial}{\partial t} - \nu\Delta \right) \Delta + N^2 \Delta_{\perp} \right) \Delta_{\perp} \Phi = 0$$

$$\left( \frac{\partial}{\partial t} - \nu\Delta \right) \Delta_{\perp} \Psi = 0$$

$$\left( \left( \frac{\partial}{\partial t} - D\Delta \right) \left( \frac{\partial}{\partial t} - \nu\Delta \right) \Delta + N^2 \Delta_{\perp} \right) \Delta_{\perp} S = 0$$

## Complete solution construction

### Expansions for auxiliary scalar functions $\Phi$ , $\Psi$ , $S$

$$\Phi = e^{-i\omega t} \sum_{j=1}^3 \int_{-\infty}^{+\infty} A_j(k_\xi, k_\eta) \exp(ik_j \zeta + ik_\xi \xi + ik_\eta \eta) dk_\xi dk_\eta$$

$$\Psi = e^{-i\omega t} \int_{-\infty}^{+\infty} B(k_\xi, k_\eta) \exp(ik_4 \zeta + ik_\xi \xi + ik_\eta \eta) dk_\xi dk_\eta$$

$$S = -\frac{\rho_0}{\Lambda} e^{-i\omega t} \sum_{j=1}^3 \int_{-\infty}^{+\infty} \frac{(k_\xi \cos \varphi - k_j \sin \varphi)^2 + k_\eta^2}{i\omega - D(k^2 + k_j^2)} A_j(k_\xi, k_\eta) \times$$

$$\times \exp(ik_j \zeta + ik_\xi \xi + ik_\eta \eta) dk_\xi dk_\eta$$

Coefficients are defined from boundary conditions



## Wave numbers are solutions of the dispersion equation

Dispersion relation of the 8-th order  $\omega = \text{const}$

$$\left( \nu \kappa_s (k^2 + k_j^2)^3 - i\omega(\nu + \kappa_s)(k^2 + k_j^2)^2 - \omega^2(k^2 + k_j^2) + N^2 \left( (k_\xi \cos \varphi - k_j \sin \varphi)^2 + k_\eta^2 \right) \right) \left( k_j^2 + \frac{\omega}{2i} + k^2 \right) = 0$$

Stokes type solution

$$\left( k_j^2 + \frac{\omega}{i\nu} + k^2 \right) = 0, \quad k_j = \pm \sqrt{\frac{i\omega}{\nu} - k^2} \quad \delta_N^{(\nu)} = \sqrt{\frac{\nu}{N}} \quad \delta_N^{\kappa_S} = \sqrt{\frac{\kappa_S}{N}}$$

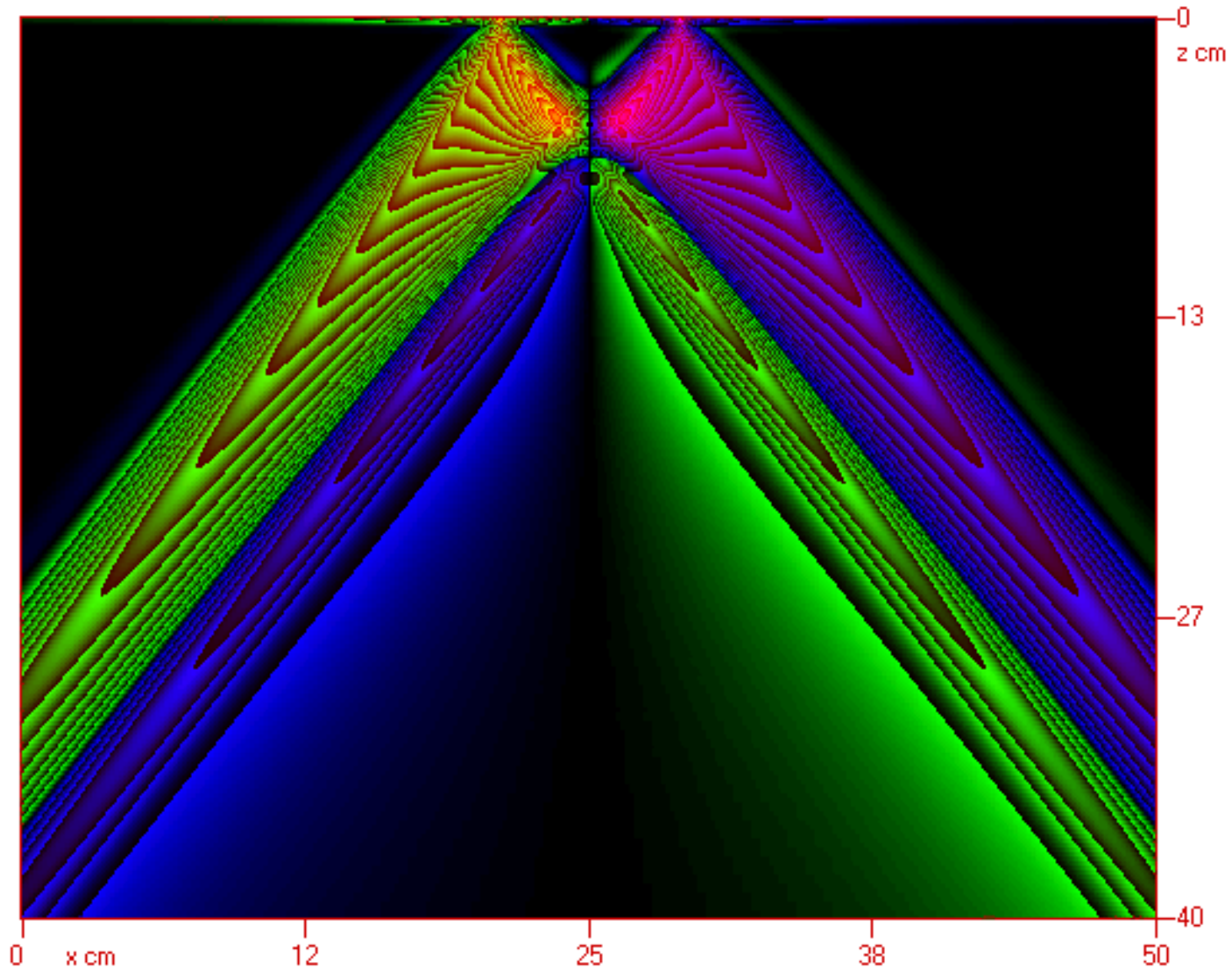
Definition of spectral coefficients  $A, B, C$  –  
substitution of solution into boundary conditions

$$\begin{cases} A \left( k_\eta^2 \sin \varphi + k_1 \beta_1 \right) + B \left( k_\eta^2 \sin \varphi + k_2 \beta_2 \right) + C i k_\eta \cos \varphi = U_\xi \\ -A k_\eta \gamma_1 - B k_\eta \gamma_2 + i C \gamma_3 = U_\eta \\ A \left( k_\eta^2 \cos \varphi - k_\xi \beta_1 \right) + B \left( k_\eta^2 \cos \varphi - k_\xi \beta_2 \right) - C i k_\eta \sin \varphi = U_\zeta \end{cases}$$

$$\mathbf{U} = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \mathbf{u}(\xi, \eta) e^{-i k_\xi \xi - i k_\eta \eta} d\xi d\eta$$

$$\beta_i = k_i \sin \varphi - k_\xi \cos \varphi \quad \gamma_i = k_i \cos \varphi + k_\xi \sin \varphi$$

## Динамика поля горизонтальной компоненты скорости жидкости





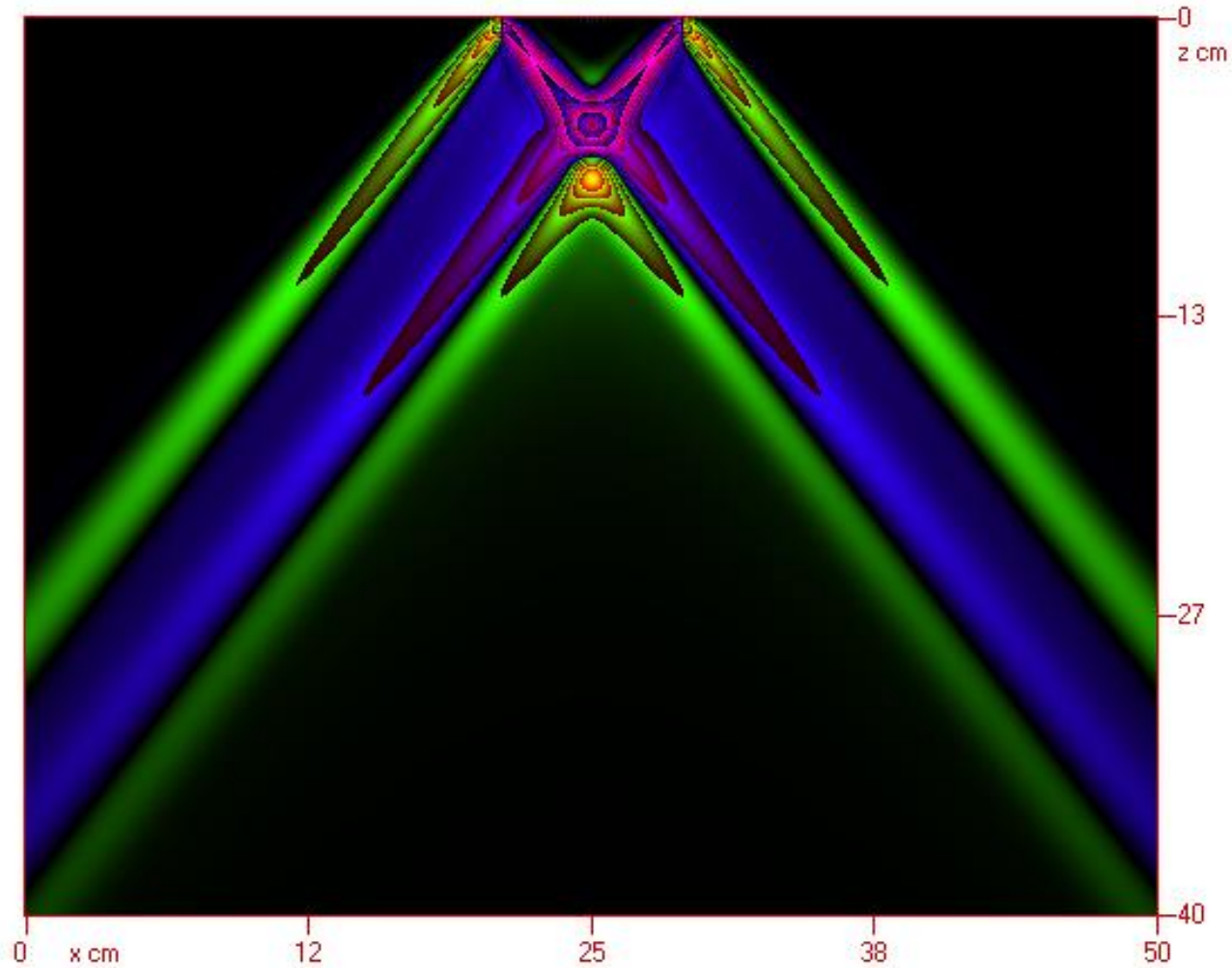
## Оболочки 3D би-модальных волновых пучков – «мерцающие» внутренние пограничные течения

$$N = 1.2 \text{ s}^{-1}$$

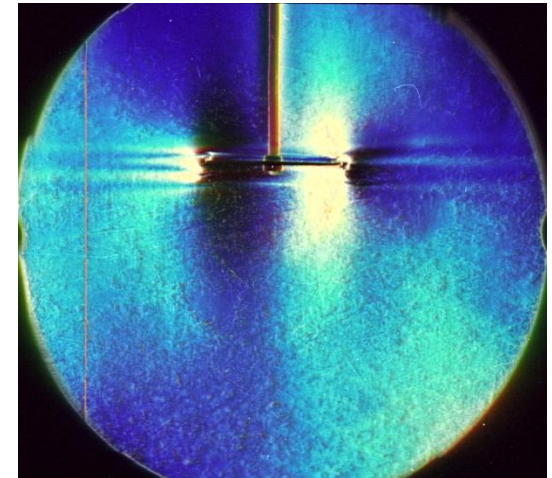
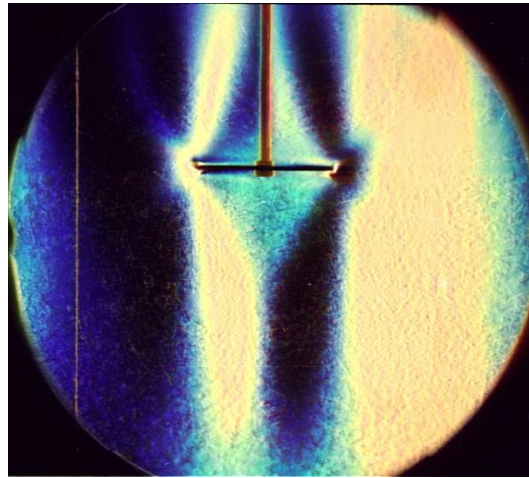
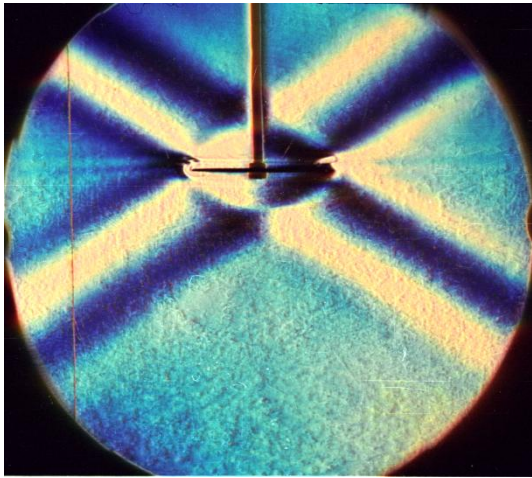
$$R = 4 \text{ cm}$$

$$\omega = 1.0 \text{ s}^{-1}$$

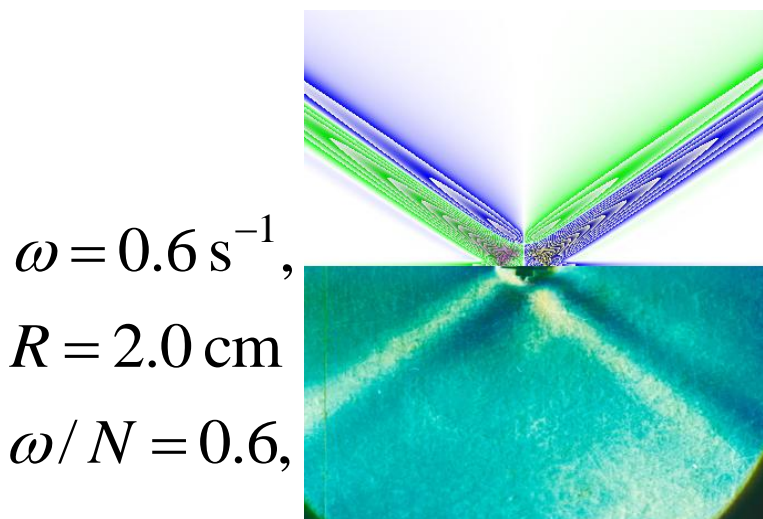
$$u = 0.25 \text{ cm/s}$$



Дифференциальный анализатор – картина сдвига скорости  $\partial v_z / \partial r$



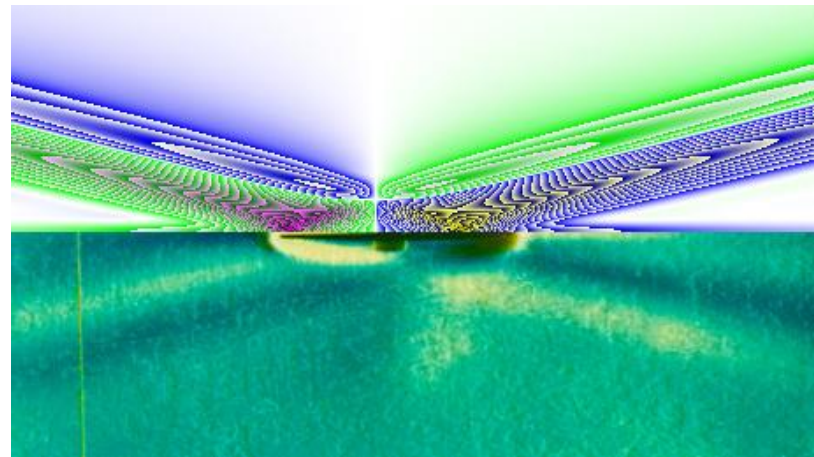
$$L_{cr} = \sqrt[3]{g\nu/N} \quad \omega/N = 0.55; 0.97; 1.27. \quad D = 5 \text{ cm} \quad T_b = 6.4 \text{ s},$$



$$\omega = 0.6 \text{ s}^{-1},$$

$$R = 2.0 \text{ cm}$$

$$\omega/N = 0.6,$$



$$\omega = 0.3 \text{ s}^{-1},$$

$$R = 5.0 \text{ cm}$$

$$\omega/N = 0.3,$$

$$2R < L_{cr}$$

Пучки периодических внутренних волн

$$2R > L_{cr}$$



Pattern of periodic internal waves wedges contacting with conductivity probes in a fluid with variable buoyancy



## Propagation of internal wave beams in an arbitrarily stratified liquid

**M.S. Paoletti, Harry L. Swinney**

Propagation and evanescent internal wave in a deep ocean model

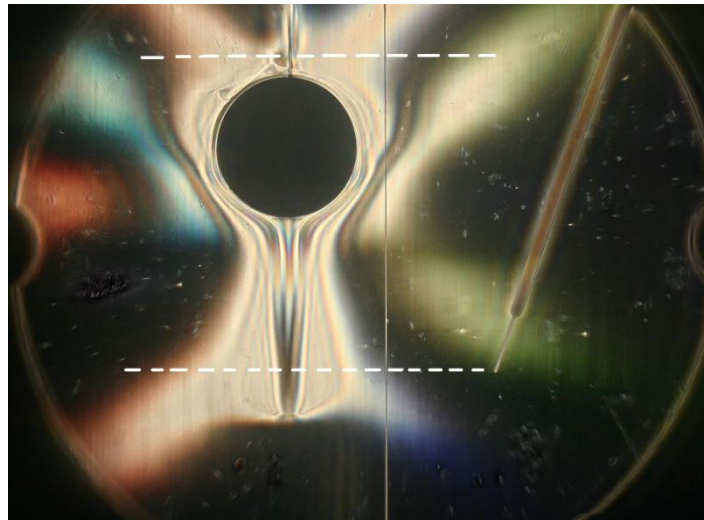
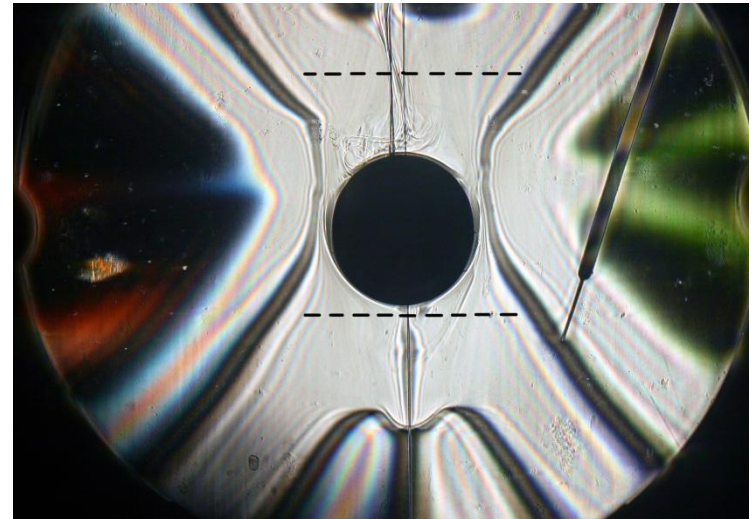
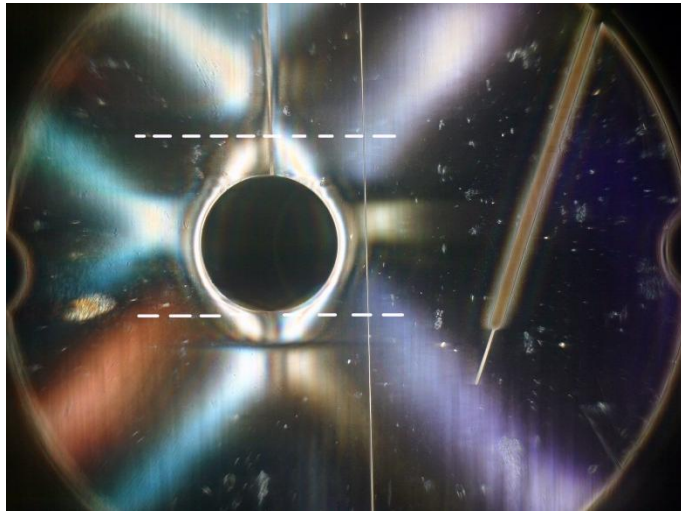
Journal of Fluid Mechanics, 2012. V. 706, P. 571–583. doi:10.1017/jfm.2012.284c

We present experimental and computational studies of the propagation of the internal waves in a stratified fluid with an exponential density profile that models the deep ocean. The buoyancy frequency smoothly varies by nearly an order of magnitude over the fluid depth, rather than being constant or piece-wise constant as in prior studies. In addition to being non-uniform the stratification is characterized by a turning depth is equal to the wave frequency. Internal wave reflects from the turning depth and became evanescent below.

The vertical velocity fields on the incoming and reflected waves above the turning depth agree within a few percent with previously untested theory for a fluid with arbitrary stratification

[Kistovich Yu.V., Chashechkin Yu.D. Linear theory of the propagation of internal wave beams in an arbitrarily stratified liquid // J. Appl. Mech. Tech. Phys. V. 39, 729-737 (1998).

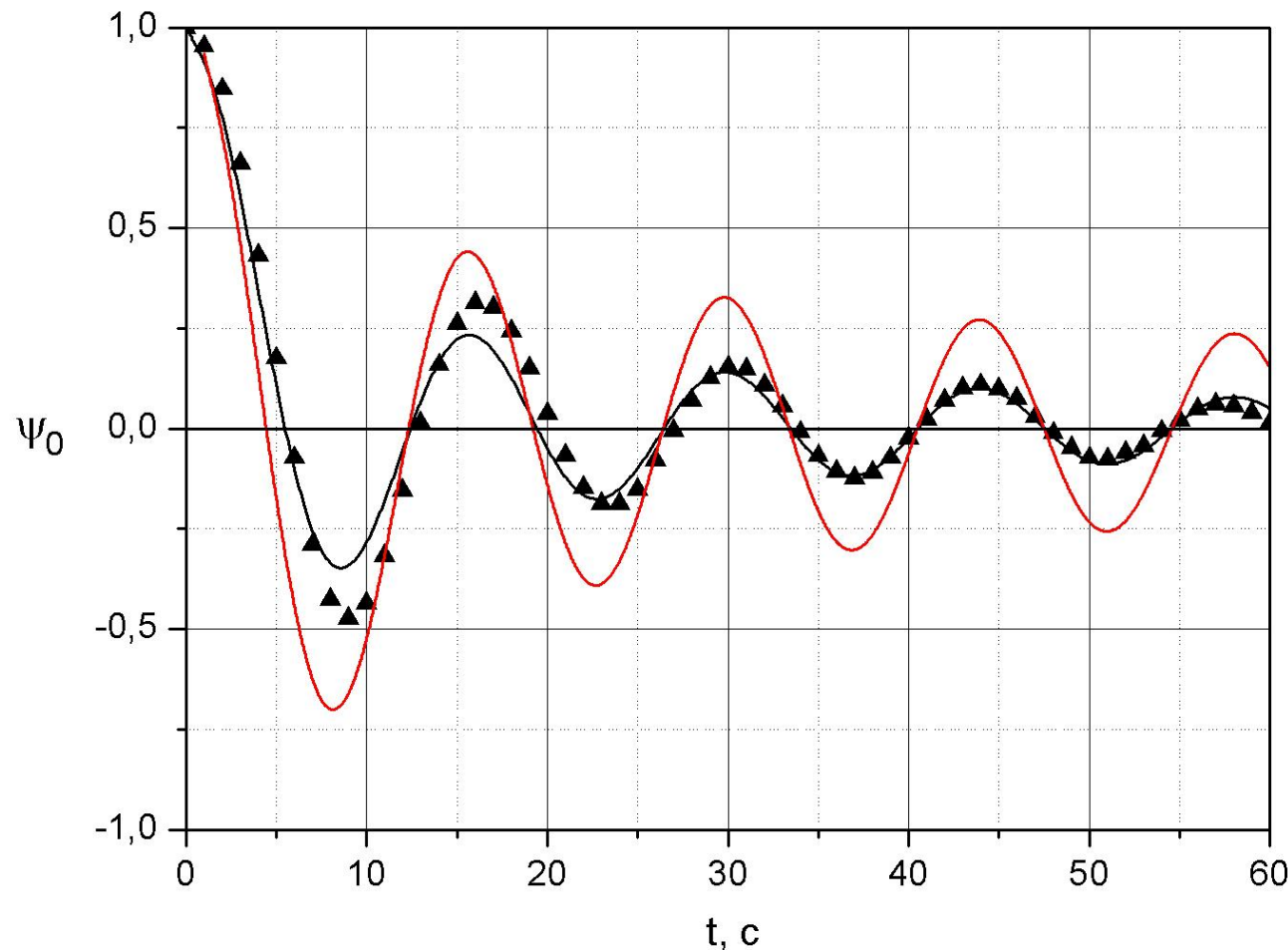
Formation of singular components inside the periodic wave cone  
with increasing amplitude of the the source oscillations




$$T_b = 11.2 \text{ s} \quad D = 4.5 \text{ cm} \quad A = 2.7 \text{ cm} \quad \omega / N = 0.68$$

Singular envelopes and auto cumulative jets in the periodic internal wave cone

## Neutral buoyancy sphere motion: calculations and experiment



Васильев А.Ю., Чашечкин Ю.Д.  
Затухание свободных колебаний  
шара нейтральной плавучести в  
вязкой непрерывно  
стратифицированной жидкости  
// Прикладная математика и  
механика. 2009. Т.73. Вып. 5.  
С.776-786

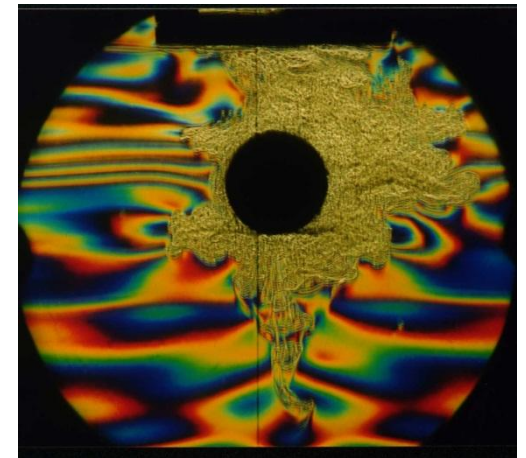
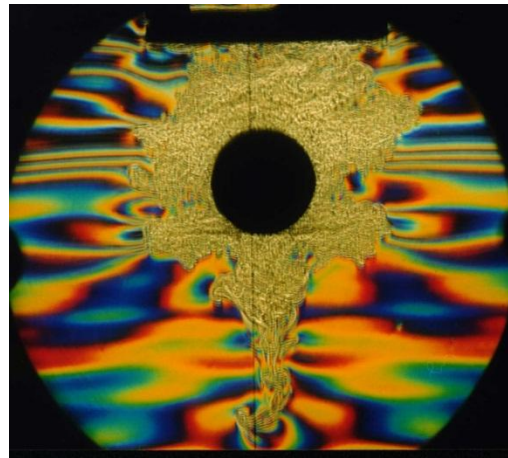
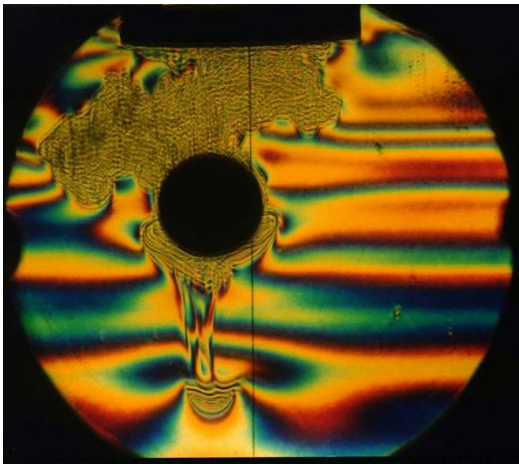
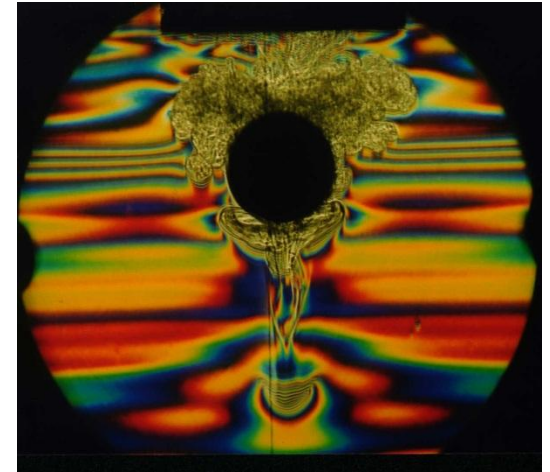
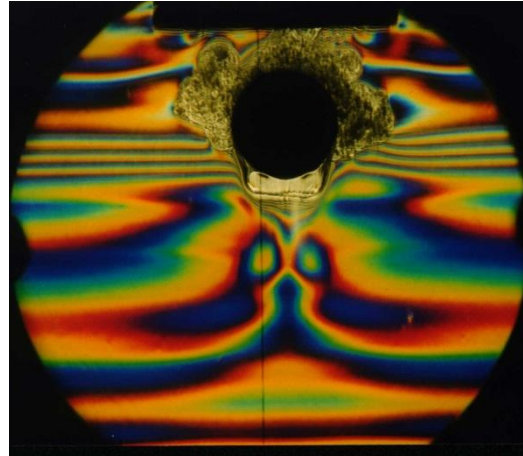
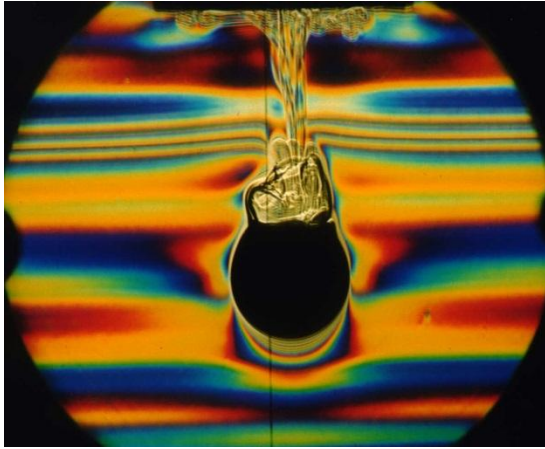
**эксперимент**   $N = (0.47 \pm 0.02) \text{ c}^{-1}$   $R = 3.35 \text{ см}$

⑨  $N = 0.46 \text{ c}^{-1}; a = 0.14$

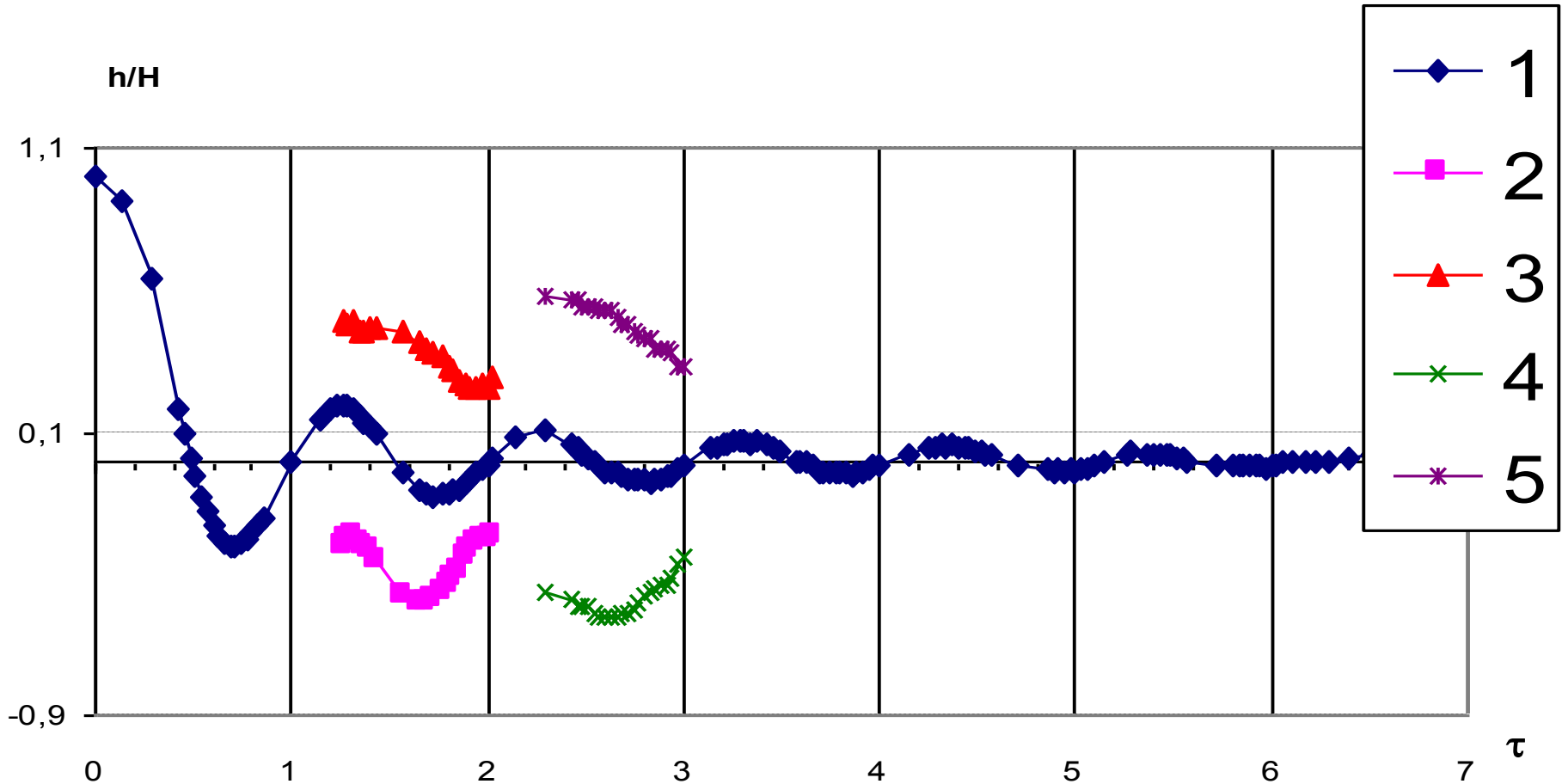
⑨ Larsen



Schlieren image of flow around sphere sinking on a horizon of neutral buoyancy,  
( $D = 4.5$  cm,  $T_b = 7,9$  с,  $H = 12$  cm)



a, b) – sinking and first ascending; c, d) and e, f) – formation of the first and decay of secondary autocumulative jets



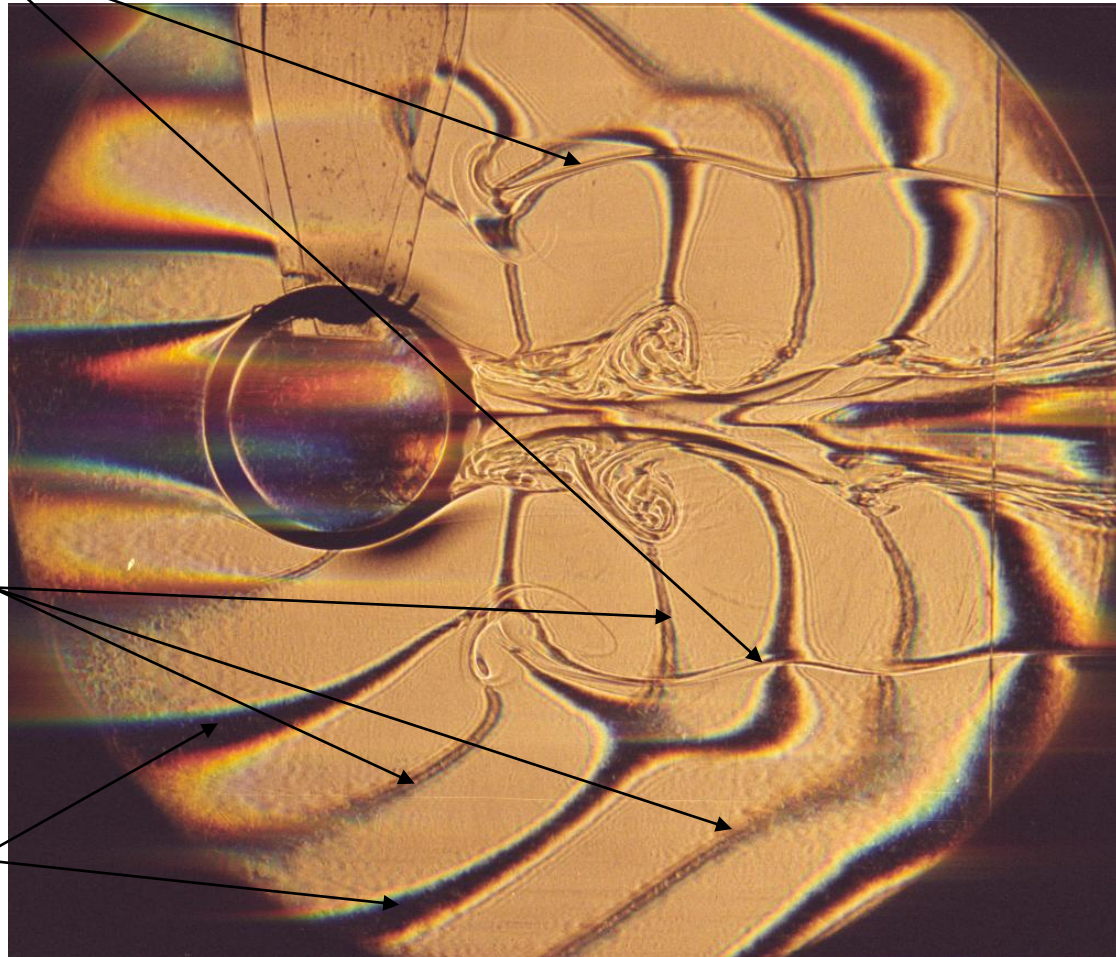
Oscillations of vertical cylinder ( $D = 3.1$  cm,  $L = 5.1$  cm), (curve 1) и poles of autocumulative jets in two frames: laboratory (curves 2, 4) and attached to the body (curve 3, 5), ( $T_b = 7$  с,  $H = 11$  см)

## Soaring sidics in the internal wave field past cylinder in uniformly stratified fluid

Maksoutov's  
Slit-Thread  
schlieren  
method

Double gray  
curves are troughs  
of internal waves

Dark curves are  
crests of internal  
waves



Formation of  
interior  
interfaces and  
soaring  
vortices  
after the  
beginning  
of the cylinder  
motion

$$\tau = 5.8$$

$$D = 5 \text{ cm}, U = 0,35 \text{ cm/c}, T = 13 \text{ c}, Fr = 0.14, Re = 165$$

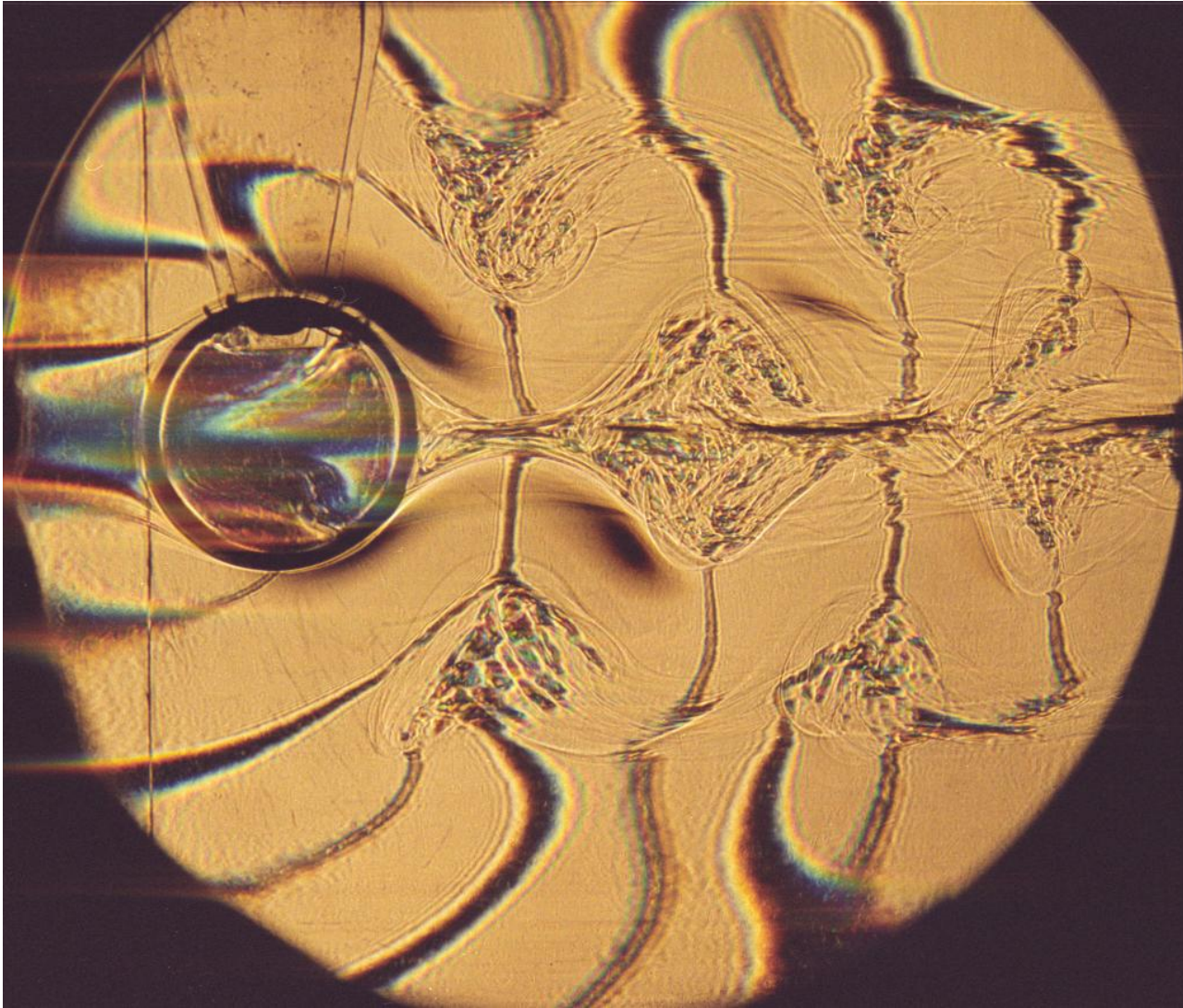




Вид на лентикулярисы с террасы дома Ф.Фрони: Тулон, 2008 г.



## Formation of soaring vortices after beginning of motion



Maksoutov's Slit-Thread schlieren method

$$D = 5 \text{ cm}, U = 0,35 \text{ cm/c}, T = 13 \text{ c}, Fr = 0.14, Re = 165 \quad \tau = 65$$



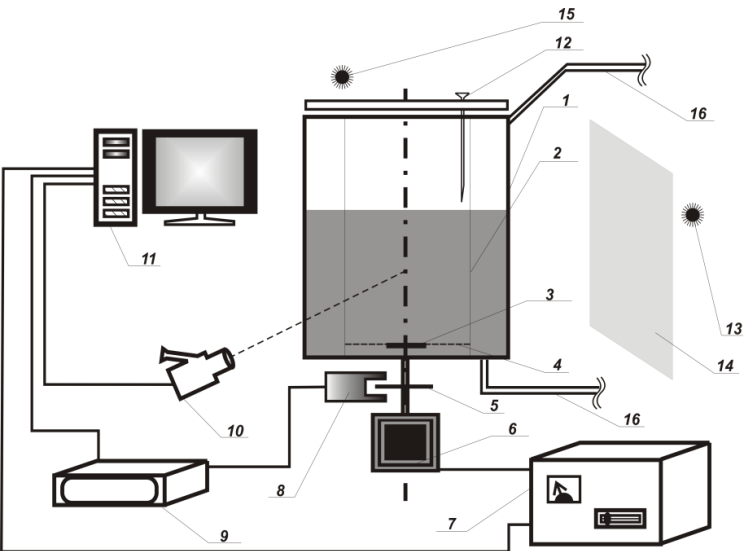
Лентикулярисы над зданием NCAR (Boulder, Co, November 12, 2012)



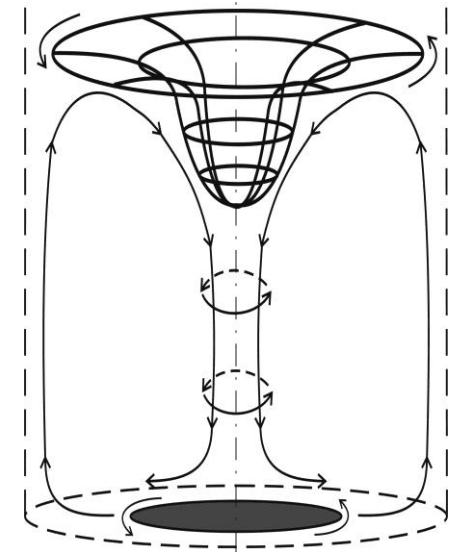


Spiral structure of floating debris in Vityaz' Bay (Pacific ocean).  
Photo O. G. Konstantinov (POI FED RAS).

## Compound vortex flow in a homogeneous fluid produced by a rotating disc placed at the bottom of cylindrical container

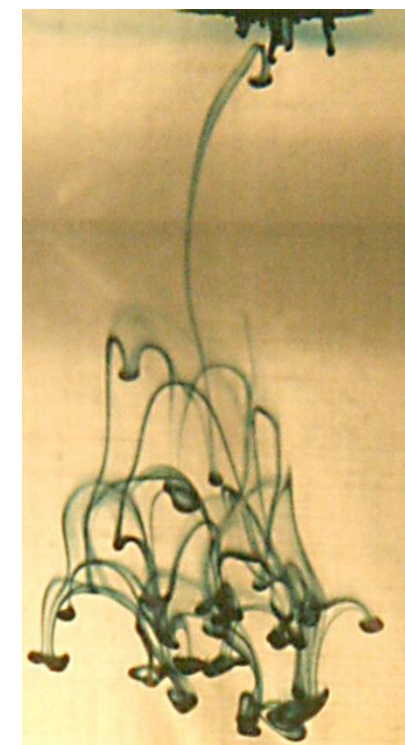
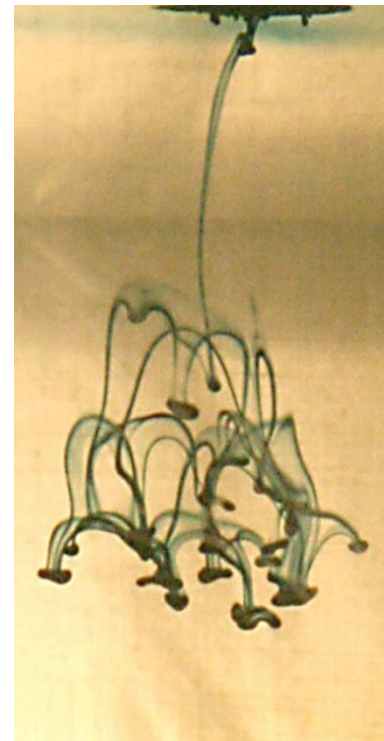
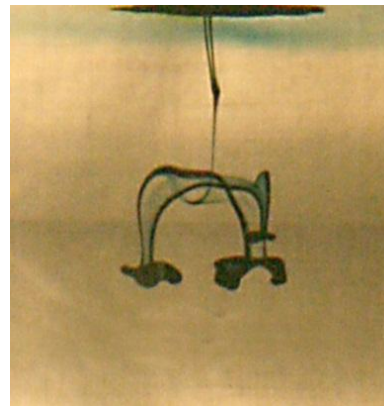
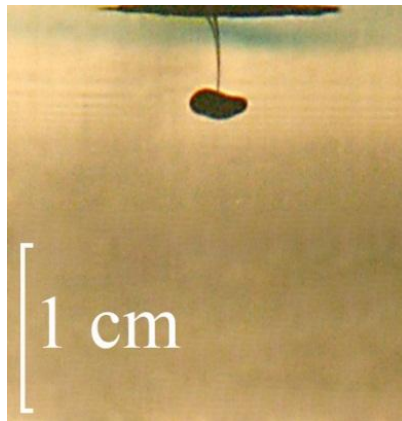


Scheme and Photo  
of experimental facility

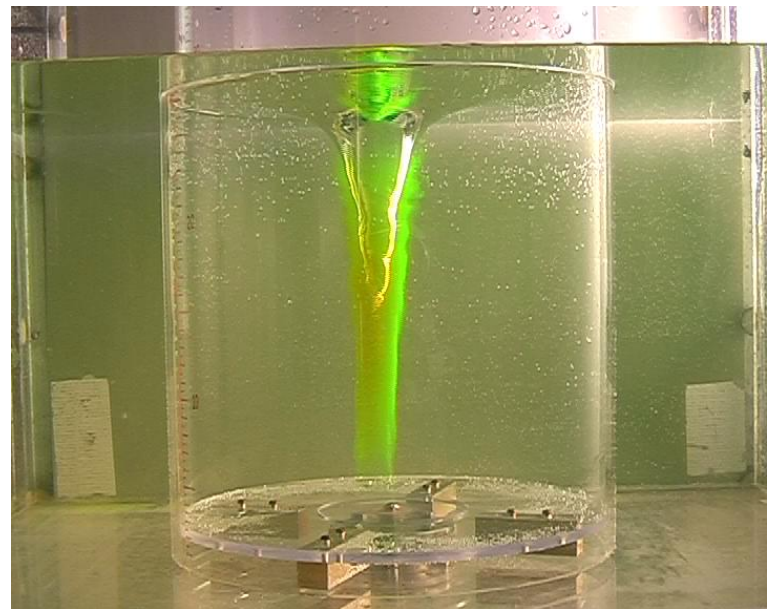
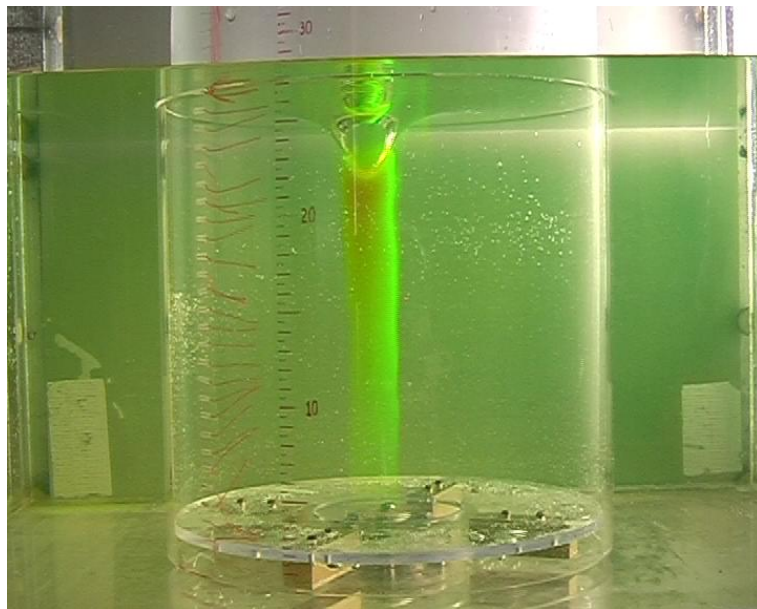
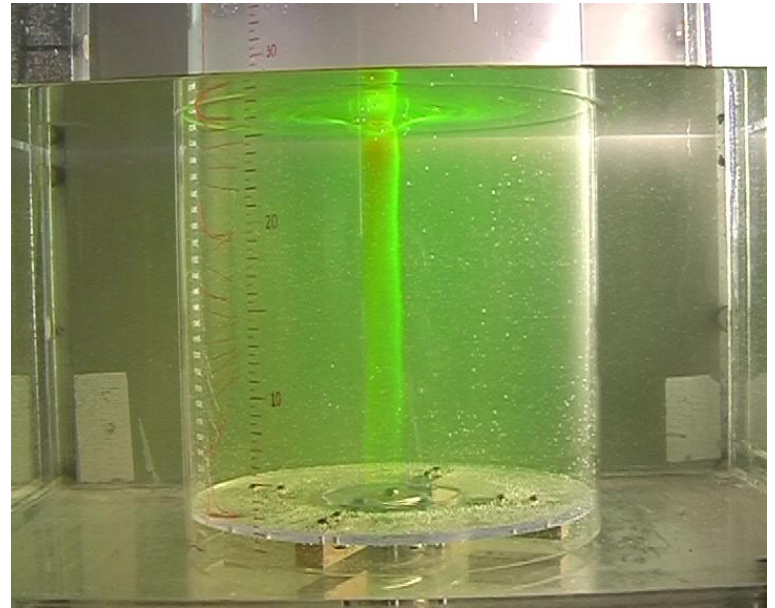
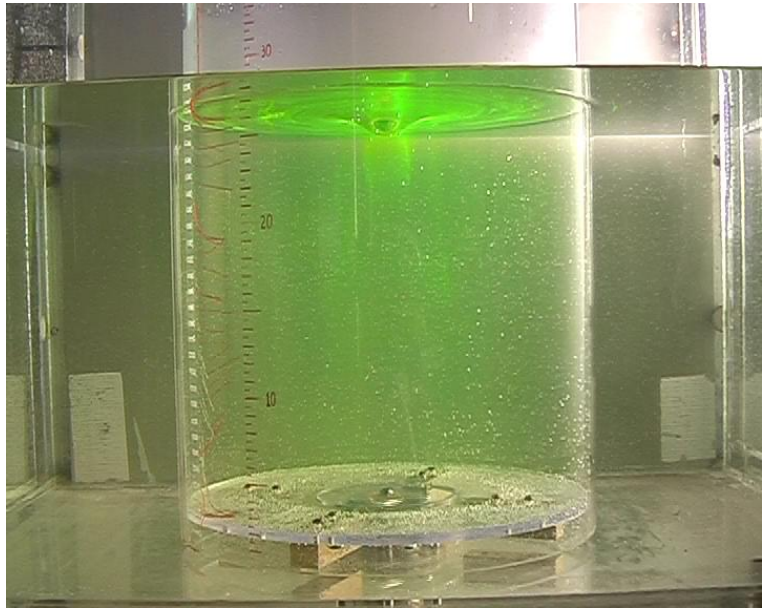


Scheme of the  
rotating disc  
induced flow

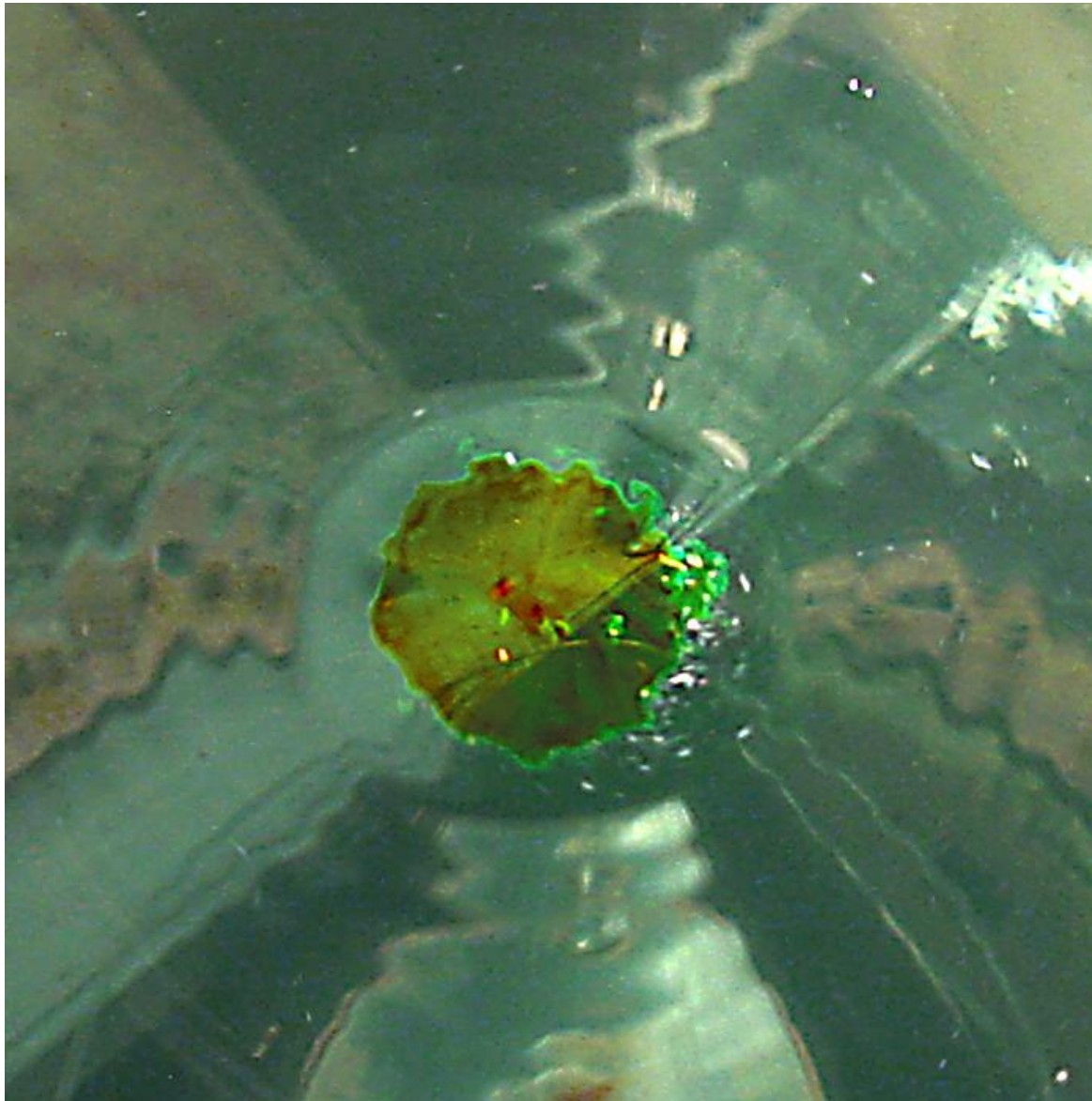
W.B. Rogers, MIT, 1858





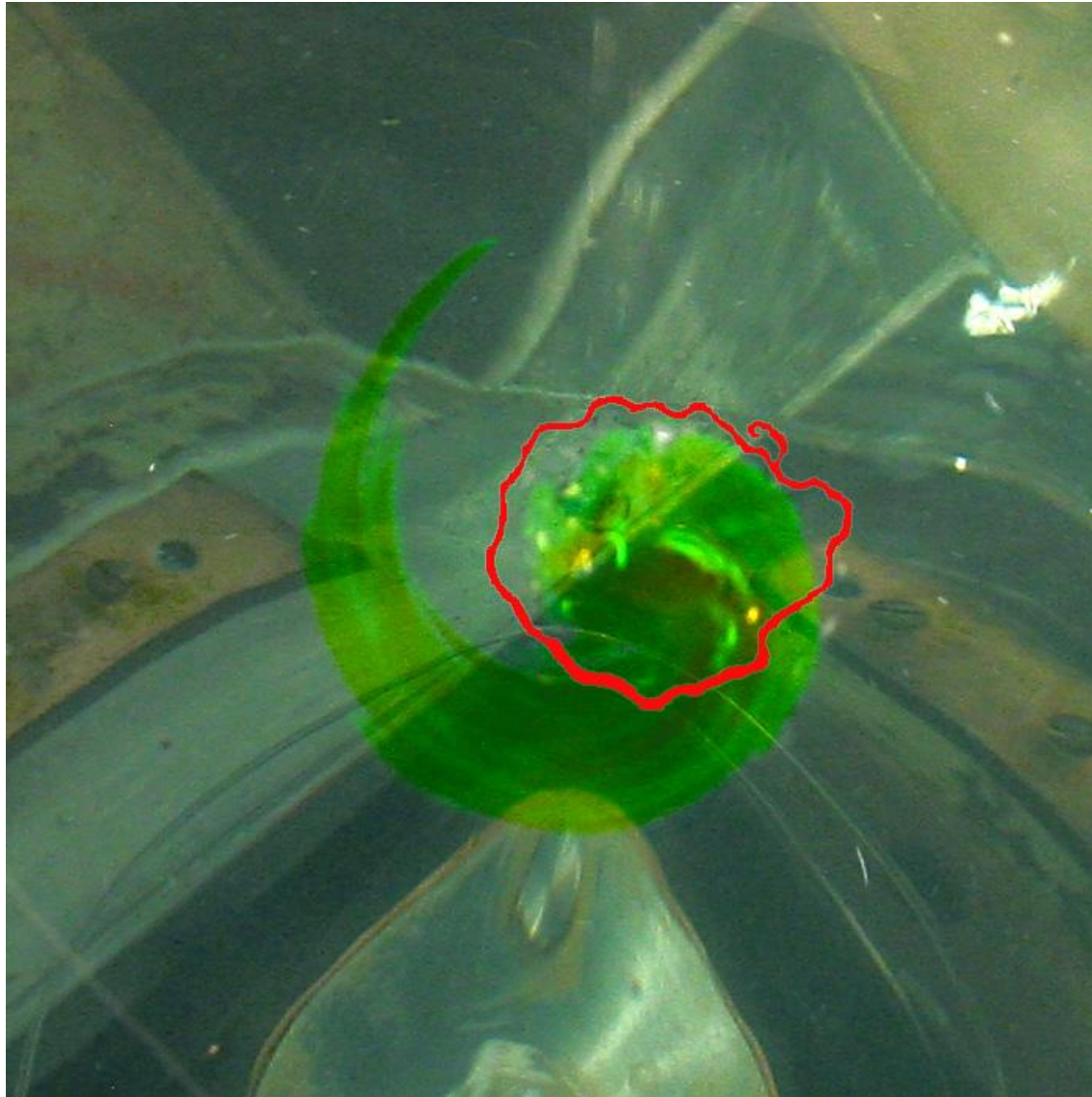


Transport of a dye inside compound vortex



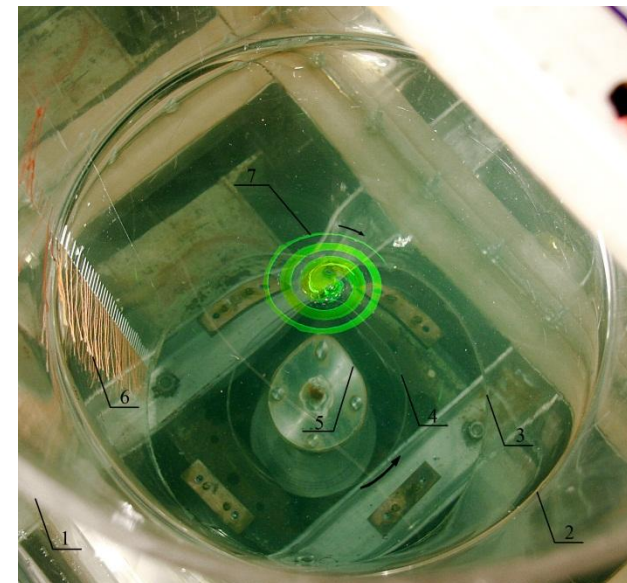
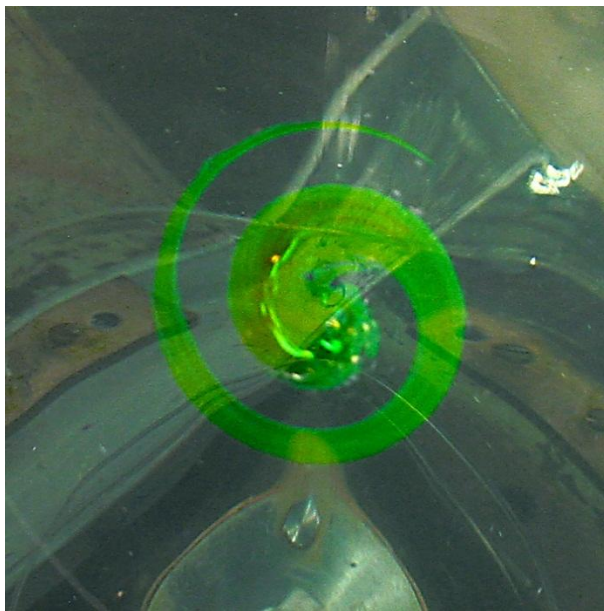
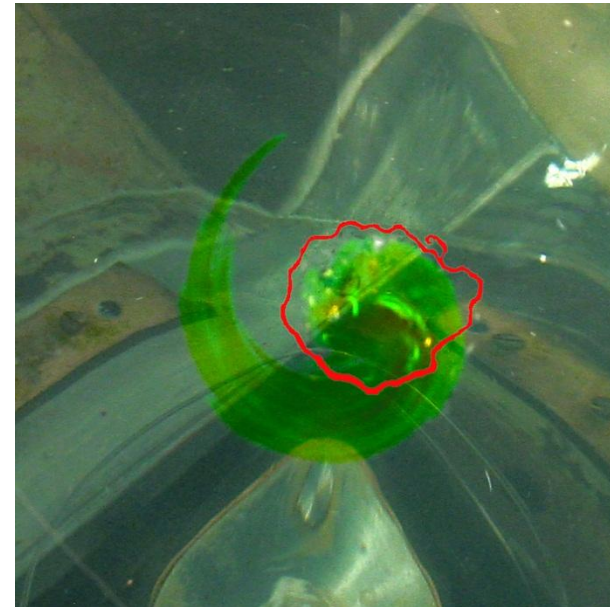
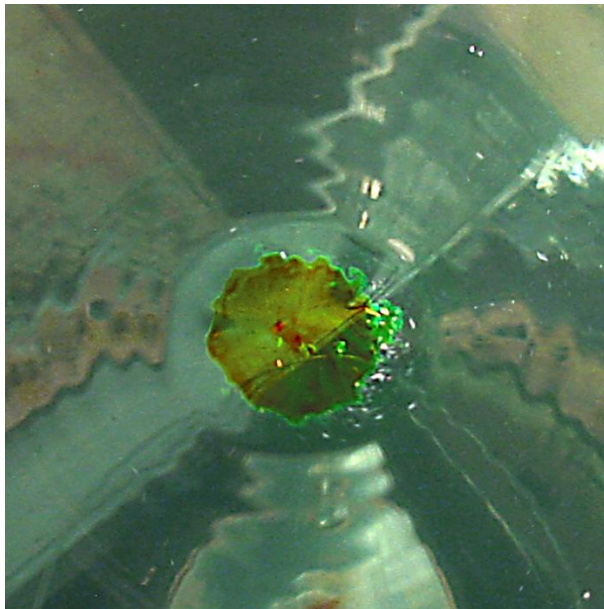
Round dye patch produced by droplet of URANIL solution fallen in the center of the surface trough in the compound vortex



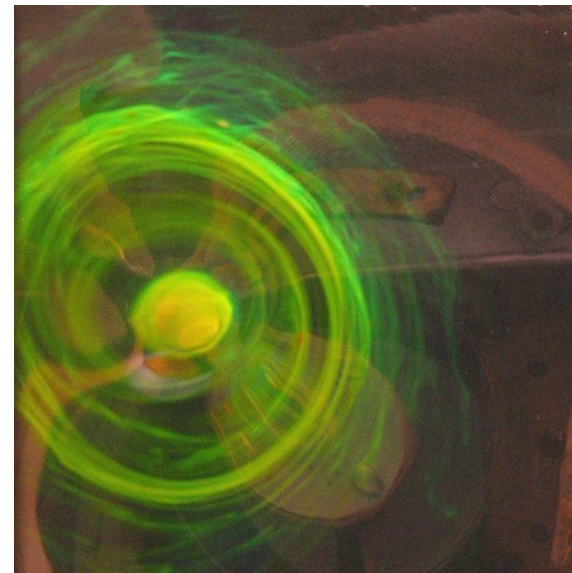
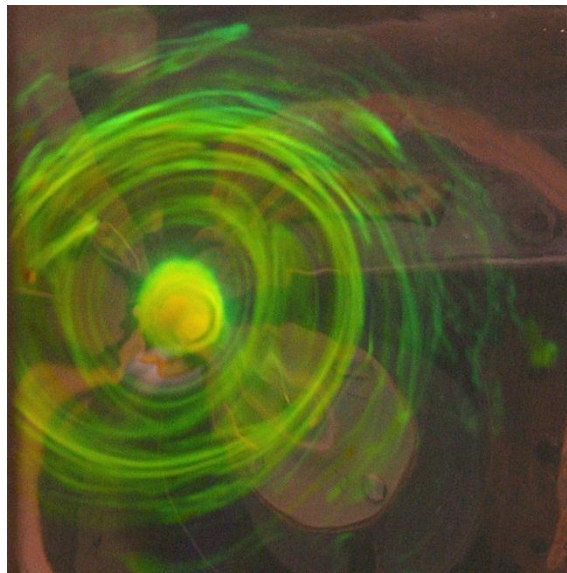
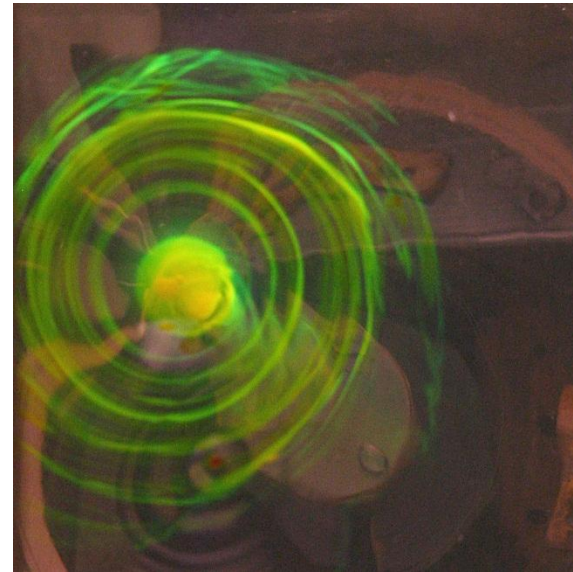
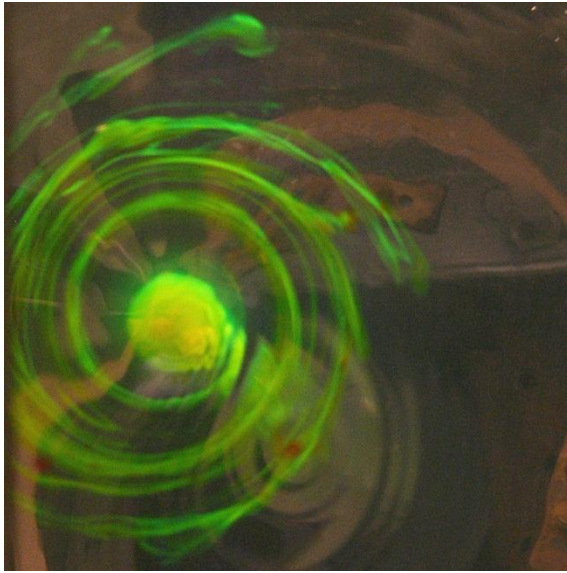


Spinning of a spiral arm from the patch produced by droplet of URANIL solution fallen in the center of the surface trough in the compound vortex





Spinning of filaments from a dye spot of the surface of compound vortex



$H = 40$  cm, 820 revo,  $R = 7,5$  cm:  $t = 1, 6, 12, 14$  s .

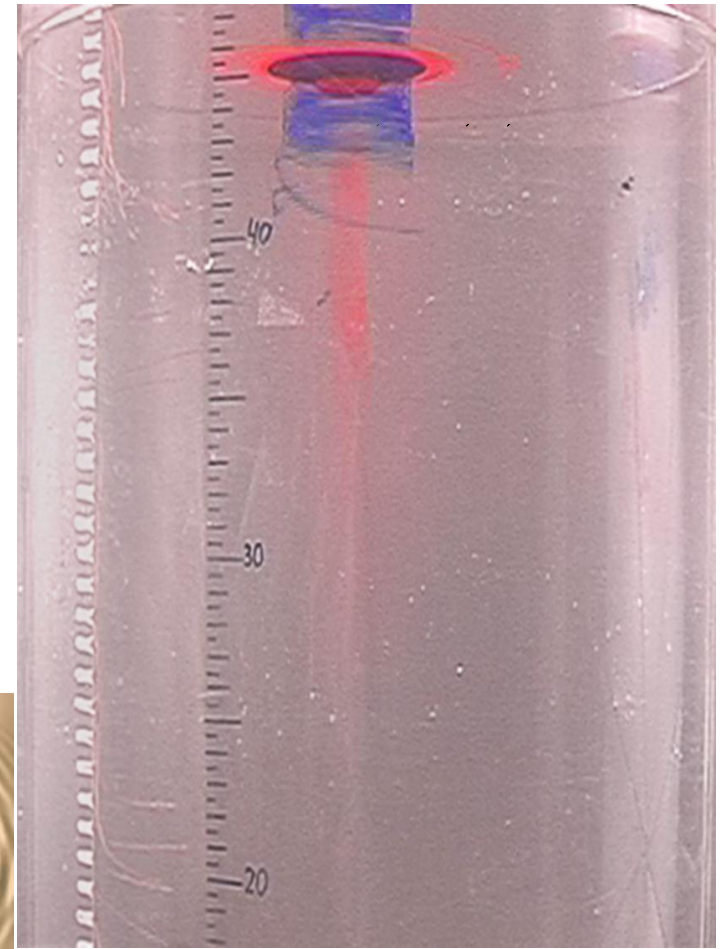
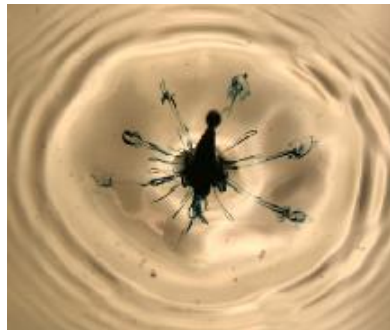


## Structural stability of a dye transport pattern in compound vortex



$\Omega = 200 \text{ RPM}$  ( $3.3 \text{ 1/s}$ )

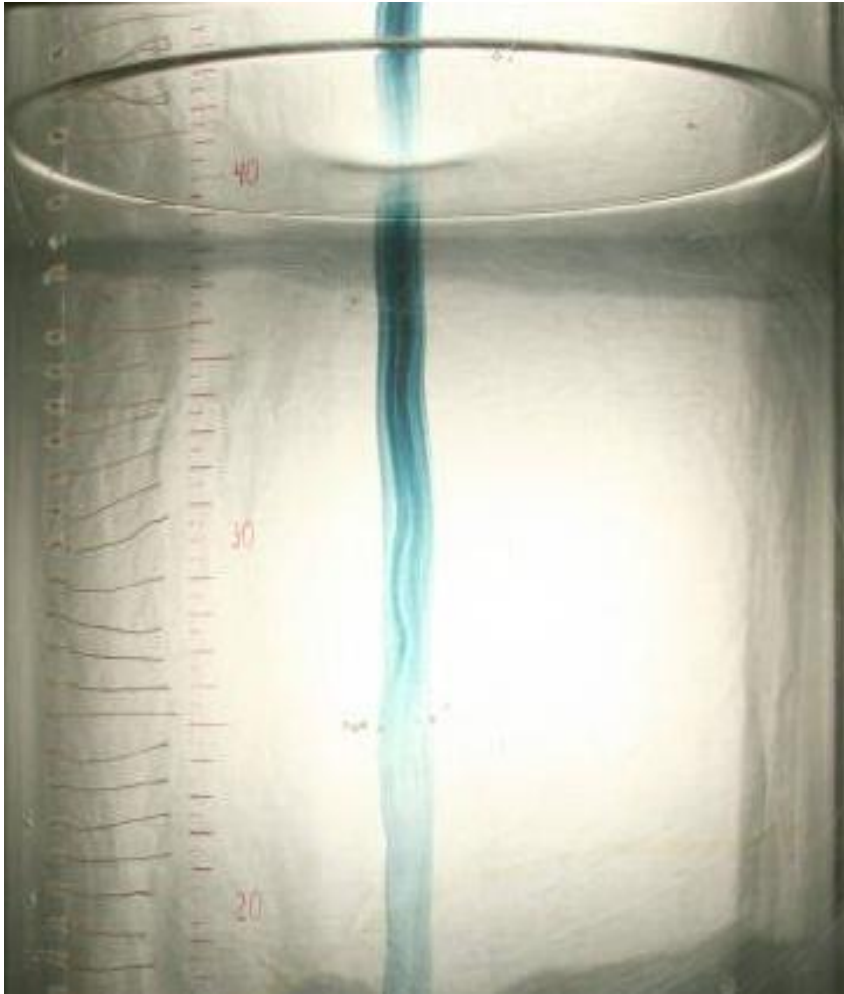
$\omega = 1.0 \text{ s}^{-1}$   
 $H = 40 \text{ cm}$



$\Omega = 800 \text{ RPM}$  ( $13.3 \text{ 1/s}$ )

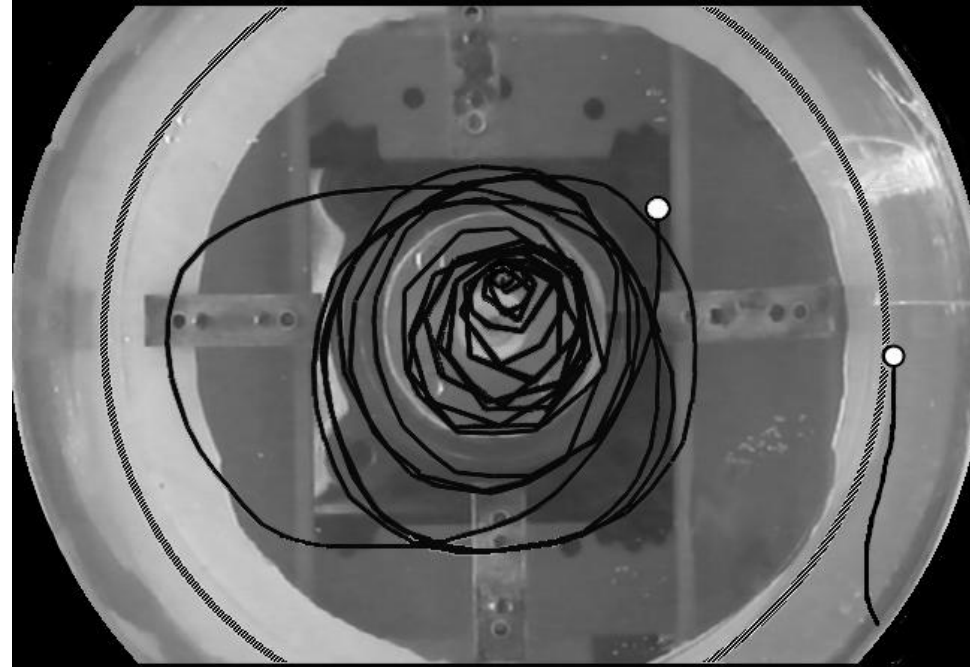
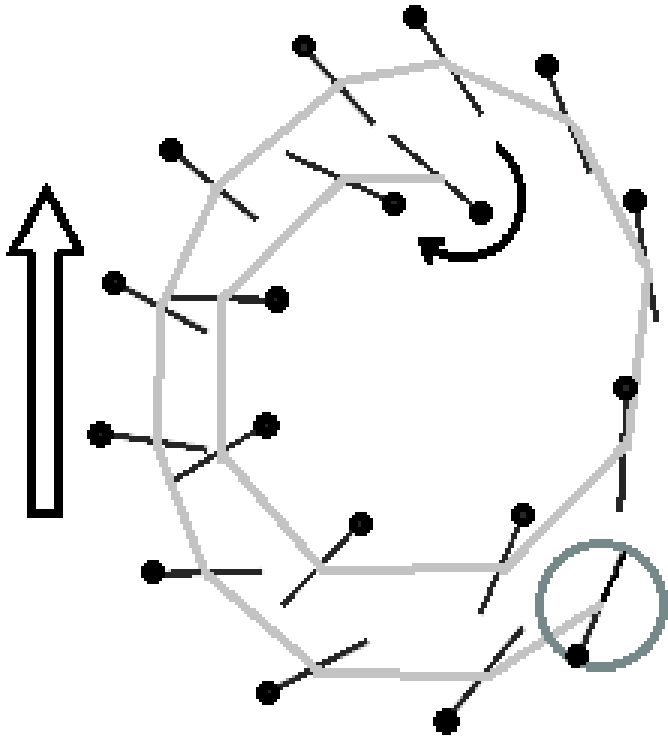


## Transport of dye and oil inside compound vortex

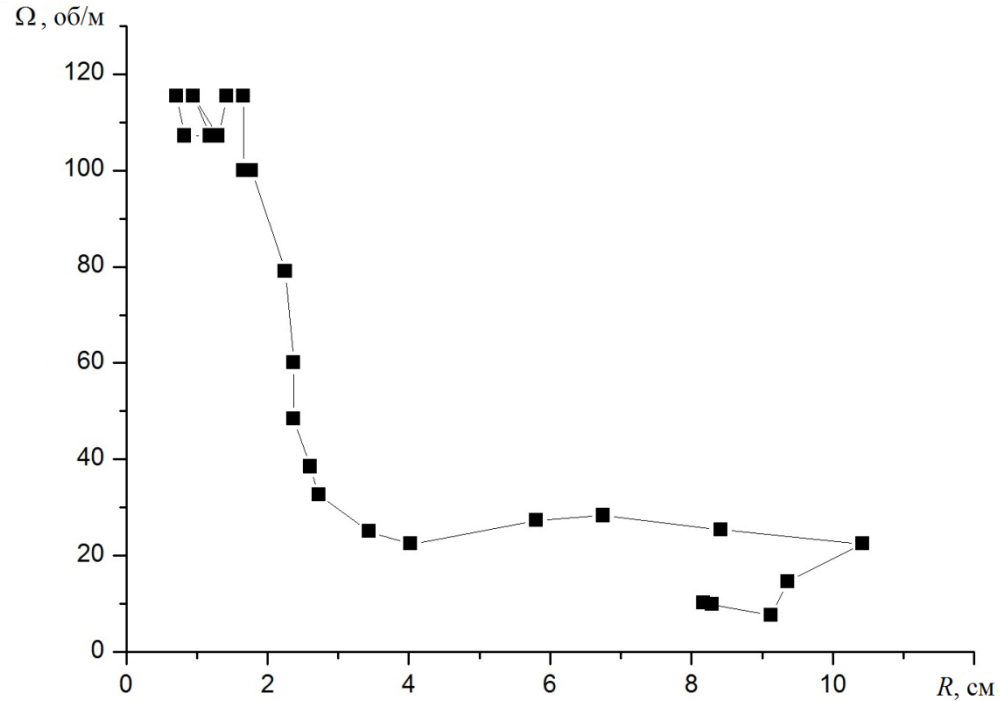
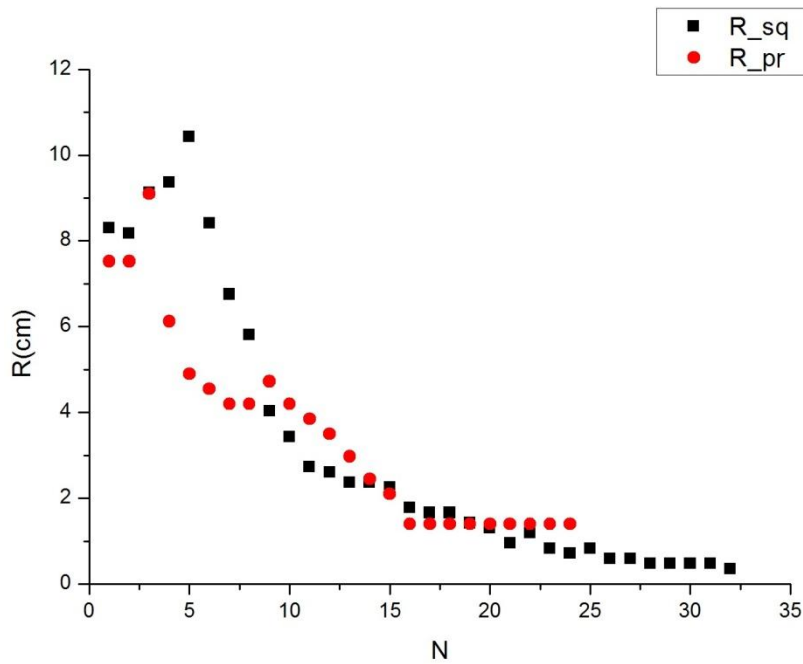


Inertial waves in the vortex trough

## Rotation and swirling of a solid marker in compound vortex



## Rotation and swirling of a solid marker in compound vortex



Расстояния от центра маркера до центра вихря в зависимости от номера оборота вокруг центра.

R\_sq – маркер-квадрат, опущенный в покоящуюся жидкость.

R\_pr – маркер-пробирка, опущенный в установившийся вихрь

Horizontal profile of marker angular velocity of general rotation  $\Omega$



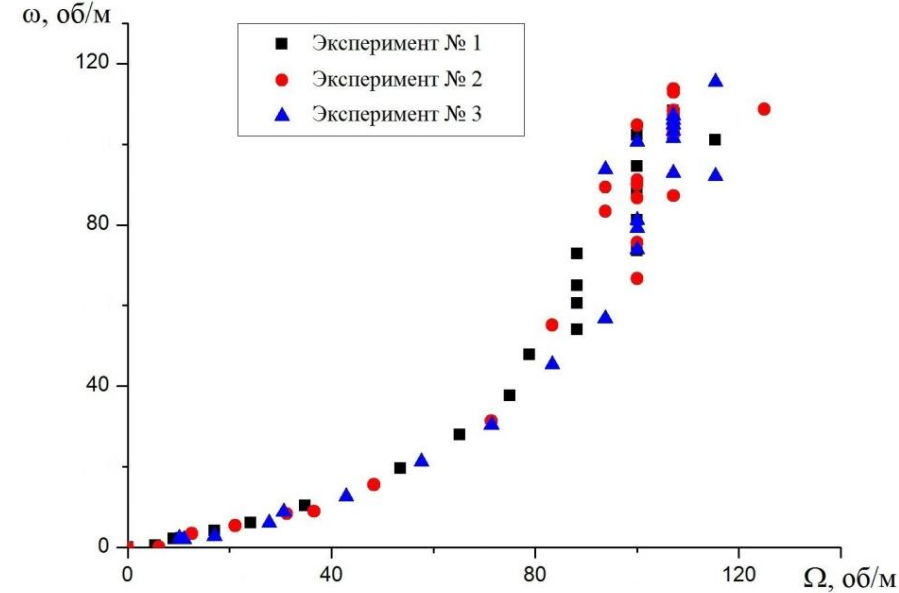
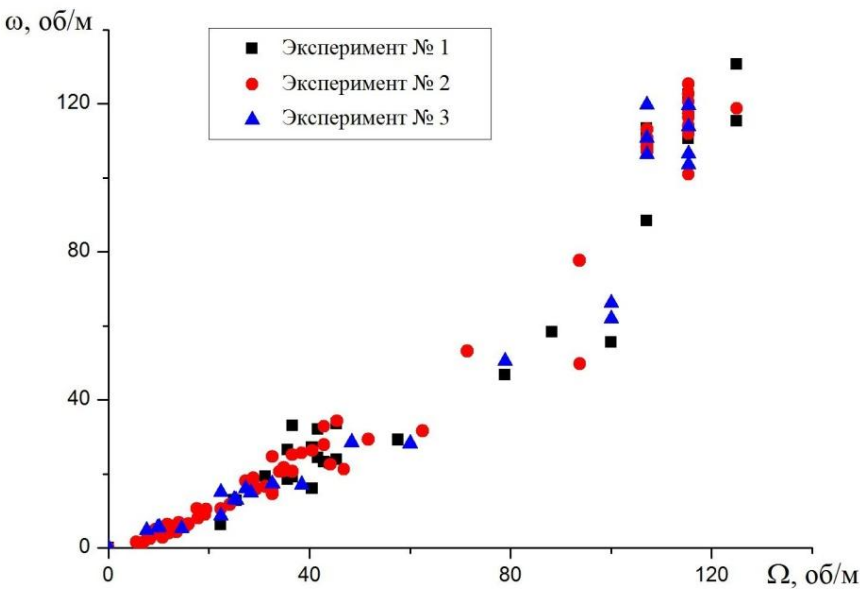
# Rotation and swirling of solid markers in compound vortex

$H = 40$  cm

$\Omega = 500$  revo

3D marker

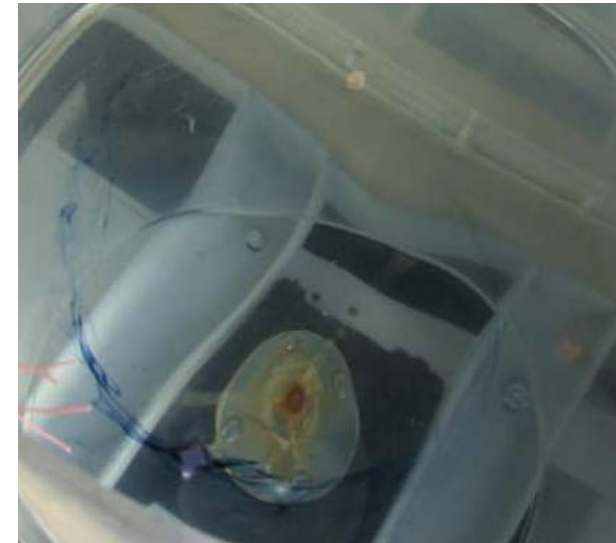
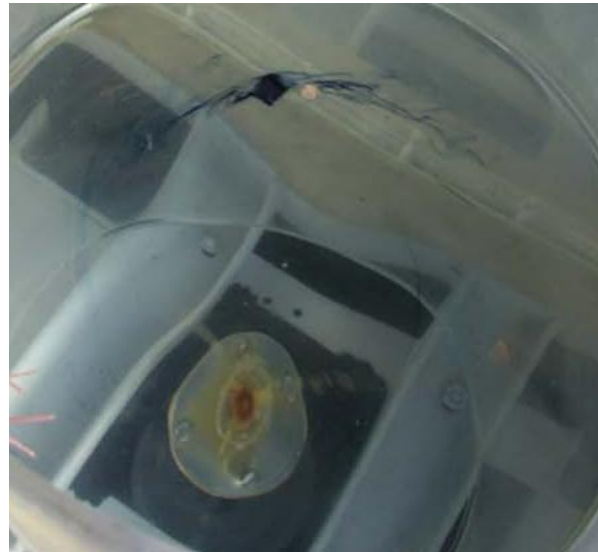
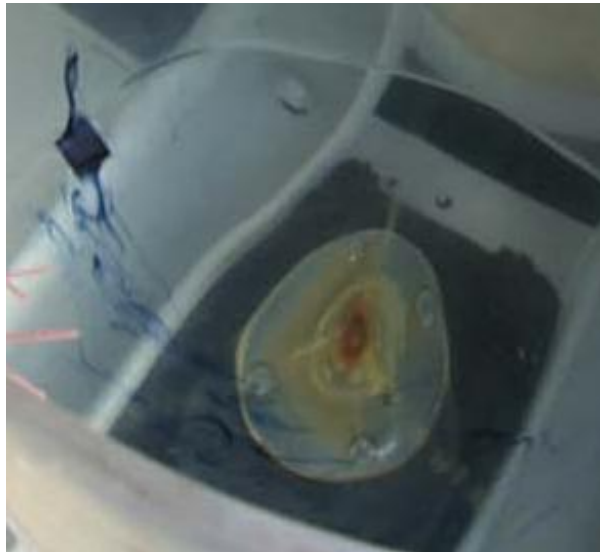
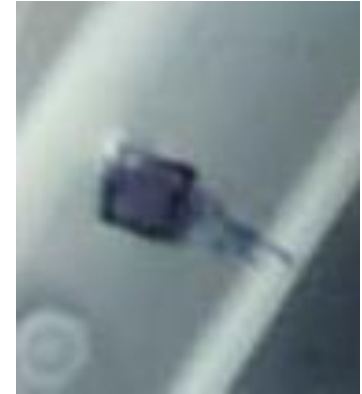
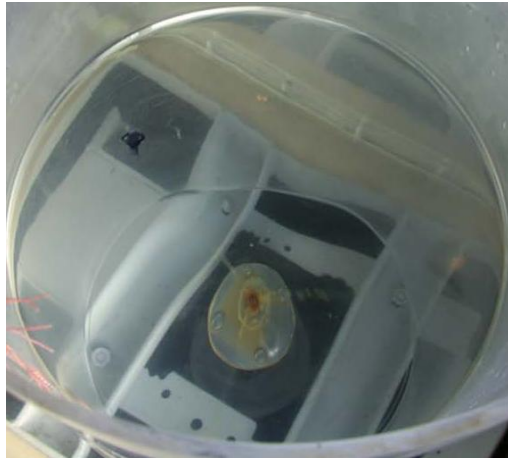
2D marker  
Plane square



$\Omega$  is general rotation frequency

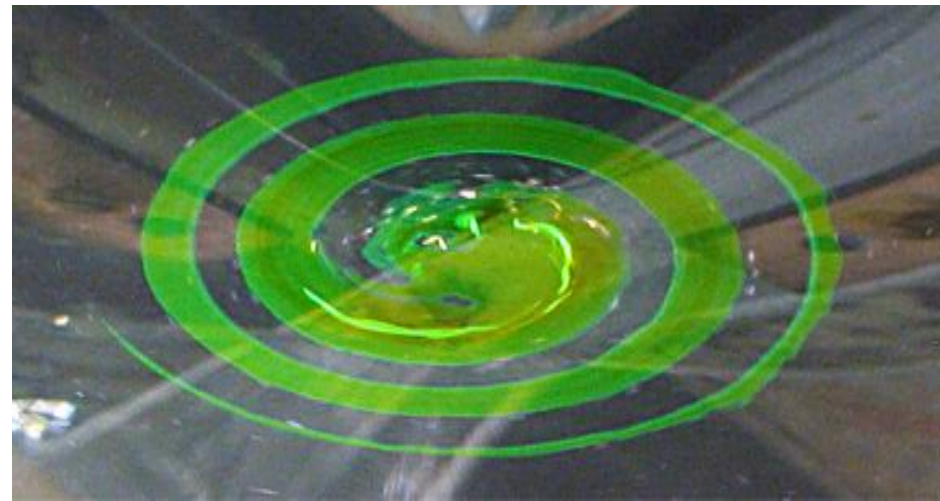
$\omega$  is swirling frequency

## Transport of a dye from plain free moving floating marker





Spiral structure of floating debris in Vityaz' Bay (Pacific ocean).



Spiral arm formed from Uranium patch in the surface trough of compound vortex



*Remarks on the Mathematical Classification of Physical Quantities,*  
by J. CLERK-MAXWELL, LL.D., F.R.S.

The first part of the growth of a physical science consists in the discovery of a system of quantities on which its phenomena may be conceived to depend. The next stage is the discovery of the mathematical form of the relations between these quantities. After this, the science may be treated as a mathematical science, and the verification of the laws is effected by a theoretical investigation of the conditions under which certain quantities can be most accurately measured, followed by an experimental realisation of these conditions, and actual measurement of the quantities.

It is only through the progress of science in recent times that we have become acquainted with so large a number of physical quantities that a classification of them is desirable.

Начальный этап развития физических наук включает определение системы величин, от которых зависит изучаемое явление. Следующий этап состоит в открытии математических форм соотношений между этими величинами. После чего наука может трактоваться как математическая наука. Подтверждение истинности законов требует теоретического определения условий, при которых некоторые величины могут быть наиболее точно измерены с последующей экспериментальной реализацией этих условий и действительным измерением указанных величин. Благодаря развитию наук в последнее время мы ознакомились с таким большим числом физических величин, что их классификация становится необходимой.

Maxwell J.C. Remarks on mathematical classification of physical quantities // Proceedings of the London Mathematical Society 1869. V. 1-3(1). P. 224-233.

## Fundamental Laws of Conservations and Thermodynamics

$$\frac{\partial \rho}{\partial t} + \frac{\partial p_i}{\partial x_i} = 0 \quad \mathbf{p} = \rho \mathbf{v} \quad \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0 \quad \mathbf{v} = \frac{\mathbf{p}}{\rho}$$

$$\frac{\partial \rho e}{\partial t} + \operatorname{div}(e \mathbf{p} + \mathbf{J}^{(e)}) = 0 \quad \Pi_{ij} = \rho v_i v_j + P \delta_{ij} - \sigma_{ij}$$

$$\frac{\partial p_i}{\partial t} + \frac{\partial \Pi_{\alpha i}}{\partial x_\alpha} = \rho f_i \quad \frac{\partial \rho s}{\partial t} + \operatorname{div}(\rho s \mathbf{v} + \mathbf{J}^{(s)}) = P^{(s)}$$

$$\frac{\partial \rho^{(n)}}{\partial t} + \operatorname{div}(\mathbf{p}^{(n)}) = 0 \quad \mathbf{v} = \rho^{-1} \sum \rho^{(n)} \mathbf{v}^{(n)}$$

$$\rho = \rho(T, P, c) = \sum \rho^{(n)} \quad + \text{boundary conditions: no-slip, no-flux,}$$

attenuation with a distance from a source

$$e = e(T, P, c)$$

### Definition:

Fluid Flow is transport of momentum supplemented by self-consistent variations of observable physical quantities and is described by the fundamental set of equations

D'Alembert – Navier – Stokes – Fourier – Fick – Mendeleev system  
is fundamental set of governing fluid flow equations (complete and self consistent)

$$\rho = \rho(p, T, S_i), \quad e = e(\rho, p, T, S_i) \quad \mathbf{p} = \rho \mathbf{v} \quad \rho(z) = \rho_o \exp(-z/\Lambda),$$

$$\frac{\partial \rho}{\partial t} + \mathbf{div} \rho \mathbf{v} = 0 \quad \mathbf{j}_T = -\kappa_T \nabla \rho T \quad \Lambda = |d \ln \rho / dz|^{-1}$$
$$\mathbf{j}_S = -\kappa_S \nabla \rho S \quad N = \sqrt{g / \Lambda} \quad T_b = 2\pi / N$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \rho \mathbf{v} = -\nabla p + \rho \mathbf{g} + \mu \Delta \mathbf{v} + \mathbf{f}$$

$$\frac{\partial \rho T}{\partial t} + (\mathbf{v} \nabla) \rho T = \nabla (\kappa_T \nabla \rho T) + Q_i$$

$$\frac{\partial \rho S}{\partial t} + (\mathbf{v} \nabla) \rho S = \nabla (\kappa_T \nabla \rho S) + Q_S$$

+ conventional boundary conditions (no-slip, no-flux, attenuation of disturbances at infinity distance from the source)

Compatibility condition defines the set rank and order of the coupled fundamental set

Landau L.D., Lifshits E.M. Theoretical physics. V.6. Hydrodynamics. M.: Nauka. 1944. 646 p.

Muller P. The equations of oceanic motions. Cambridge: CUP. 2006. 292 p.



## Д. И. Менделеев = D. Mendeleeff

1. Об упругости газов. СПб.: 1875. 262 с. (On resilience of gases).
2. Введение и редактирование перевода монографии Г. Мона «Метеорология или учение о погоде» СПб., 1876. (G.Mohn “Meteorology or studies of weather”).
3. О сопротивлении жидкостей и о воздухоплавании. СПб. 1880. (On drag in fluids and aeronautics).
4. Исследование водных растворов по удельному весу). 1887. 520 с. (Studies of water solutions on specific gravity).
5. Mendeleeff D. The variation in density of water with temperature // Philosophical Magazine. 1892. V. 33. № 200 (S.5). P. 99-132.

Landau L.D., Lifshits E.M. Theoretical physics. V.3. Hydrodynamics. M.: Nauka. 1944, 626 p., 1986. 736 p.

Muller P. The equations of oceanic motions. Cambridge: 2006. 292 p.

Vallis G.K. Atmospheric and oceanic fluid mechanics. Fundamentals and large-scale circulation. Cambridge: University Press. 2007.

## Continuous groups theory is instrument for comparison of symmetries for different differential equations

Ten parametric Galilelian groups of point transformations of the complete set of Fluid Mechanics Equations

$\rho = \rho(T, p, S)$  Adequate form of state equation

$X_1 = \partial_t$   $X_{2-4} = \partial_{x_i}$   $Y = \partial_p$  Shifts – homogeneity of time, space and pressure

Rotations – isotropy of space

$$X_5 = y\partial_x - x\partial_y + v\partial_u - u\partial_v$$

$$X_6 = \left( z + \frac{gt^2}{2} \right) \partial_x - x\partial_z + (w + gt)\partial_u - u\partial_w$$

$$X_7 = \left( z + \frac{gt^2}{2} \right) \partial_y - y\partial_z + (w + gt)\partial_v - v\partial_w$$

$$X_{8-10} = t\partial_{x_i} + \partial_{u_i}$$

Galilelian transformations

## Claude-Louis-Marie-Henri (M.) Navier.

Mémoire sur les Lois du Mouvement des Fluids // Mém. d  
l'Acad. des Sciences. 1822. V. 6. P. 389.

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \rho\mathbf{g} + \mu\Delta\mathbf{v} \quad \frac{\partial \rho}{\partial t} + \operatorname{div} \rho\mathbf{v} = 0$$

Interpretation of negligent P.-S. Girard's (1816) Experiments leads C.M. Navier to posing wrong boundary conditions

$$E \mathbf{v} + \mu \frac{\partial \mathbf{v}_{\parallel}}{\partial \mathbf{n}_{\perp}} = 0$$

Mr. Navier could present his basic principles in form of hypotheses which must be proved experimentally. However, if common equations of fluid mechanics are so difficult for analysis what can we receive from this new ones even more complicated equations? Antoine Cournot, 1828.

"Useless equations"



$$\operatorname{div} \mathbf{v} = 0, \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \rho \mathbf{v} \Delta \mathbf{v} + \rho \mathbf{g} \quad p' = \frac{p}{\rho} - U \quad \mathbf{g} = \nabla U$$

Generators groups of shifts

$$X_1 = \partial_t \quad X_3 = \partial_y \quad X_2 = \partial_x \quad X_4 = \partial_z$$

Expansion of Galileian principle of relativity

$$Y_{\chi_2} = \chi_2 \partial_y + \dot{\chi}_2 \partial_v - \rho \ddot{\chi}_2 y \partial_p \quad Y_{\chi_1} = \chi_1 \partial_x + \dot{\chi}_1 \partial_u - \rho \ddot{\chi}_1 x \partial_p$$

$$Y_{\chi_3} = \chi_3 \partial_z + \dot{\chi}_3 \partial_w - \rho \ddot{\chi}_3 z \partial_p$$

Infinite sub-algebra of pressure shifts

$$p' = p + \pi(t) \quad Y_1 \rightarrow Y_\pi = \pi(t) \partial_p$$

Extensions (boundary layer type solutions)

$$Z_1^* = 2t \partial_t + \mathbf{r} \partial_{\mathbf{r}} - \mathbf{v} \partial_{\mathbf{v}} - 2p' \partial_{p'}$$

# Turbulence equations

$(k - \varepsilon - \tau - \vartheta)$  – МОДЕЛЬ

$$\operatorname{div} \mathbf{v} = 0$$

$$\frac{dv_i}{dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( v \frac{\partial v_i}{\partial x_j} - w_{ij} \right) + g_i \alpha T$$

$$\frac{dT}{dt} = \frac{\partial}{\partial x_i} \left( \kappa_T \frac{\partial T}{\partial x_i} - q_i \right)$$

$$\frac{dw_{ij}}{dt} = \frac{\partial}{\partial x_m} \left( \frac{v_t}{\sigma_k} \frac{\partial w_{ij}}{\partial x_m} \right) + P_{ij} - \frac{2}{3} \delta_{ij} \varepsilon - c_1 \frac{\varepsilon}{k} \left( w_{ij} - \frac{2}{3} \delta_{ij} k \right) - c_2 \left( P_{ij} - \frac{2}{3} \delta_{ij} P \right)$$

$$\frac{dk}{dt} = \frac{\partial}{\partial x_i} \left[ \frac{(v + v_t)}{\sigma_k} \frac{\partial k}{\partial x_i} \right] + \Pi - \varepsilon$$

$$\frac{dq_i}{dt} = \frac{\partial}{\partial x_j} \left( \frac{v_t}{\sigma_\vartheta} \frac{\partial q_i}{\partial x_j} \right) + (1 - c_{2T}) P_{iT} - w_{ij} \frac{\partial T}{\partial x_j} - c_{1T} \frac{\varepsilon}{k} q_i$$

$$\frac{d\zeta}{dt} = \frac{\partial}{\partial x_i} \left( \frac{v_t}{\sigma_T} \frac{\partial \zeta}{\partial x_i} \right) - 2q_i \frac{\partial T}{\partial x_i} - c_T \frac{\varepsilon}{k} \zeta$$

$$\frac{d\varepsilon}{dt} = c_\varepsilon \frac{\partial}{\partial x_i} \left( \frac{v_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_i} \right) + c_{\varepsilon 1} \frac{\varepsilon}{k} \left( -w_{ij} \frac{\partial v_i}{\partial x_j} + \beta g_i q_i \right) - c_{\varepsilon 2} \frac{\varepsilon^2}{k}$$

## Poor set of symmetries

$$X = \partial_t \quad X = \partial_T \quad X = \pi(t) \partial_p$$

$$X_i = \chi_i(t) \partial_{x_i} + \dot{\chi}_i(t) \partial_{v_i} - x_i \ddot{\chi}_i(t) \partial_p$$

Linearized form of D'Alembert – Navier – Stokes – Fourier – Fick – Mendeleev set

$$\rho(z) = \rho_o \exp(-z / \Lambda), \quad \Lambda = |d \ln \rho / dz|^{-1} \quad N = \sqrt{g / \Lambda} \quad T_b = 2\pi / N$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \mathbf{v} = 0$$

$$\mathbf{p} = \rho \mathbf{v}$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} = -\nabla p + \rho \mathbf{g} + \mu \Delta \mathbf{v} + \mathbf{f}_e$$

$$\mathbf{j}_T = -\kappa_T \nabla \rho T$$

$$\mathbf{j}_S = -\kappa_S \nabla \rho S$$

$$\frac{\partial \rho T}{\partial t} = \nabla (\kappa_T \nabla \rho T) + Q_i$$

$$\frac{\partial \rho S}{\partial t} = \nabla (\kappa_S \nabla \rho S) + Q_S$$

+ conventional boundary conditions (no-slip, no-flux, attenuation of disturbances at infinity distance from the source)

Compatibility condition for coupled of equations defines the rank and order of the set as well as power of characteristic (dispersion) equation



## Наблюдаемость физических величин

Наблюдаемыми являются только инварианты -- физические величины, для которых существуют законы сохранения.

Движение – преобразование пространства в себя. Преобразование конфигурационного пространства в себя эквивалентно перемещению твердого тела.

Течение жидкости – преобразование физического пространства высокой размерности в себя. Наблюдаемы пространственно-временные интервалы (расстояния и промежутки времени), масса (плотность), объем, импульс, механическая и внутренняя энергия, концентрация веществ, материальные и производные термодинамические величины – коэффициенты оптического и акустического преломления, скорость звука, теплоемкость,....

Жидкая частица не идентифицируема. Скорость жидкости – не наблюдаема.

Основной наблюдаемый параметр течений – Импульс, задающий силовое действие потока на препятствие и расход (объемный, массовый).

### Определение

Течение жидкости – поток импульса, сопровождающийся самосогласованными изменениями термодинамических параметров среды, описывается фундаментальной системой уравнений.

Смысл физических величин определяется связями с другими величинами. Нетождественные преобразования изменяют смысл физических величин, обозначаемых одинаковыми символами.

Landau L.D., Lifshits E.M. Theoretical physics. V.6. Hydrodynamics. M.: Nauka. 1986. 736 p. Part 2. Viscous fluid. Problem 2. To define motions in gravitational wave on a fluid with large viscosity:

$$\rho = \text{const} \quad \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \quad (v \geq \omega \lambda^2)$$

$$\frac{\partial v_x}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial z^2} \right)$$

Linearized set of singular perturbed equations of the 4-th order

$$\frac{\partial v_z}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial z^2} \right) - g$$

$$\sigma_{zz} = -p + 2\mu \frac{\partial v_z}{\partial x} = 0, \quad \sigma_{xz} = \mu \left( \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) = 0 \quad \Big|_{z=\zeta} \Rightarrow \Big|_{\zeta=0} \quad \text{Linearized boundary conditions}$$

$$\left( 2 - i \frac{\omega}{\nu k^2} \right)^2 + \frac{g}{\nu^2 k^3} = 4 \sqrt{1 - i \frac{\omega}{\nu k^2}} \quad \nu k^2 \ll \sqrt{gk} \quad \omega = \pm \sqrt{gk} - 2i \nu k^2$$

$$\nu k^2 \gg \sqrt{gk} \quad \omega = -ig/2\nu k \quad \text{Regular disturbed solution only}$$

## § 79. Sound absorption

## Results

Frequency of waves is a real and positive  $\omega = \text{Re } \omega = \omega_0 + \mathbf{k} \mathbf{U}; \omega_0 > 0$

Theory of PDE with small coefficients at the highest derivatives - singular disturbed equations distinguishes *redics* and *sidics*. *Method of solution is* expansions in series

$$v_i, p, \rho \sim A_i \exp i\Theta = A_i e^{i(\mathbf{k}\mathbf{x} - \omega t)} \quad \omega = \omega(\mathbf{k}), \quad \omega = \omega(\mathbf{k}, A\mathbf{k}) \quad \mathbf{k} = \mathbf{k}_1 + i\mathbf{k}_2$$

*Redics* (regular disturbed components) are solutions with imaginary part proportional to dissipative factors and describe waves, vortices, jets and so on

$$p = p_0 + \varepsilon p_1 + \varepsilon^2 p_2,$$

$$\mathbf{v}, p, \rho \sim A \exp i\Theta, \quad \text{Re } \Theta \gg \text{Im } \Theta, \quad \text{Im } \Theta \sim \nu^\alpha, \quad \alpha > 0$$

*Sidics* (singular disturbed components) are solutions with imaginary part inverse proportional to dissipative factors. They describe thin elongated features, forming sets of boundary layers on contact surfaces and their analogues in a fluid interior (fine structure of stratification, envelopes of vortices, jets, wakes).

$$p = \varepsilon^{-\gamma} \left( p_0 + \varepsilon p_1 + \varepsilon^2 p_2 \right), \quad \gamma > 0$$

$$\mathbf{v}, p, \rho \sim A \exp i\Theta, \quad \Theta \sim \sqrt{i}, \quad \text{Re } \Theta \sim \text{Im } \Theta, \quad \text{Im } \Theta \sim \nu^{-\gamma}, \quad \gamma > 0$$

There are at least **two** distinguished singular on viscosity components, one is Stokes' type and second one is before unknown internal type.



Complete solution of LINEARISED FUNDAMENTAL SET OF NIN-HOMEGENEOUS FLUID MECHANICS EQUATIONS SYSTEM is based on summarizing of dispersion equation ALL ROOTS

$$A = \sum_j \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} a_j(k_x, k_y) \exp\left(i\left(k_{zj}(k_x, k_y)z + k_x x + k_y y - \omega t\right)\right) dk_x dk_y$$

dispersion equation for basic system is factorized equation of the tenth order

$$D_v(k, \omega) \cdot F(k, \omega) = 0$$

Factorized singular disturbed roots correspond to extended thin flow components (Stokes type)

$$D_v(k, \omega) = -i\omega + \nu k^2 \quad D_{\kappa_T}(k, \omega) = -i\omega + \kappa_T k^2 \quad D_{\kappa_S}(k, \omega) = -i\omega + \kappa_S k^2$$

Location of wave field and singular disturbed solutions is defined by Geometry of the Problem

$$F(k, \omega) = -D_v(k, \omega) D_{\kappa_T}(k, \omega) D_{\kappa_S}(k, \omega) \left( k^2 + i \frac{k_z (\Lambda_T + \Lambda_S)}{\Lambda_T \Lambda_S} \right) +$$

$$+ D_{\kappa_T}(k, \omega) \left( \frac{\omega k_z}{\Lambda_S} D_v(k, \omega) - N_S^2 k_{\perp}^2 \right) + D_{\kappa_S}(k, \omega) \left( \frac{\omega k_z}{\Lambda_T} D_v(k, \omega) - N_T^2 k_{\perp}^2 \right)$$

$$k^2 = k_x^2 + k_y^2 + k_z^2 \quad k_{\perp}^2 = k_x^2 + k_y^2$$

## Thickness of Stokes and internal fine flow components

$$\delta_{St} = \delta_N \sqrt{2/\sin \Theta_\omega},$$

$$\delta_i = \delta_N \sqrt{\frac{2 \sin \Theta_\omega}{\left(1 - \frac{g\Lambda}{c^2}\right) \sin^2 \Theta - \sin^2 \Theta_\omega}} \approx \delta_N \sqrt{\frac{2 \sin \Theta_\omega}{\sin^2 \Theta - \sin^2 \Theta_\omega}}$$

$$\delta_N = \sqrt{\nu/N}, \quad \Theta_\omega = \arcsin(\omega/N)$$

Approximation of nonhomogeneous fluid: is underdetermined insolvable set with merged singular perturbed flow components (SPC, sidics).

Two different 3D SPC become identical and are merged into the unique one, twice degenerated sidics

$$k^2 \left( \omega + i \nu k^2 \right)^2 = 0$$

Double sidic looks like double boundary layer (transversal divergence free isobaric motion)  $\left( \omega + i \nu k^2 \right)^2 = 0$

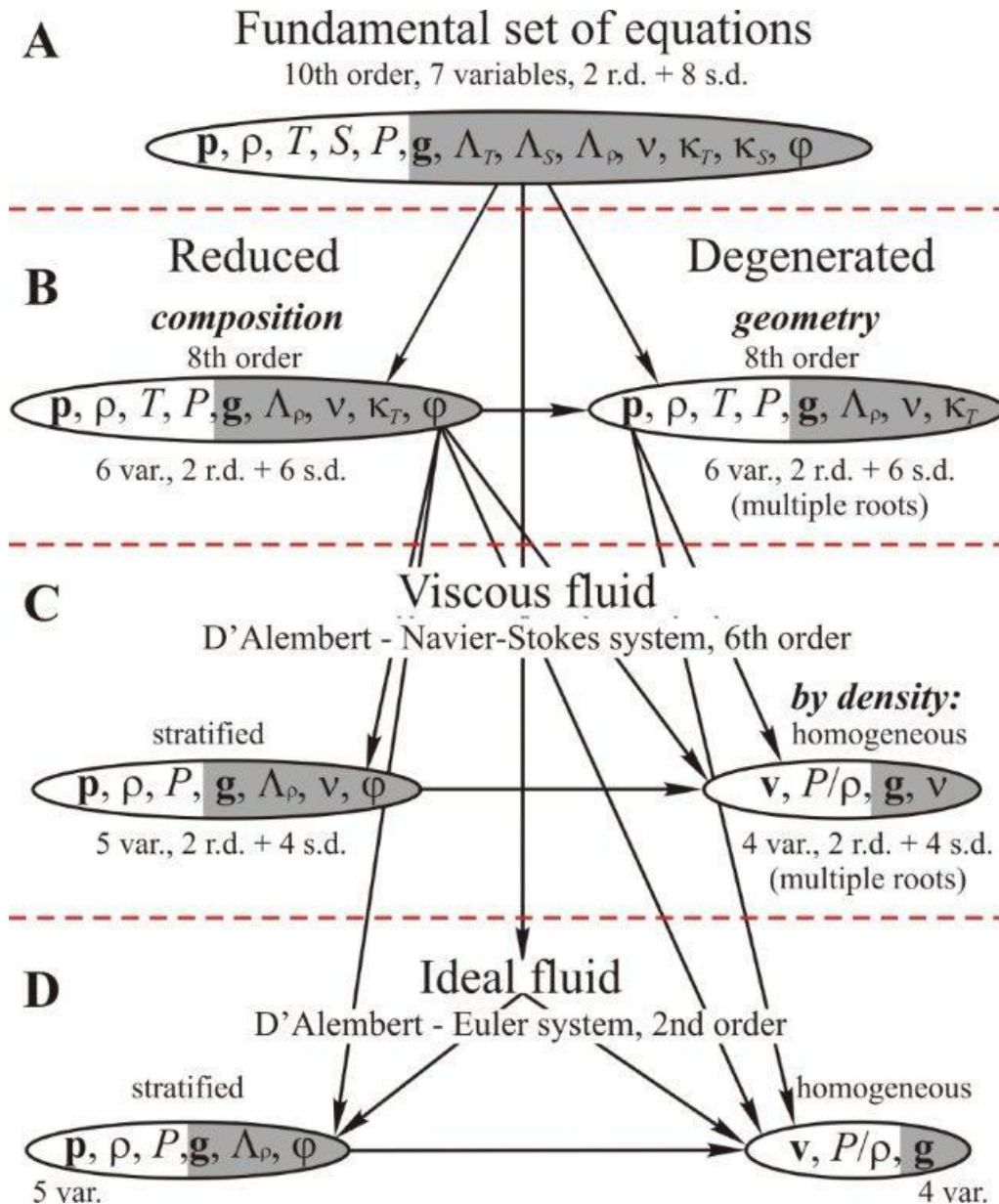
$$k_2 = -k_3 = \sqrt{\frac{i\omega}{\nu} - k_\xi^2 - k_\eta^2} \quad \mathbf{k} \cdot \mathbf{u} = 0 \quad \delta_\nu = \sqrt{2\nu/\omega}$$

$v_x = 0$ ,  $v_y$ ,  $v_z$  -- are independent components of velocity

Large scale periodic longitudinal motion  $k^2 = 0$ ,  $k_1 = i\sqrt{k_\xi^2 + k_\eta^2}$ ,  $\mathbf{v} = \mathbf{k} \sqrt{\frac{\tilde{P}_k}{\rho}}$ ,  $(\mathbf{v} \parallel \mathbf{k})$



## Hierarchy of fundamental fluid mechanics models



## A few Large scale and rich family of Fine Intrinsic Scales characterizing set of singular disturbed equations and their solutions

Density  $\rho = \rho_0 \exp(-z/\Lambda) \quad \Lambda = (d \ln \rho / dz)^{-1},$

Large (Regular) length scales

internal wave length  $\lambda = UT_b$  Size of an obstacle  $d \quad L_v = \sqrt[3]{gv} / N$

Fine (Singular) length scales:

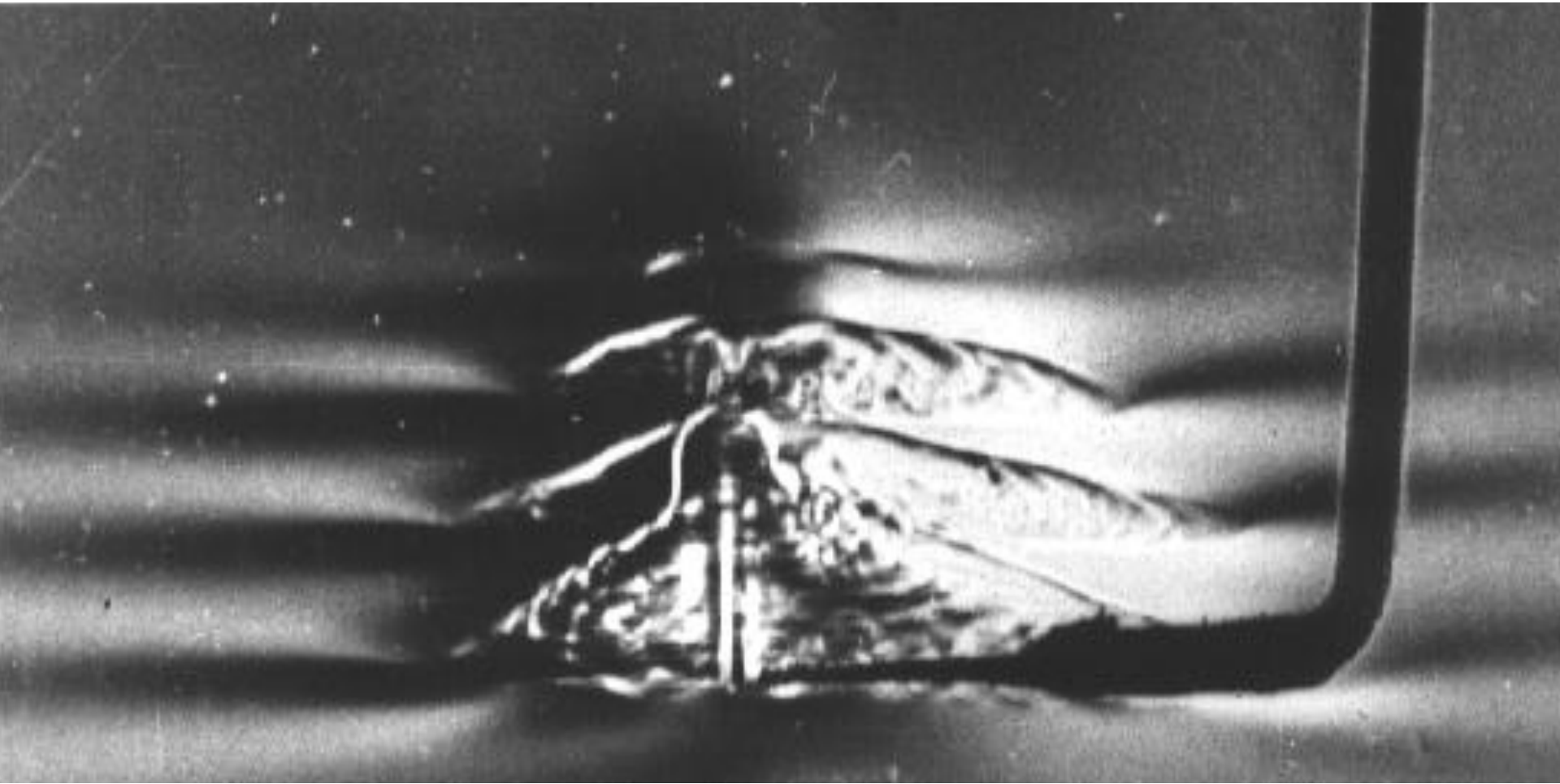
$$\delta_N^{(v)} = \sqrt{2v / N}, \quad \delta_N^{(\kappa_S)} = \sqrt{2\kappa_S / N}, \quad \delta_U^{(v)} = v / U \quad \delta_U^{(\kappa_S)} = \kappa_S / U$$

$$\Lambda \gg d \gg \delta_v, \quad \Lambda \gg L_v \gg \delta_v \quad \delta_v \gg \delta_\rho$$

**Dimensionless parameters – ratios of intrinsic scales: Re, Fr, C, Ar, St, (Pe, Sc)**

$$\text{Re} = d / \delta_U^{(v)} = U_0 d / v, \quad \text{Fr} = \lambda / 2\pi d = U_0 / Nd \quad C_\Lambda = \Lambda / d$$

$$C_\Lambda = \frac{\Lambda}{d}; \quad R_\rho^{(1)} = \frac{\alpha(T - T_0)}{\beta(S - S_0)}; \quad R_\rho^{(2)} = \frac{\alpha(\partial T / \partial \mathbf{n})}{\beta(\partial S / \partial \mathbf{n})}; \quad R_\rho^{(3)} = \frac{\kappa_T \cdot \alpha(\partial T / \partial \mathbf{n})}{\kappa_S \cdot \beta(\partial S / \partial \mathbf{n})}$$



A “New-Year tree” of Double-Diffusive Convection  
above point source heat in a continuously stratified fluid



## Заключение

- Крупномасштабные компоненты течений сосуществуют с богатым семейством тонкоструктурных компонент, формирующих семейства высокоградиентных поверхностей вблизи границ и в толще жидкостей;
- Тонкоструктурные компоненты определяют геометрию областей диссипации механической энергии, генерации завихренности и картину переноса вещества;
- Течения жидкости – перенос импульса, сопровождающийся самосогласованными изменениями термодинамических параметров, который характеризуется группой движений в расширенном физическом пространстве задачи ;
- Полная система фундаментальных уравнений для плотности, импульса, внутренней энергии, концентрации совместно с уравнением состояния и граничными условиями образует корректно поставленный, самосогласованный и разрешимый базис моделирования, предсказания и управления течениями;
- Новые экспериментальные и вычислительные инструменты, методики и коды должны быть разработаны для идентификации и крупно – и тонкоструктурных компонент и определения сценариев эволюции картины течений.

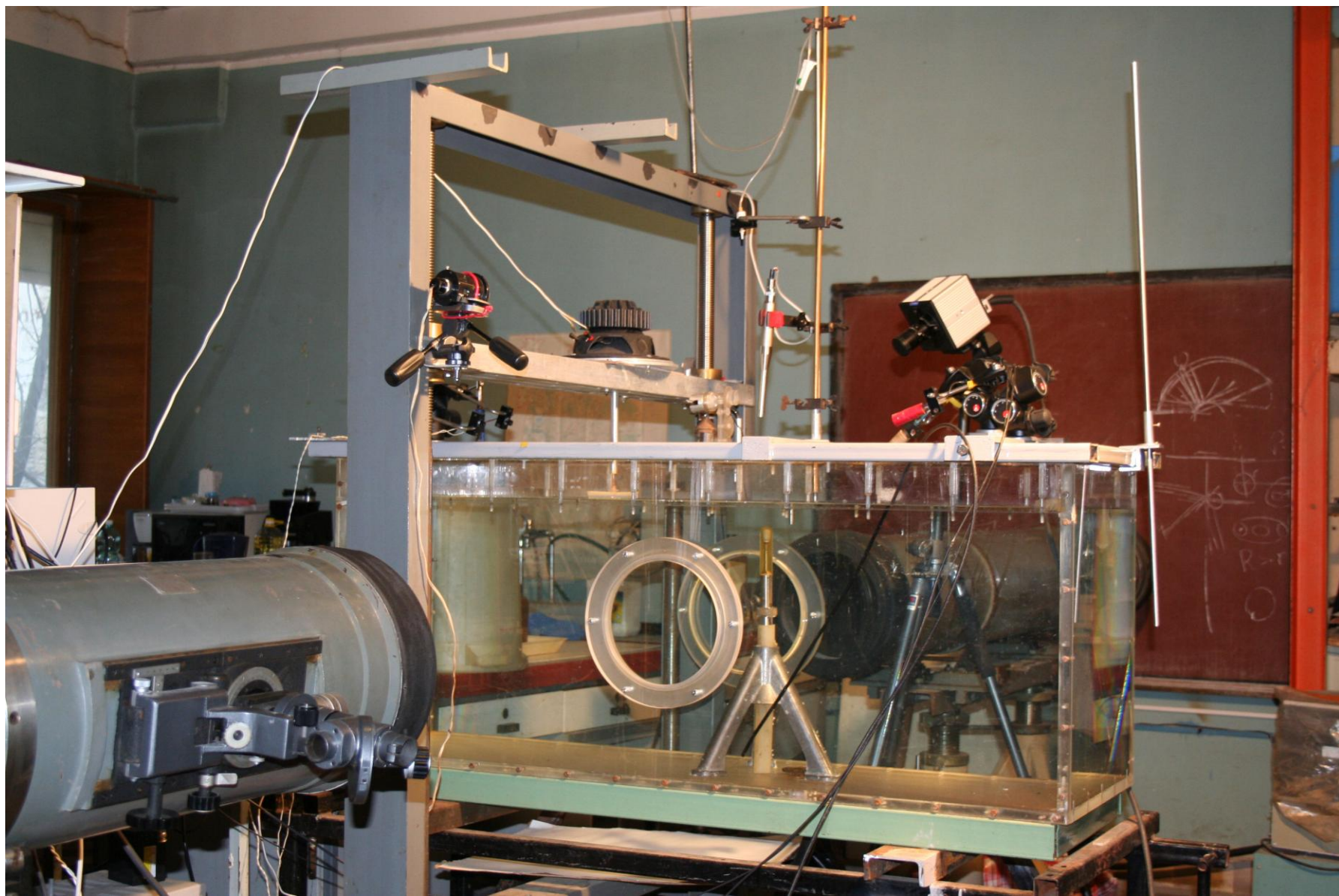
## Приглашение

Для измерения и расчета физических величин, входящих в систему фундаментальных уравнений: плотности среды, импульса и энергии механического движения, давления, концентрации ....необходимы полевые (дистанционные) и контактные инструменты высокой точности и высокого пространственного разрешения, методики контроля точности в условиях реального эксперимента.

Контроль точности инструментов и методов необходимо проводить в в ходе лабораторных и численных экспериментов на моделях изучаемых процессов.

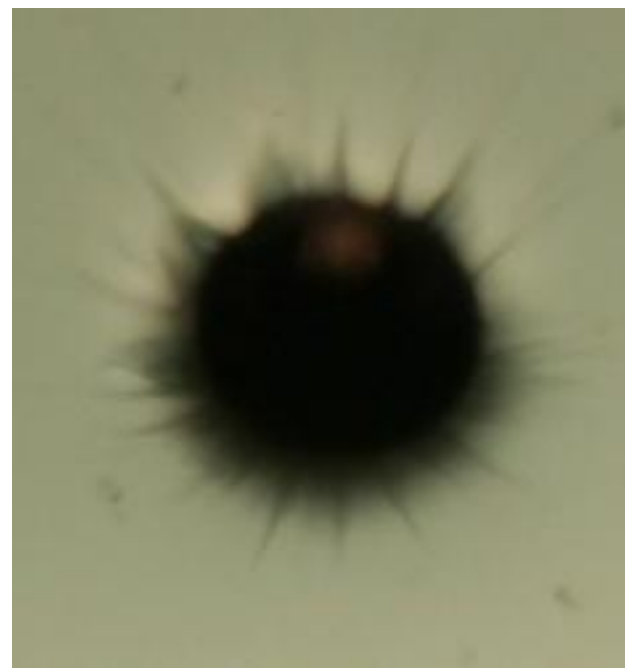
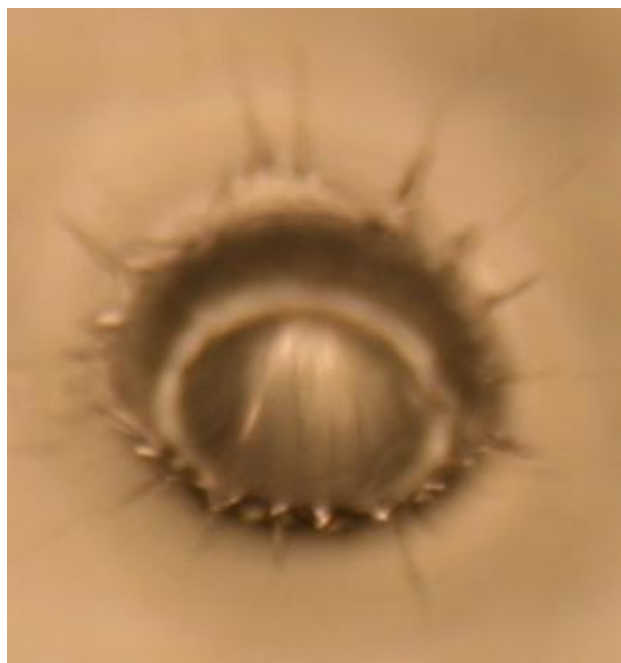
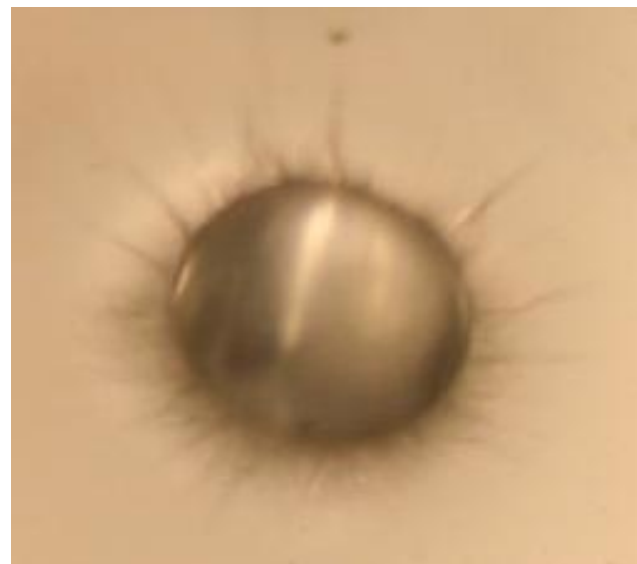
Задачи сложные,  
приглашаю к сотрудничеству во всех формах.

Программа РФФИ «Мобильность молодых ученых»



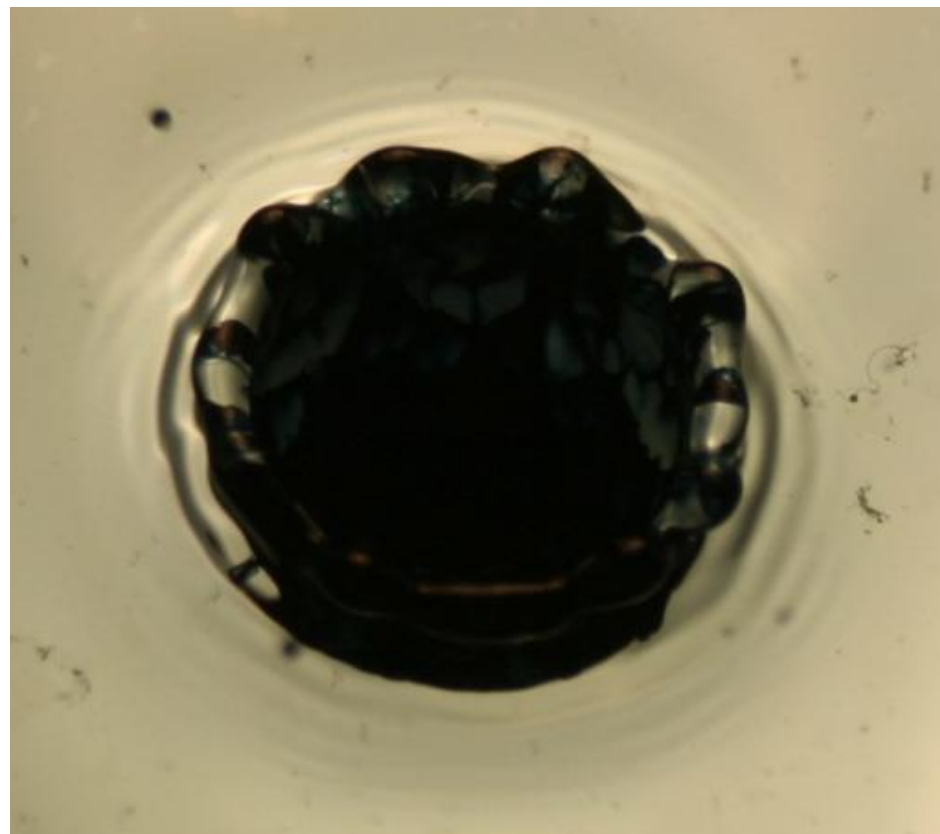
Modern experimental facility with hydrophone, microphone and high speed video camera





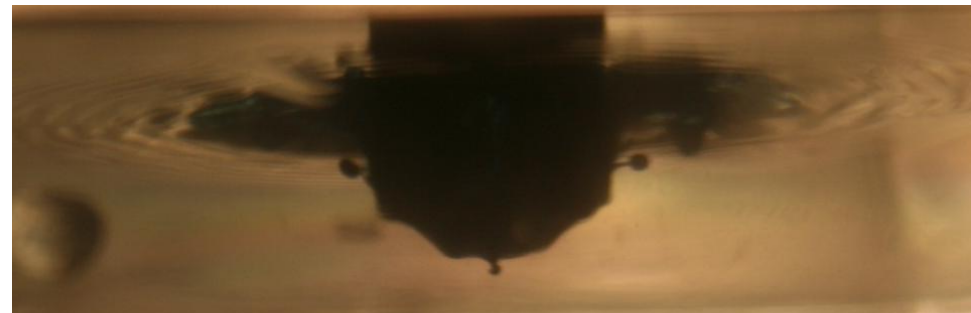
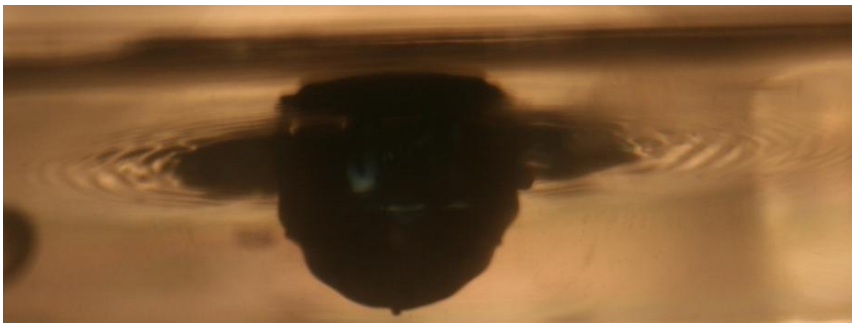
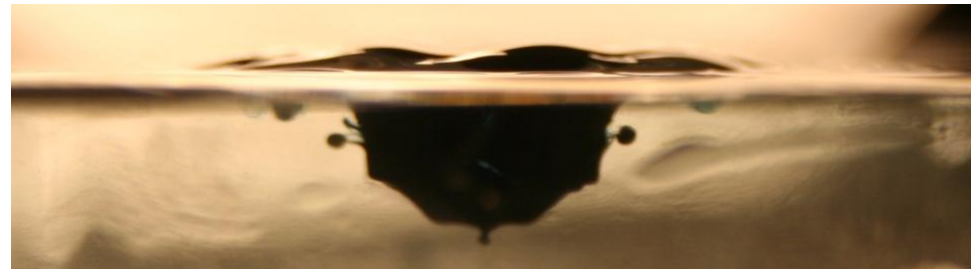
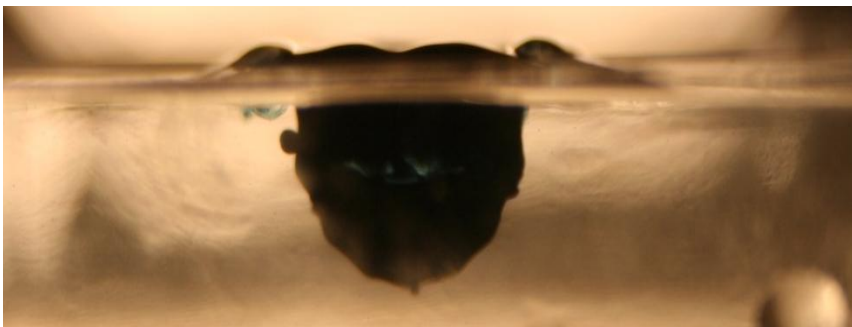
Initial contact of the drop with water surface: droplets, streamers, thin circular sheet

## Transport of matter form drop into water and in air



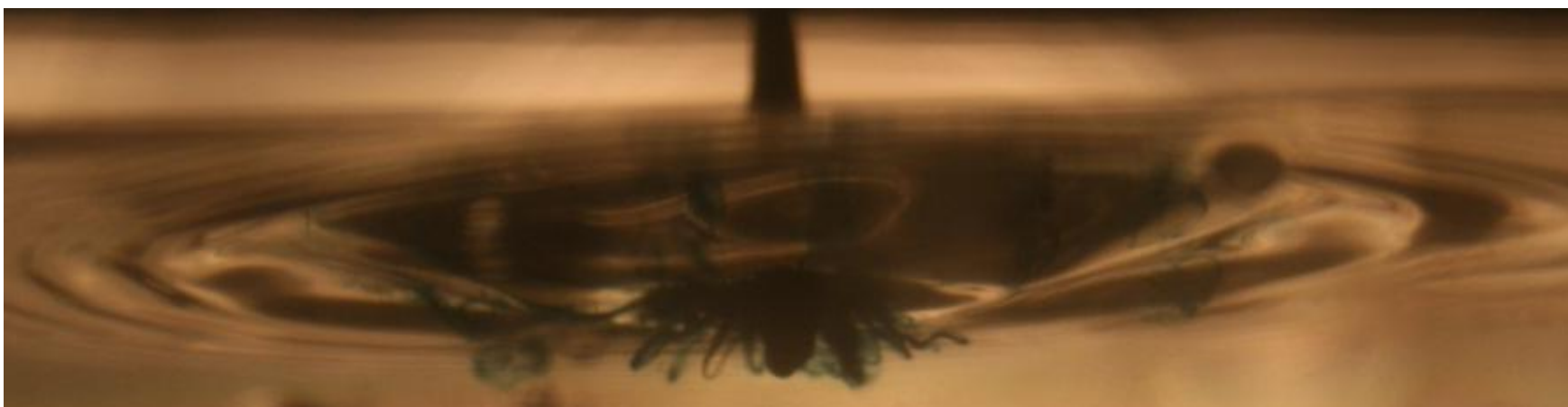
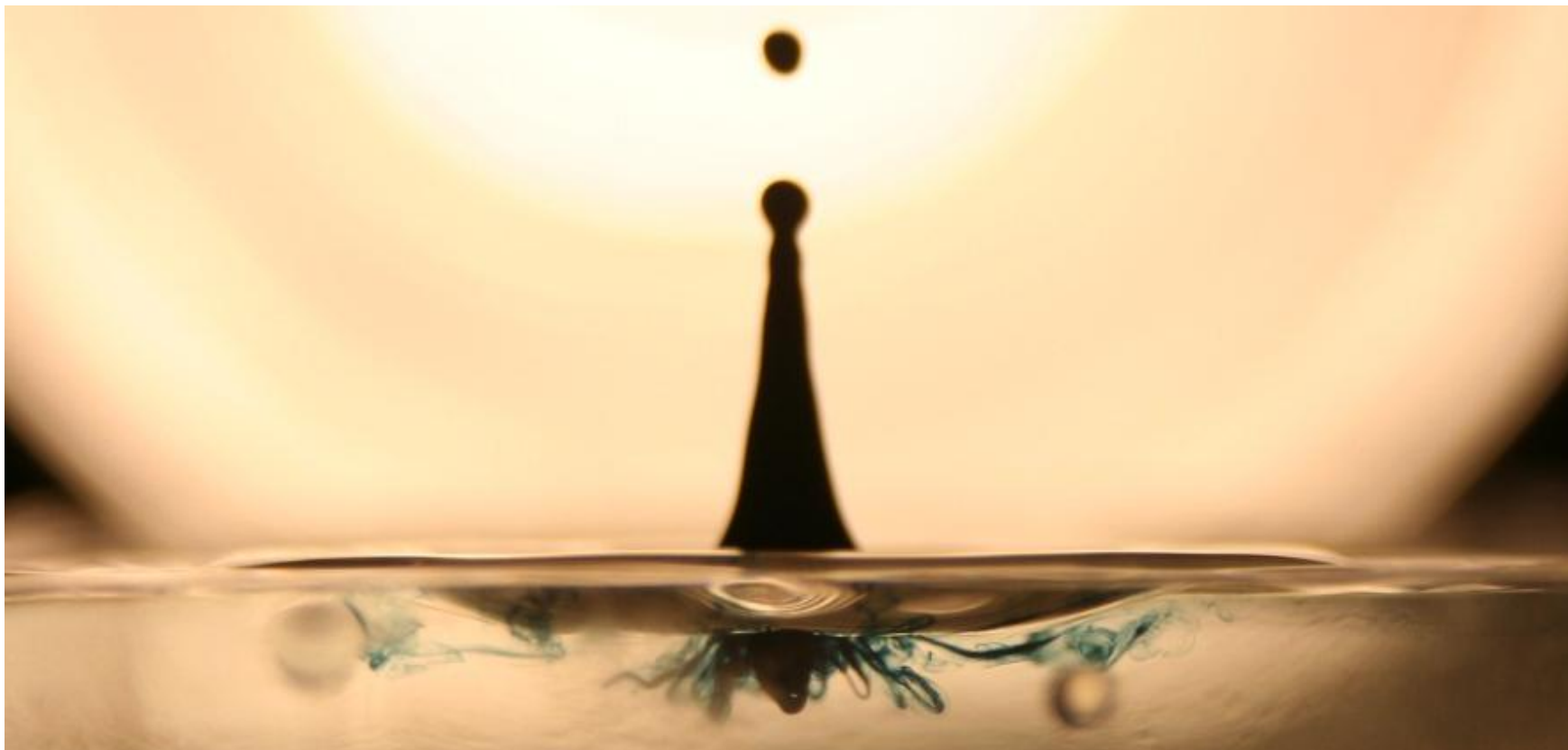
Trough and ring with edge shevron produced by a dye drop

Burst of initial contact :  
transport of matter from initial drop by small droplets into water and in air

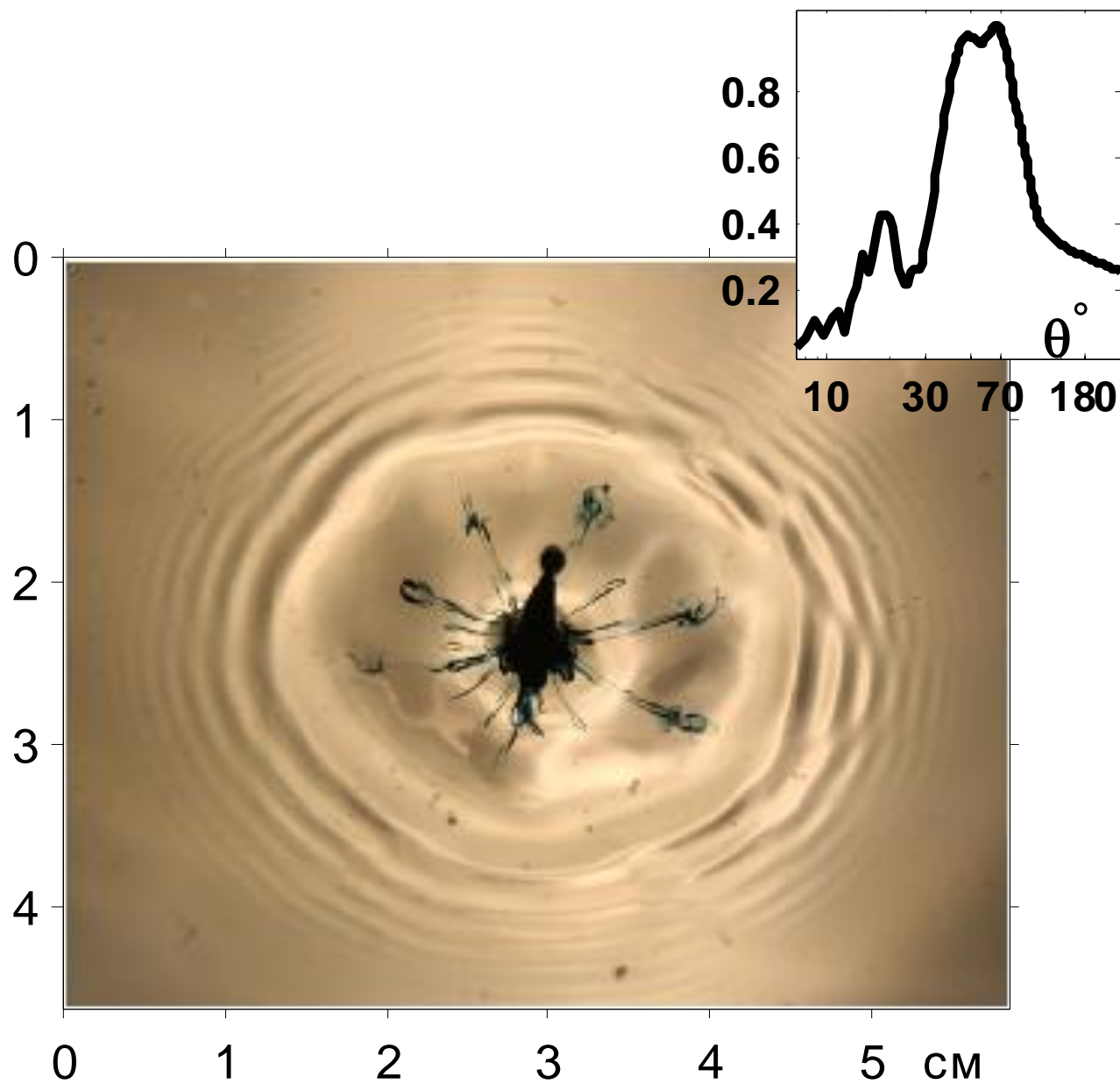


Formation of cavern with percolating jets  
with the head droplet (leading vortex ring?)

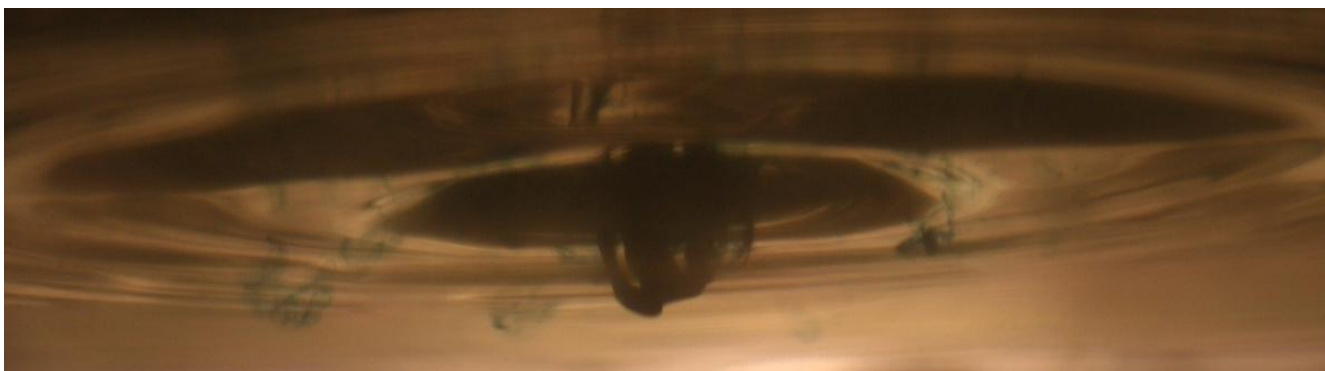
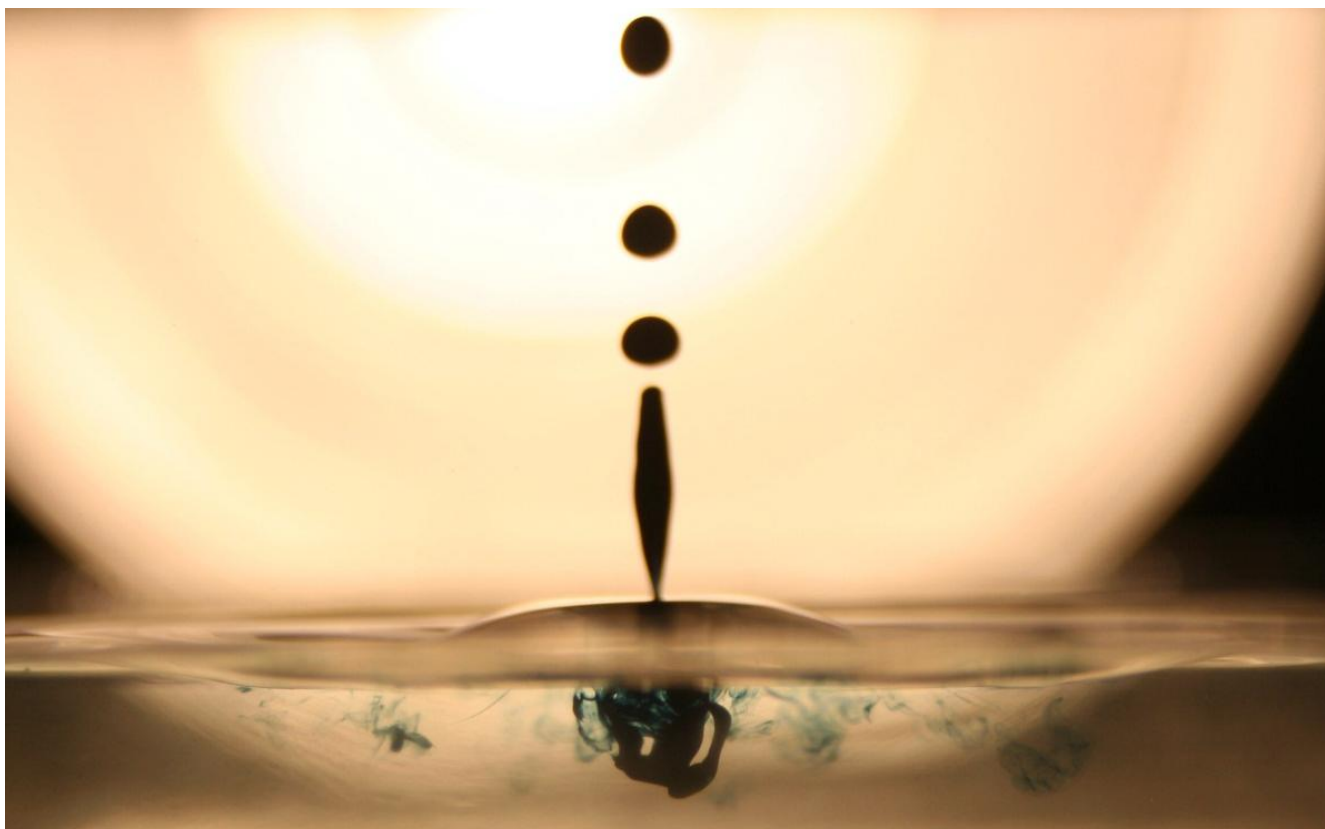




Ring capillary waves, splash and a set of radial double microjets with double filaments

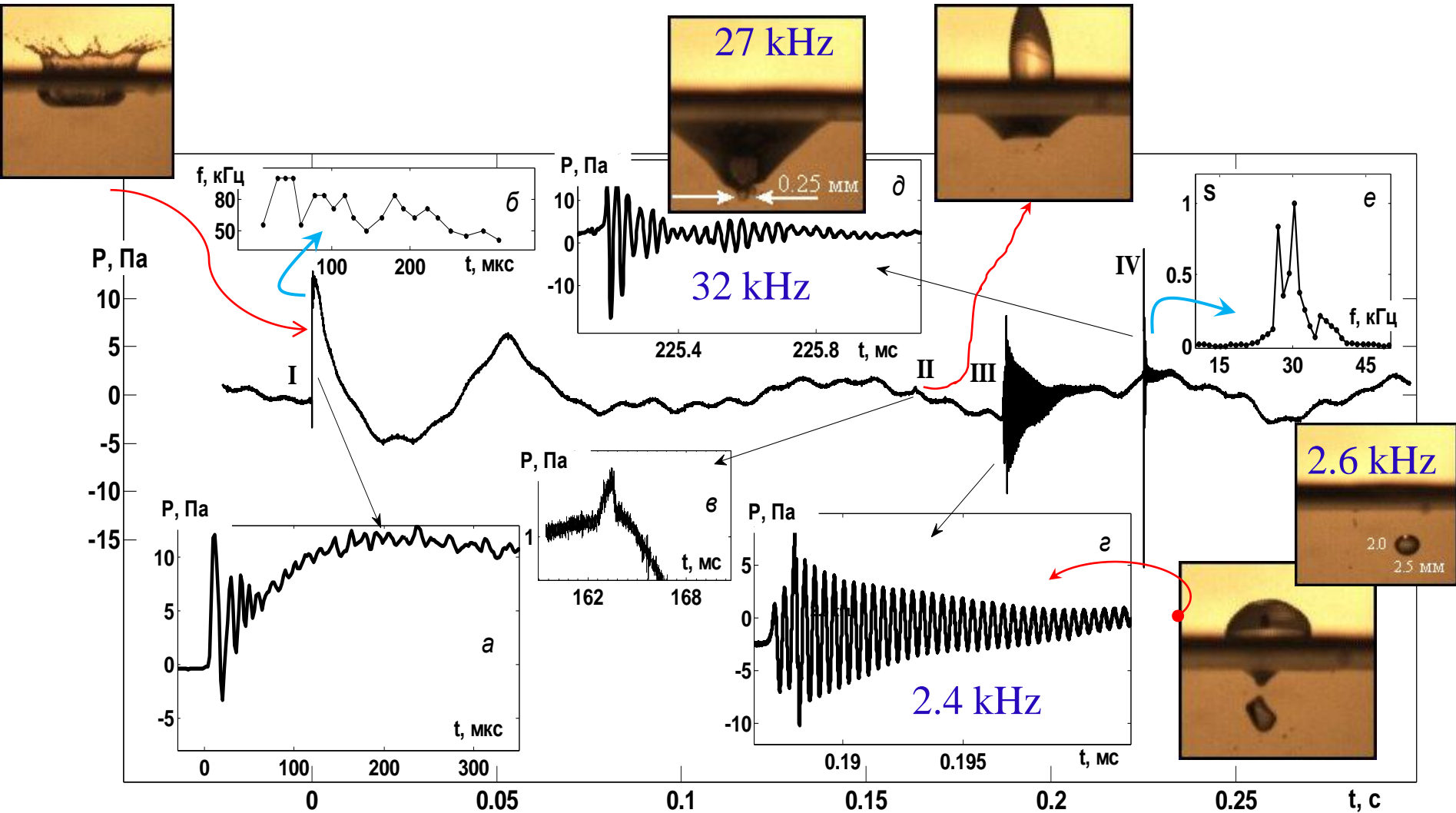


Ring capillary waves, splash and a set of radial double microjets with double tail filaments

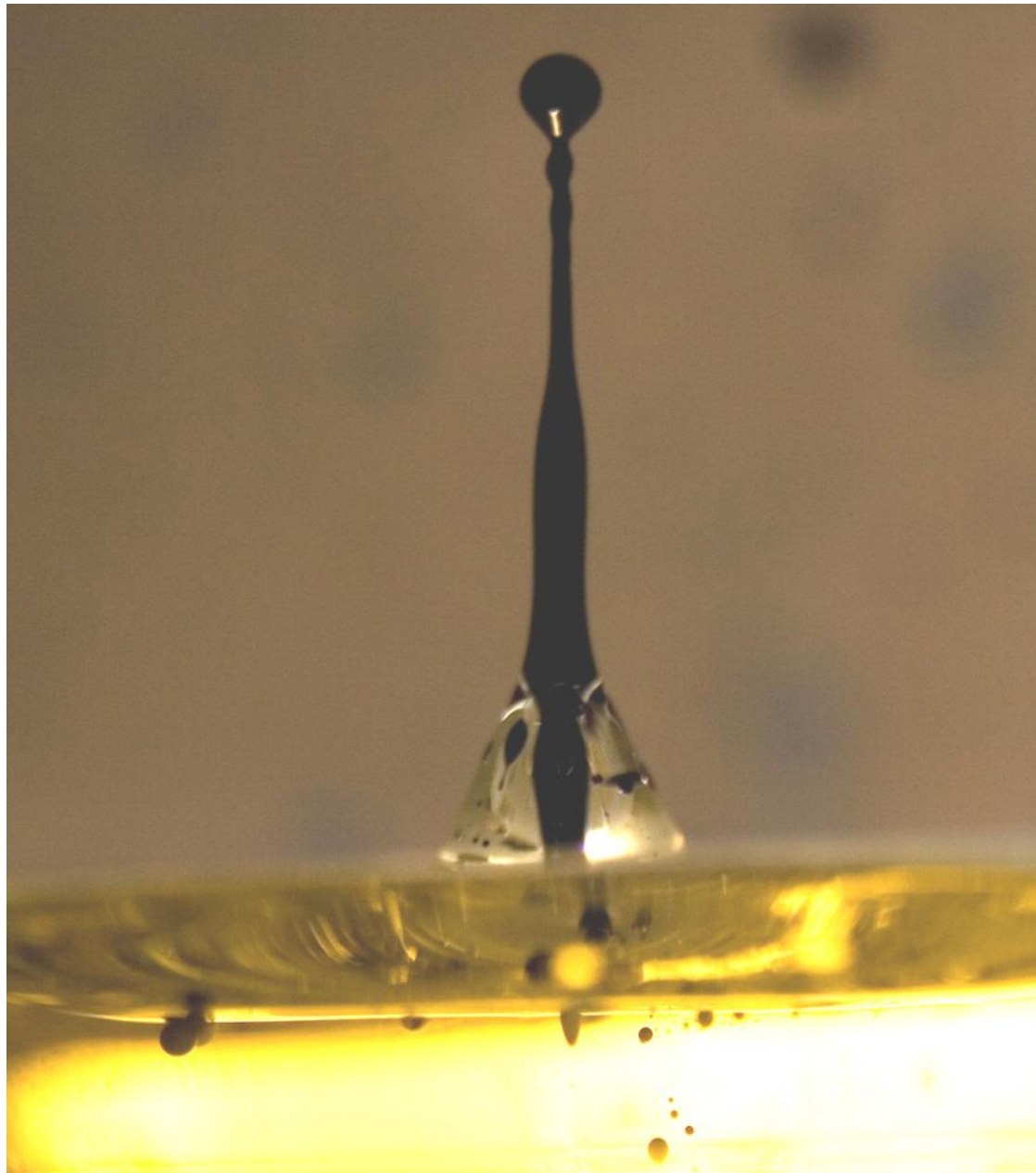


Flash and wakes of thin jets inside water layer produced by a drop of a ink solution

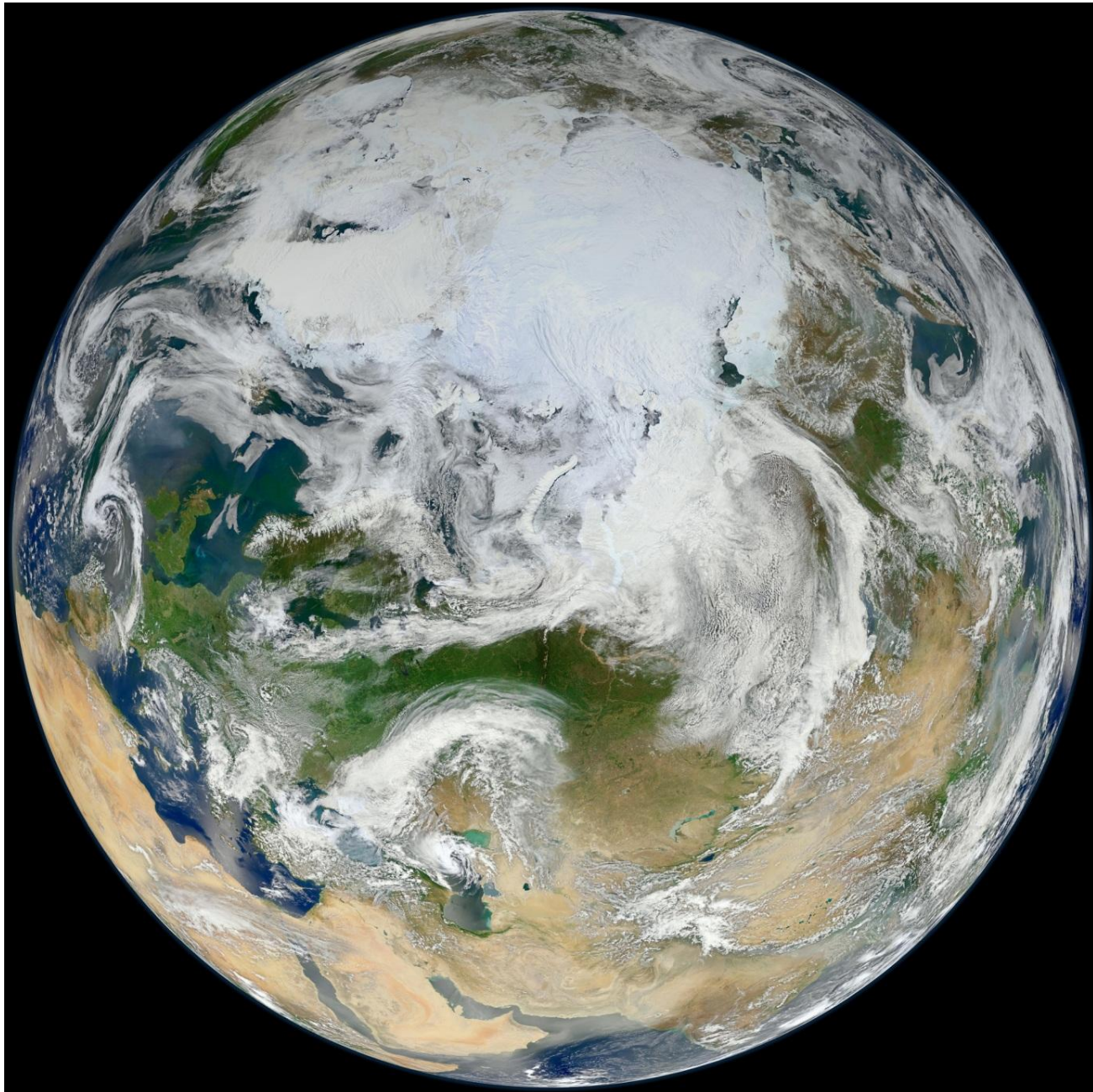
## Emitting of sound and fluid fragmentation induced by fallen drop







Splash of ink drop in sunflower oil



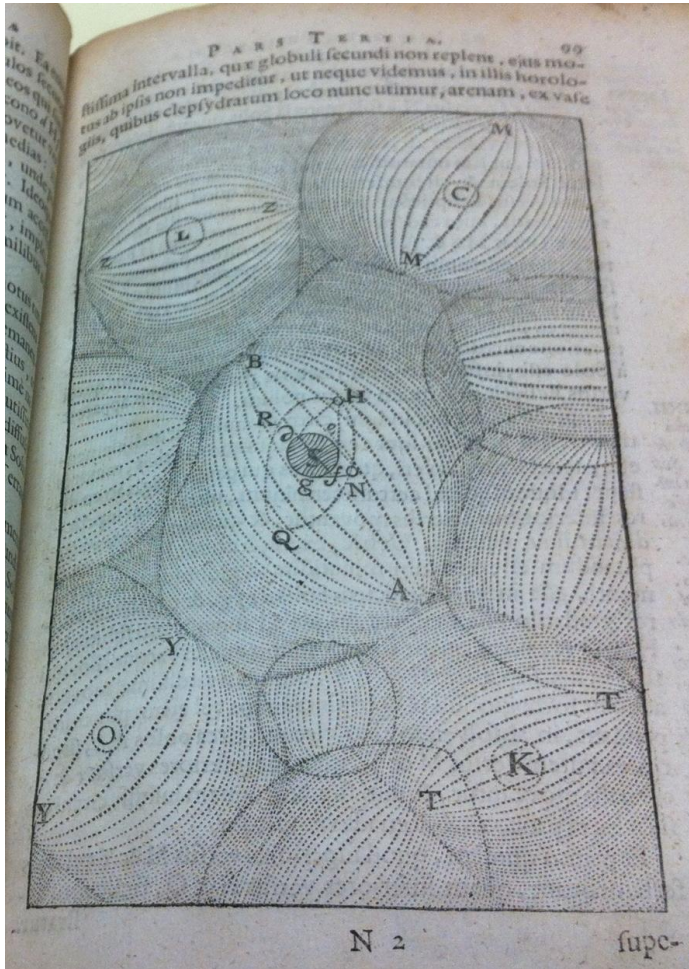




Thank you very much for your kind attention



...Бог разделил всю материю, заключенную в пространстве AEI (см. рис. 8), на огромное число мелких частей, движущихся не только каждая вокруг собственного центра, но и все вместе вокруг центра S, а все частицы в пространстве AEV двигались подобным же образом вокруг центра FI так же вращались и остальные. Частицы образовали таким путем столько вихрей, сколько ныне существует в мире светил (впредь я буду употреблять слово «вихрь» для обозначения всей материи, вращающейся таким образом вокруг каждого из подобных центров).



R. Descartes Principia Philosophiae 1644.

## Light Echo of V838 Monoceros Star flash





Theoretical Fluid Mechanics is based on concepts of

Numbers (real, complex, quaternion, hypercomplex...),

Calculation (binary, decimal ...),

Sets (manifold), metric space (Euclidian space)  $\mathbf{R}^3$

Motion (is continuous transformation 3D Euclidean space into itself with parameter *time t*)

Continuum medium (deformable)

Physical quantities characterizing media and their change

Fluid Flows – must be defined

Basic equations + initial and boundary conditions (must be specified)

OBSERVABILITY (independent measurements of Physical

Quantities by different methods with guarantee estimation of errors)

## B. The equation of motion.

### I. Kinematics and dynamics of fluid motion.

3. **Kinematical preliminaries.** Fluid flow is an intuitive physical notion which is represented mathematically by a *continuous transformation* of three-dimensional Euclidean space into itself. The parameter  $t$  describing the transformation is identified with the time, and we may suppose its range to be  $-\infty < t < +\infty$ , where  $t=0$  is an arbitrary initial instant.

In order to describe the transformation analytically let us introduce a *fixed rectangular coordinate system*  $(x^1, x^2, x^3)$ . We refer to the coordinate triple  $(x^1, x^2, x^3)$  as the *position* and denote it by  $\mathbf{x}$ . Now consider a typical point or particle  $P$  moving with the fluid. At time  $t=0$  let it occupy the position  $\mathbf{X} = (X^1, X^2, X^3)$  and at time  $t$  suppose it has moved to the position  $\mathbf{x} = (x^1, x^2, x^3)$ . Then  $\mathbf{x}$  is determined as a function of  $\mathbf{X}$  and  $t$ , and the flow may be represented by the transformation

$$\mathbf{x} = \boldsymbol{\varphi}(\mathbf{X}, t) \quad (\text{or } x^i = \varphi^i(\mathbf{X}, t)). \quad (3.1)$$

Mathematical Principles of Classical Fluid Mechanics.

By  
JAMES SERRIN.  
With 17 Figures.

ENCYCLOPEDIA OF PHYSICS

EDITED BY  
S. FLÜGGE

VOLUME VIII/1  
FLUID DYNAMICS I

CO-EDITOR  
C. TRUESDELL

WITH 186 FIGURES

## Definitions of "Motion"

Configuration space: vector metric, Dim = 3  $\mathbb{R}^3$

internal and external composition laws, associativity: for any vectors, commutativity

Evolutional space-time, Dim = 4

$\mathbb{R}^3, ct$

Velocity (or phase) space, Dim = 4

$\mathbf{V}, t (\mathbf{k}, \mathbf{w})$

Expanded physical space, Dim = 8

$\mathbb{R}^3 \mathbb{U}_3, ct, M$

**Motion** is transformation of vector metric space into itself saving the distance and mutual locations of objects and is decomposed on translation and rotation

$$\delta \mathbf{r} = \mathbf{U}_t dt + \mathbf{r} \times \mathbf{U}_\varphi dt$$

-- condition of External composition means that product of any vector on real number belongs to the same set

Parameters (Invariants) of motions

$$\mathbf{p} = M\mathbf{U} \quad E = \frac{\mathbf{p} \cdot \mathbf{U}}{2} \quad \mathbf{M} = \mathbf{r} \times \mathbf{p} \quad \frac{\mathbf{p}}{M} = \mathbf{V} \text{ (velocity)} = \mathbf{U} = \frac{d\mathbf{X}}{dt} = \mathbf{c} \text{ (celerity)}$$



# Theory of flows is based on conception of a *Number, Set, Space* and *Continuous Medium*

The set - a collection of specific and distinct objects together, conceivable as a whole.

Concept of a vector space.

Concept of a vector space.

Nonempty set for which the operations of addition and multiplication by real numbers is a **vector** or a **linear** space if the axioms hold

Laws of

- Internal composition : sum of elements belong to the set;

– *associativity*

– *commutativity*

*As well as rules:*

– algebraic addition of vectors

-- **External composition:** for any **a** and any actual number  $R$  element  $(R \mathbf{a})$  also belongs;

– *associativity* of factors product

– multiplication by a unit  $1 \mathbf{a} = \mathbf{a}$ ;

– distributivity: for any **a**, **b**, and real numbers  $M, N$   $(\mathbf{a} + \mathbf{b}) = M \mathbf{a} + M \mathbf{b}$ ,

$(M + N) \mathbf{a} = M \mathbf{a} + N \mathbf{a}$

## Flows of deformable Continuum medium

Deformable continuous media is submerged into configuration space:  $\mathbb{R}^3$

**Cauchy-Helmholtz Decomposition of fluid flows:**  
**translation, rotation and deformation of a fluid particle**

$$v_i(r_r + \delta r_k) = v_i(r_k) + \varepsilon_{ijk} \Omega_j U_k dt + \frac{\partial v_i}{\partial x_l} U_l dt$$

Shear operator connect elementary constitutions of motion and destroys independence of operators of shift and rotation.

Additional parameters must be introduced to distinguish metric space with “rigid” motion and flows of deformable media (density, thermodynamic parameters,...).

**Motion** is transformation of vector metric space into itself conserving distance: decomposition on translation and rotation

$$\delta \mathbf{r} = \mathbf{U}_t dt + \mathbf{r} \times \mathbf{U}_\varphi dt$$

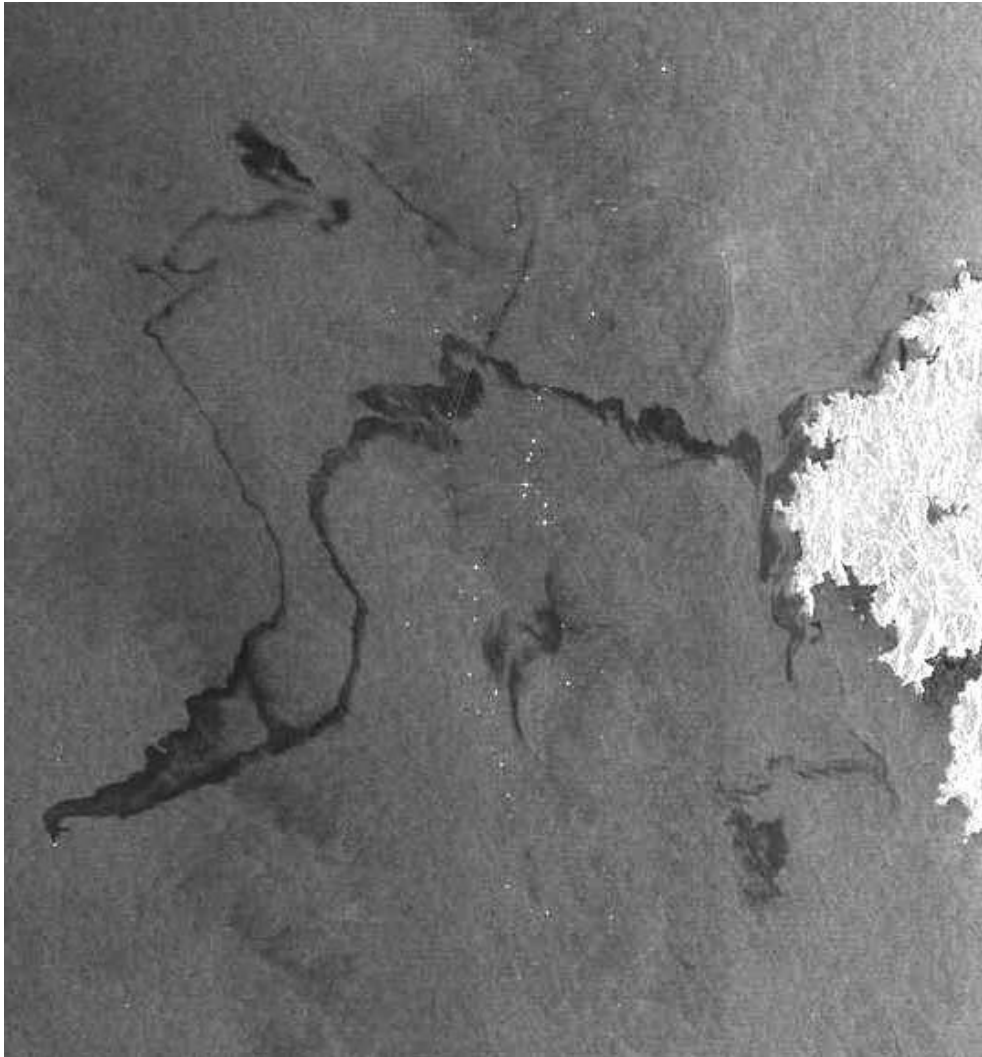
Parameters (Invariants) of motions

$$\mathbf{p} = M\mathbf{U} \quad E = \frac{\mathbf{p} \cdot \mathbf{U}}{2} \quad \mathbf{M} = \mathbf{r} \times \mathbf{p}$$

$$\frac{\mathbf{p}}{\rho} = \mathbf{V} \text{ (velocity)} \neq \mathbf{U} = \frac{d\mathbf{X}}{dt} = \mathbf{c} \text{ (celerity)}$$

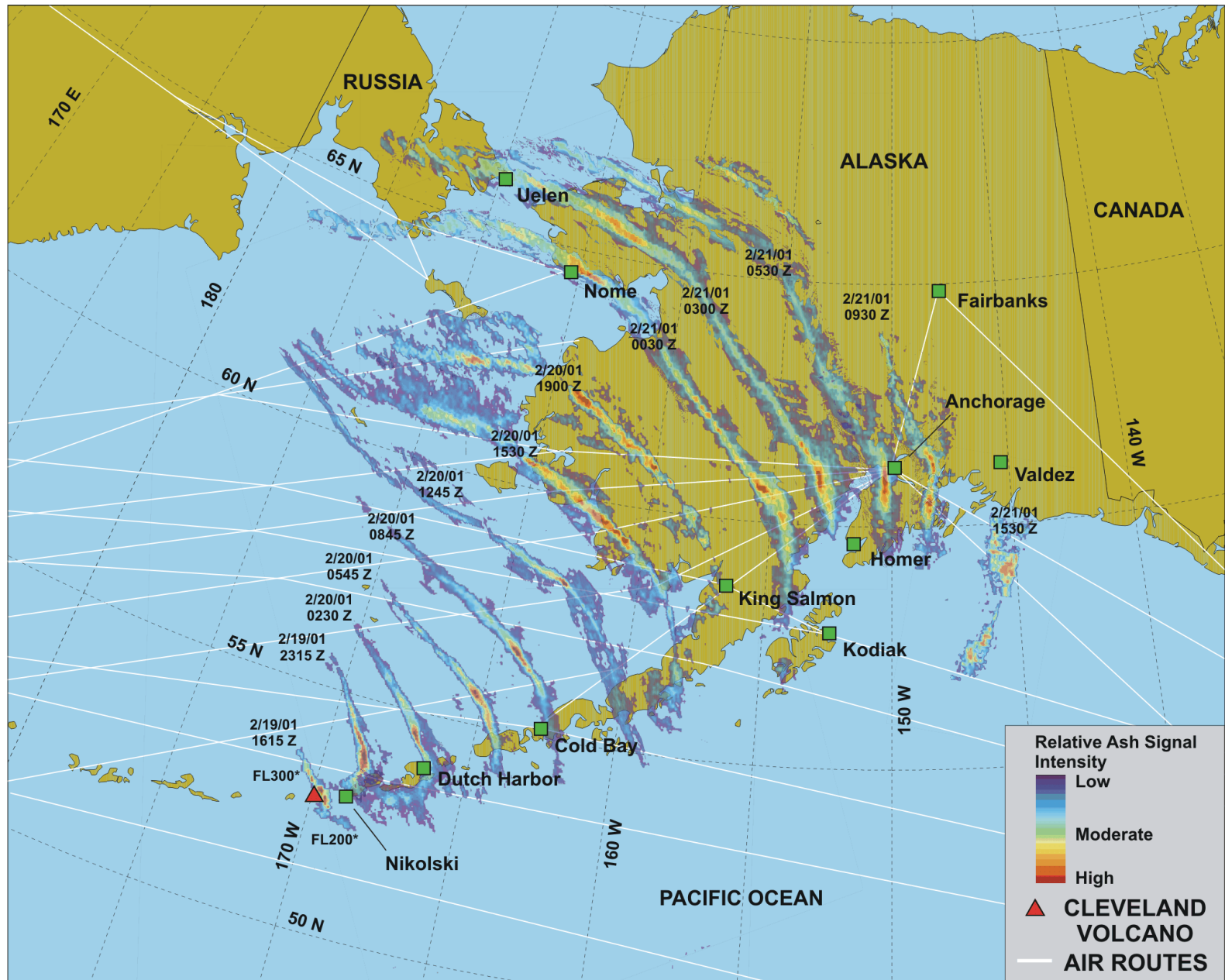
$\rho$

$$\mathbf{p} = \rho \mathbf{U} \quad E = \frac{\mathbf{p} \cdot \mathbf{U}}{2}$$

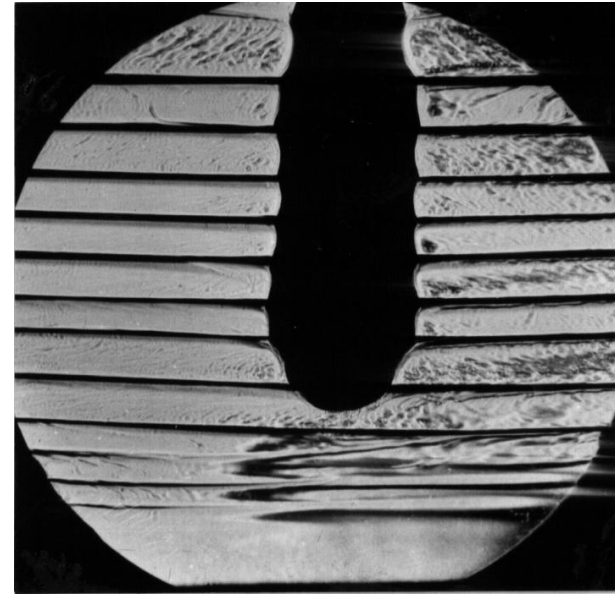
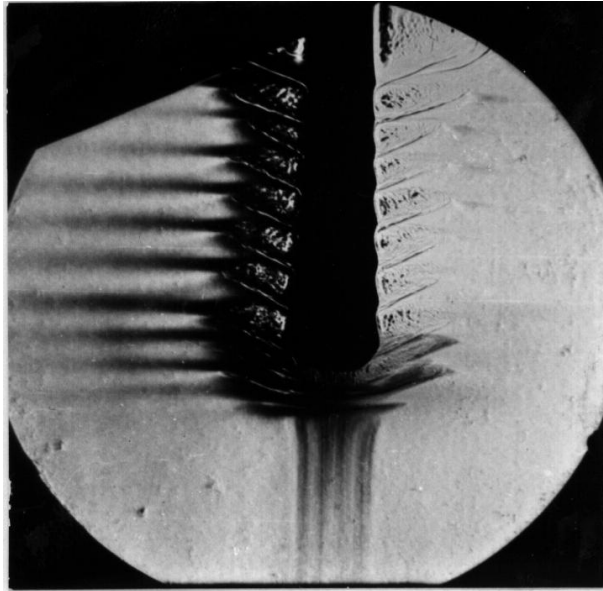


“Prestige” oil spills (SAR image on November 17, 2002)

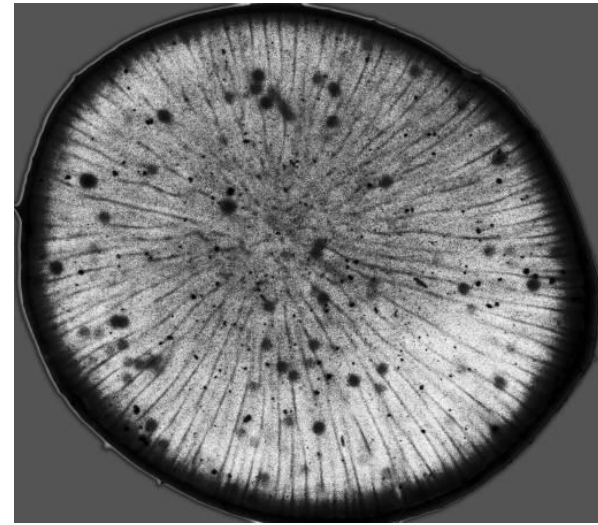
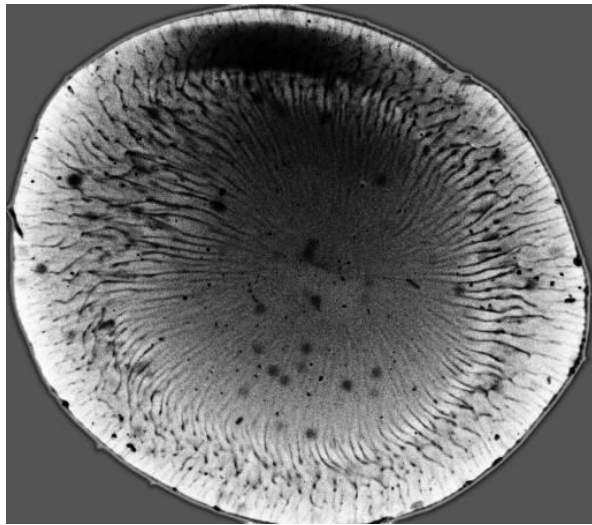




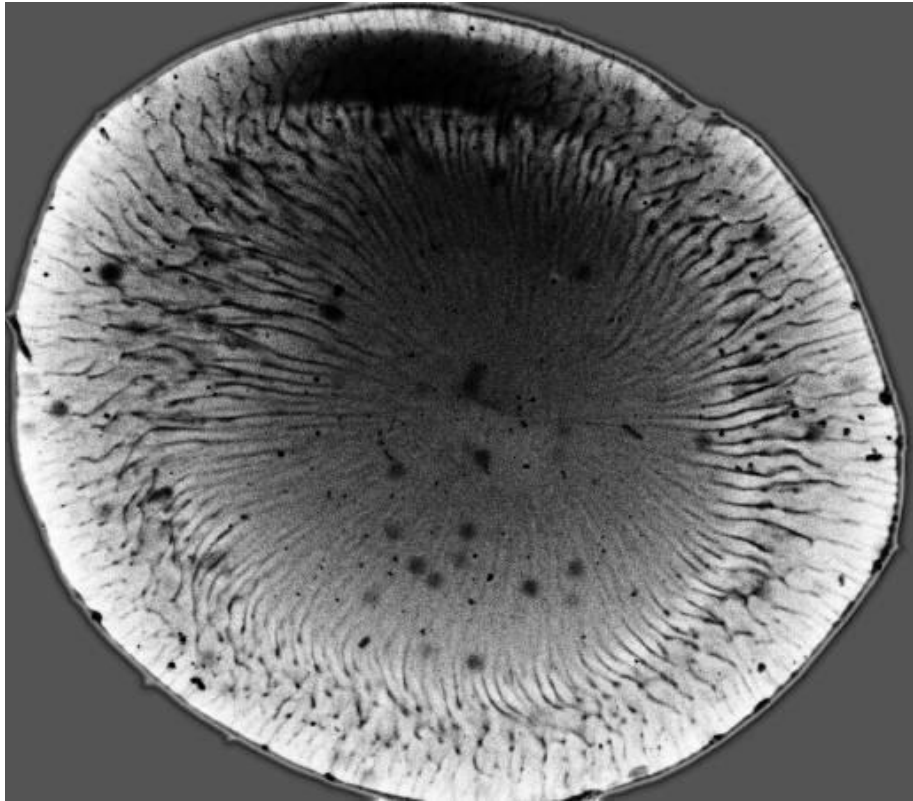
Ash plume transport after Cleveland volcano eruption



Freezing of ice on cylindrical cooler in a stratified brine



Radial structure in drying drop of quartz nanoparticles  $s$  in vodka  $D = 0.5$  cm

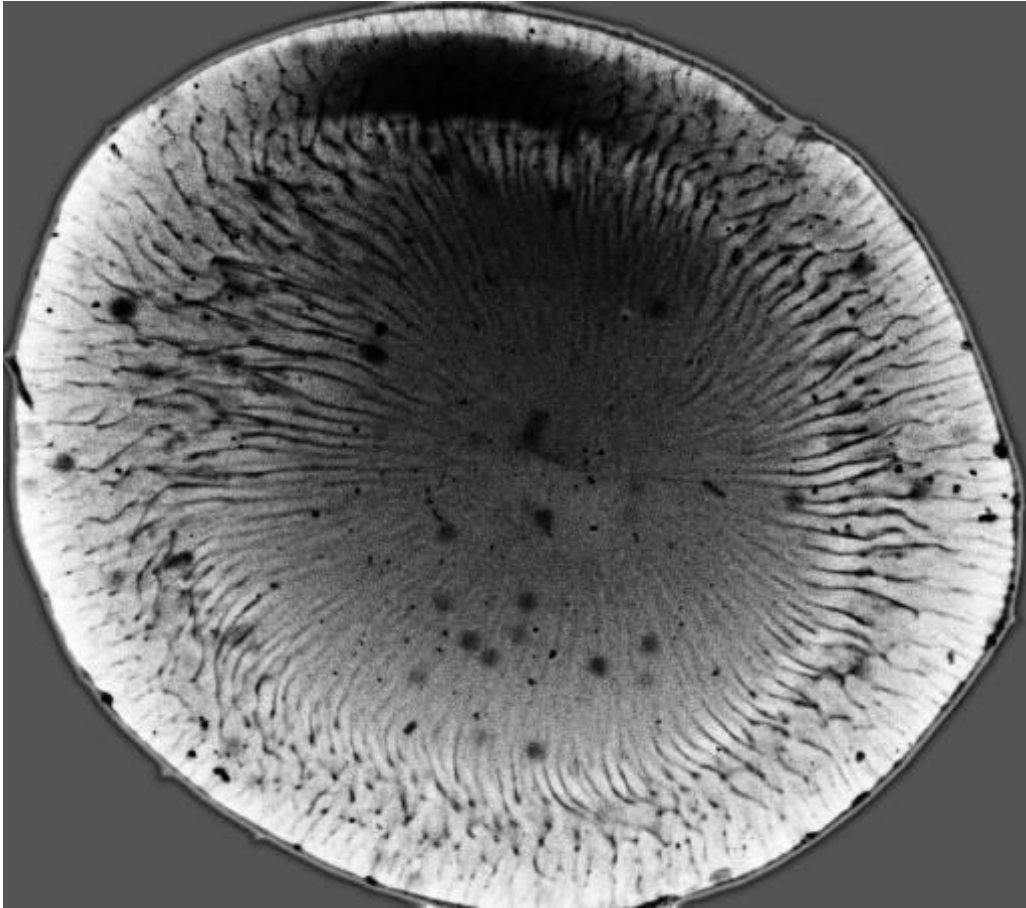


Radial structure in drying drop of  
quartz nanoparticles in vodka  $D = 0.63$  cm



Japanese sea nettles  
at the Boston Aquarium



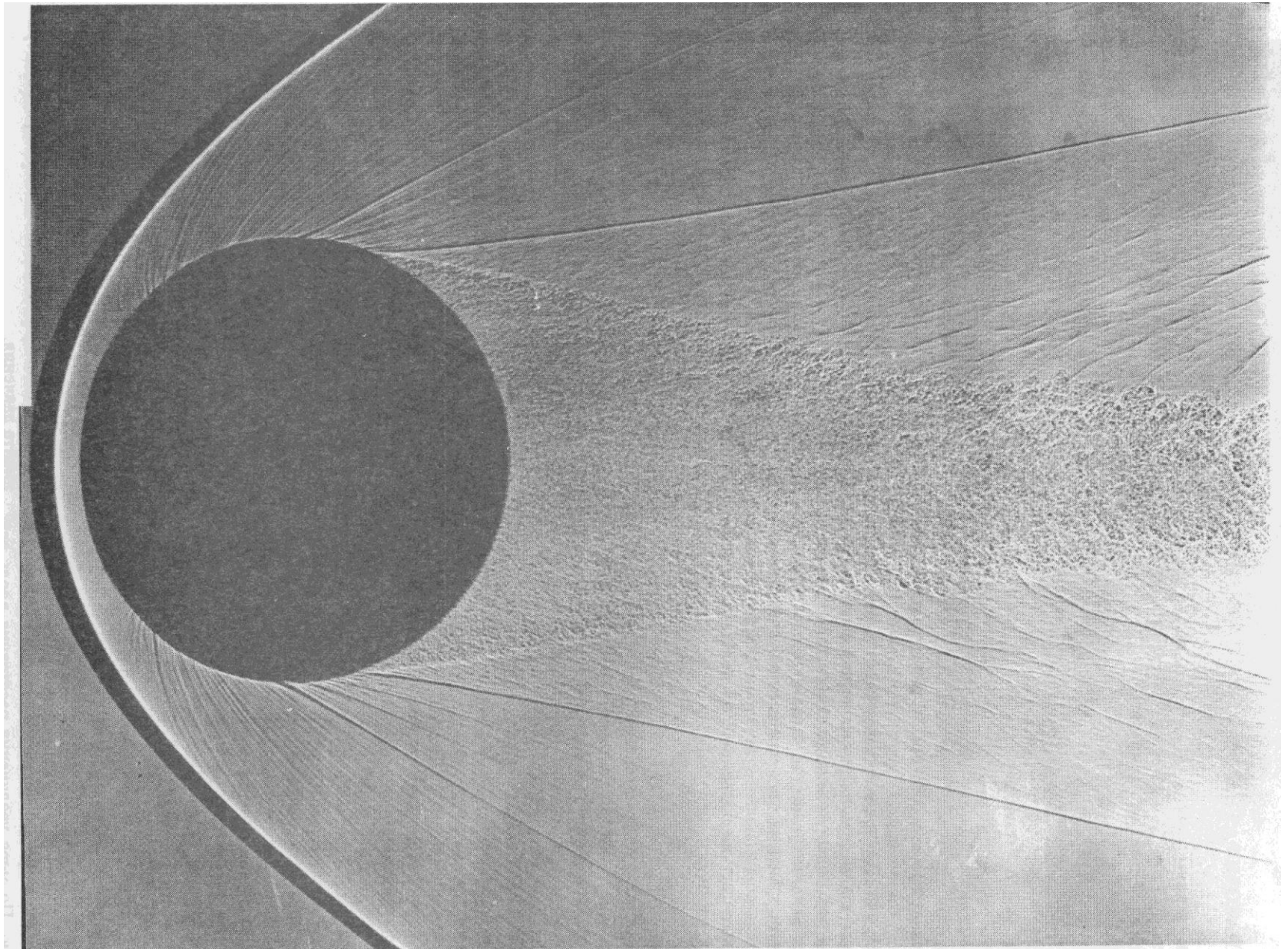


Radial structure in drying drop of quartz nanoparticles in vodka  $D = 0.63$  cm



Медуза “Японская морская крапива” в аквариуме Бостона 20 ноября 2012 г.

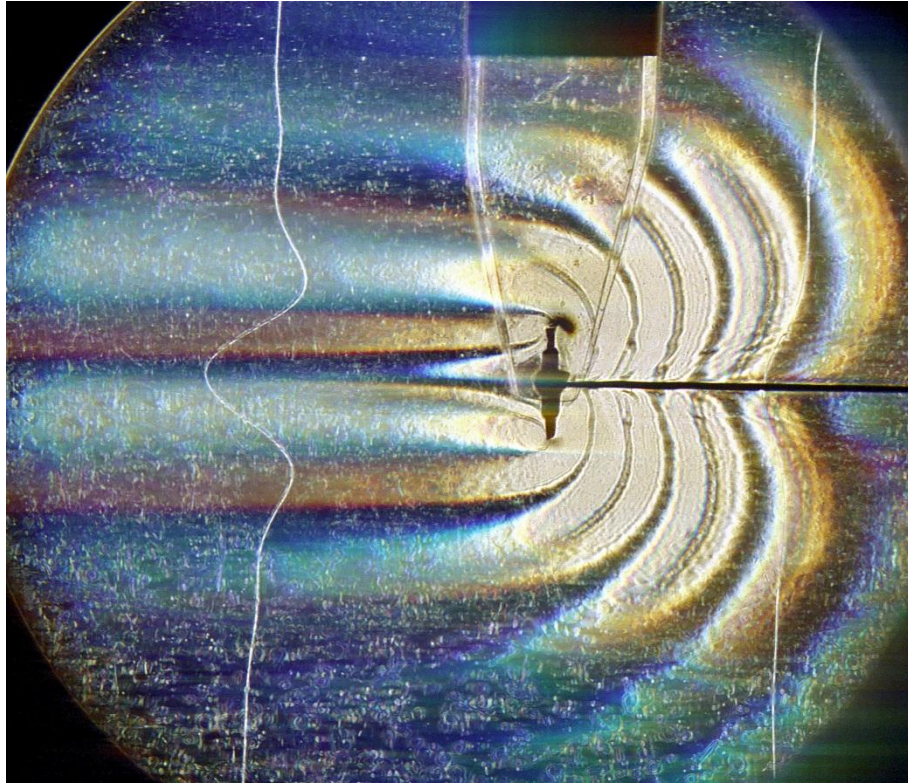




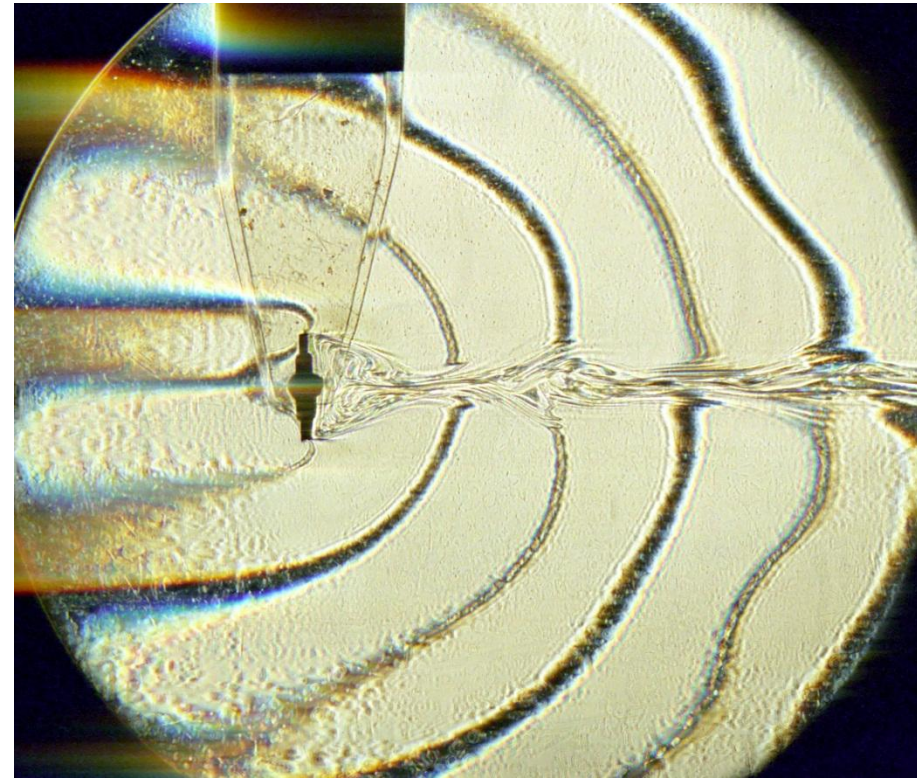
Schlieren flow of air around a sphere  $d = 8 \text{ cm}$   $M = 2.5$



$$T_b = 12.5 \text{ c},$$



$$U = 0.18 \text{ cm/ c}$$



$$U = 0.41 \text{ cm/ c}$$

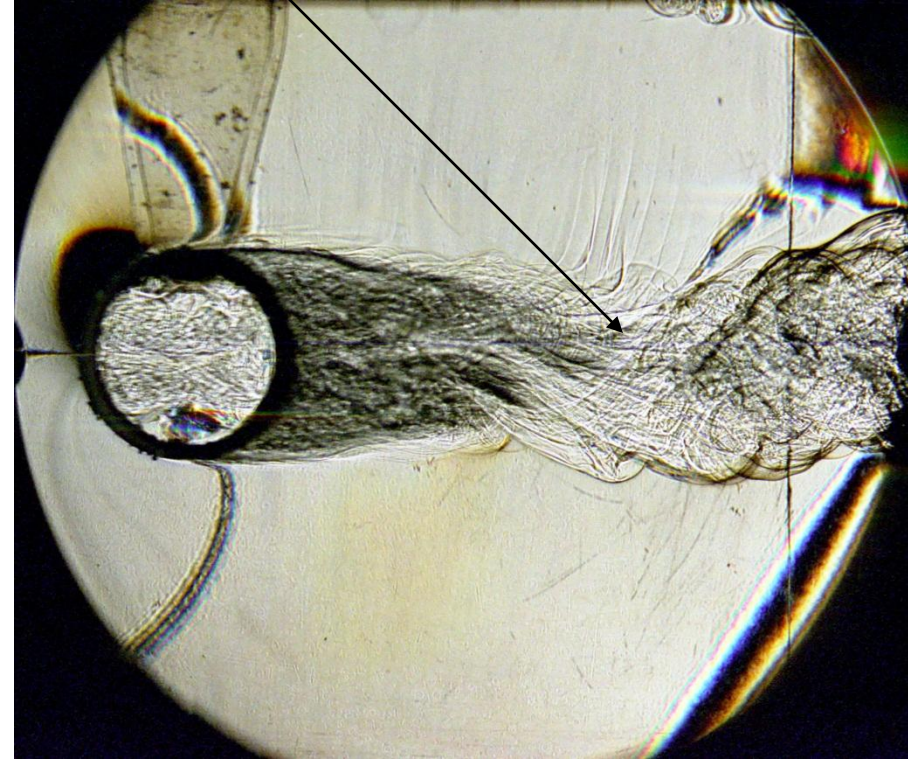
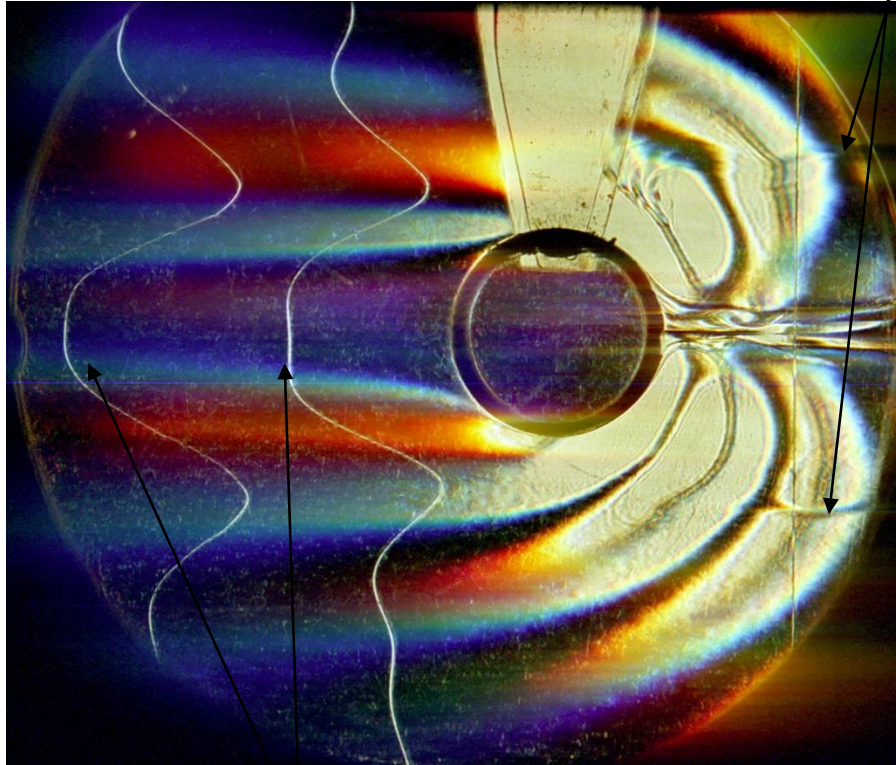
Stratified Flow around Vertical Strip



Maksoutov's  
Slit-Thread  
schlieren method

Soaring interface – singular sidic

Sidics in the density wake past cylinder



Blocked fluid visualized  
by density markers

$T_b = 20.1\text{ s}$ ,  $U = 0.1\text{ cm/s}$ ,  
 $Fr = 0,072$ ;  $Re = 54$

$T_b = 7.4\text{ s}$   $U = 3.48\text{ cm/s}$ ,  
 $Fr = 0,82$ ;  $Re = 1470$

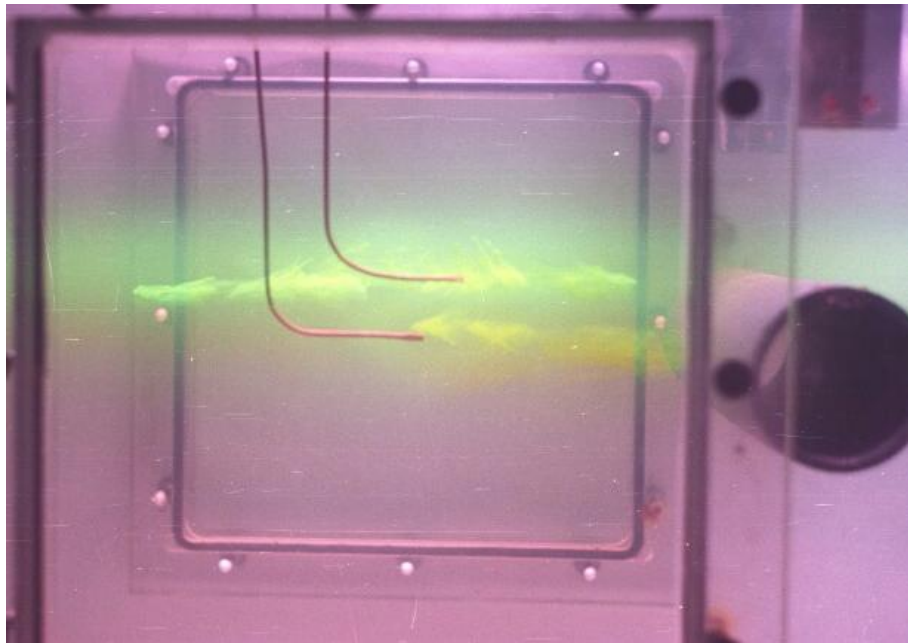
Maksoutov's Slit-Thread  
schlieren method



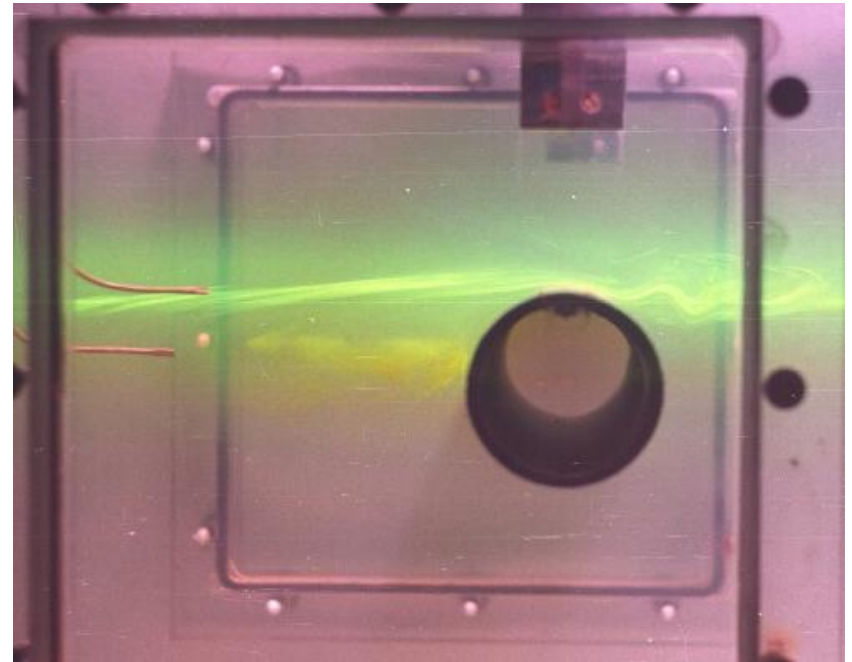
## Accumulation of a dye on the interfaces produced by moving cylinder in continuously stratified brine

Перераспределение краски в области блокировки

( $D = 7,6$  см;  $T_b = 7,0$  с;  $U = 0,24$  см/с;  $Fr = 0,035$ ;  $Re = 179$ )



Initial patter of dye

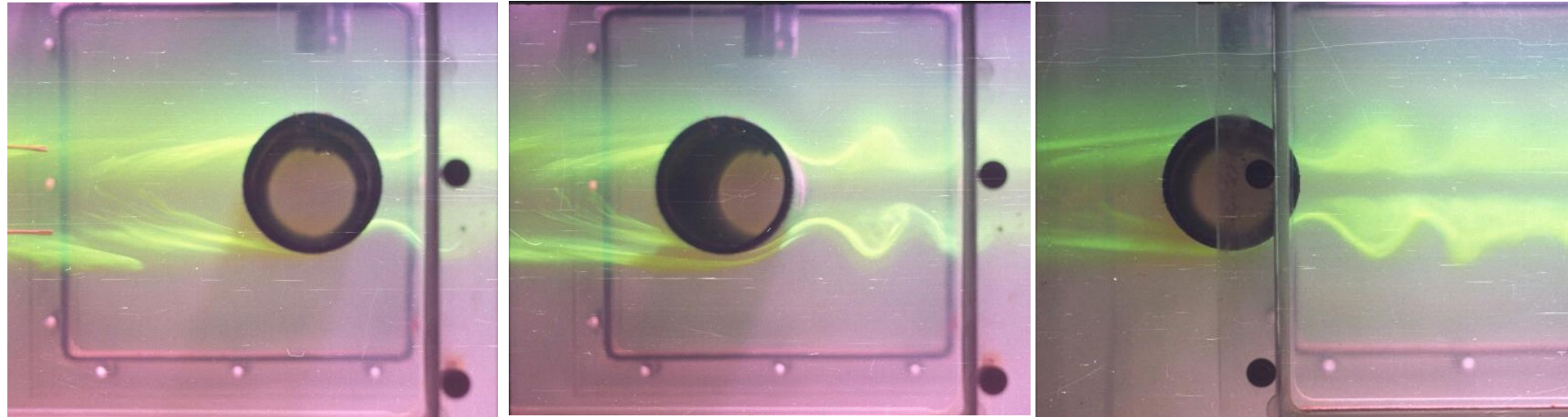


$$\tau = t / T_b = 8$$



## Accumulation of a released dye on interfaces ahead and past the cylinder

$$\tau = t/T_b = 3.6; 5.0; 8.6$$



$$D = 7.6 \text{ cm}; T_b = 7. \text{ s}; U = 0.69 \text{ cm/c}; Fr = 0.1; Re = 524$$

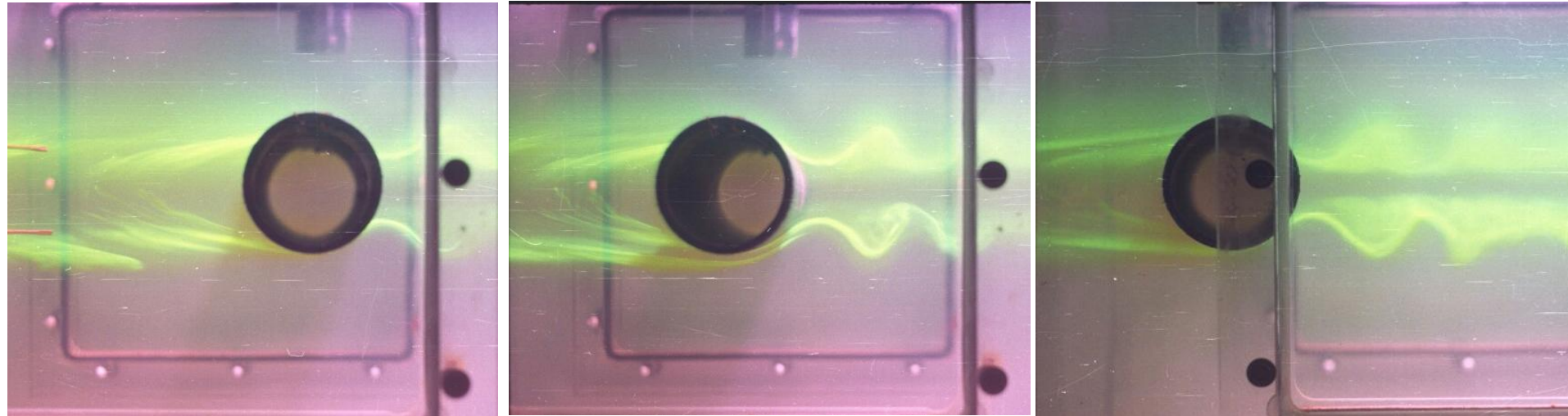
Formation of coloured envelopes on a high gradient interfaces inside the density wake.





Accumulation of a released dye on interfaces ahead and past the cylinder

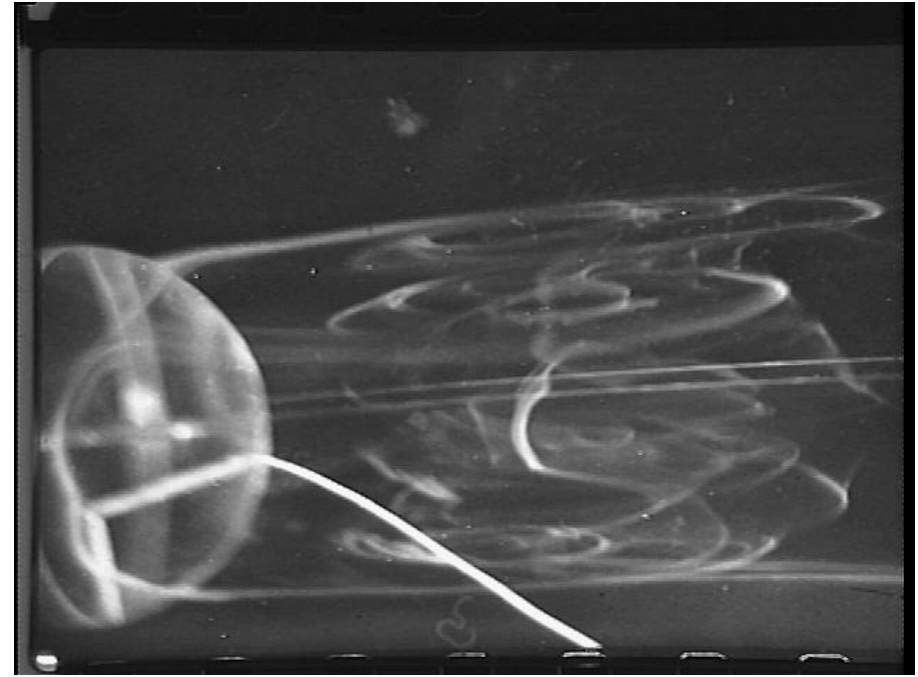
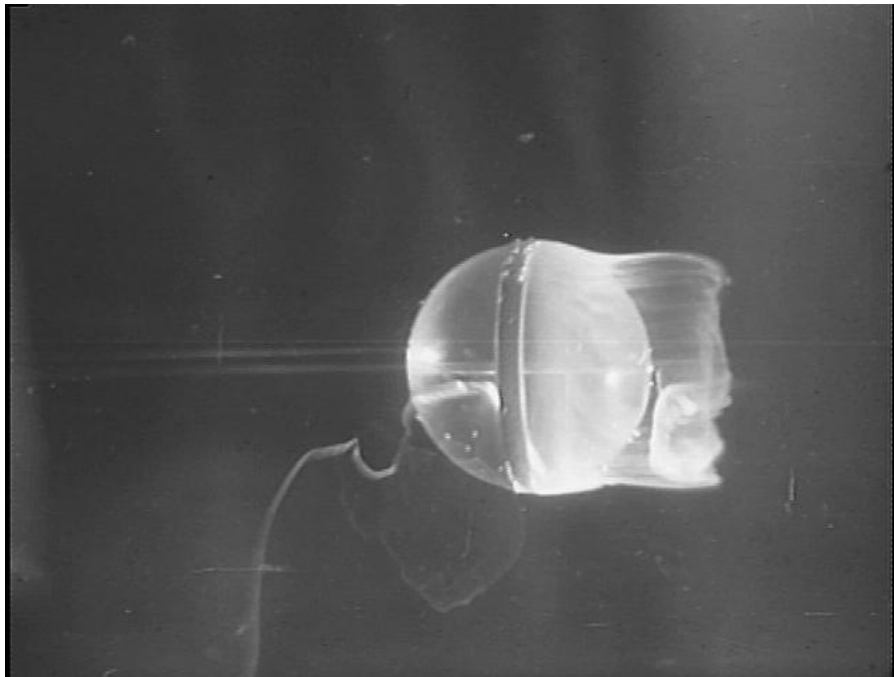
$$\tau = t/T_b = 3.6; 5.0; 8.6$$



$$D = 7.6 \text{ cm}; T_b = 7. \text{ s}; U = 0.69 \text{ cm/c}; Fr = 0.1; Re = 524$$

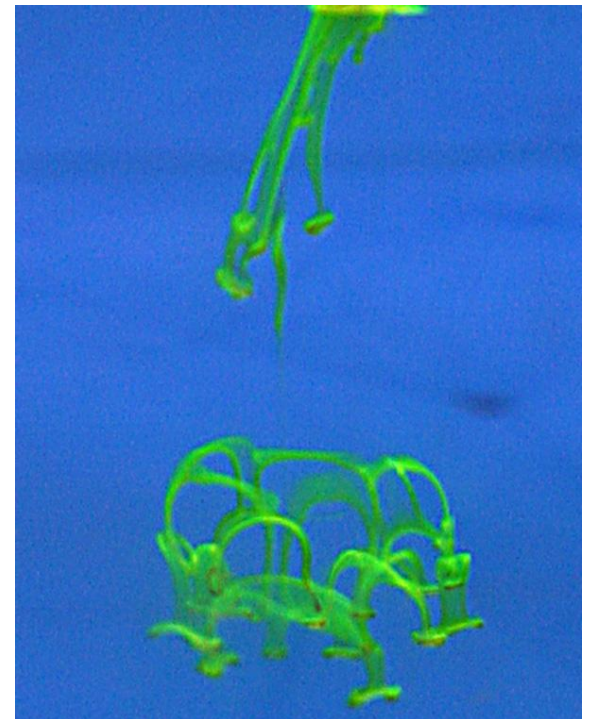
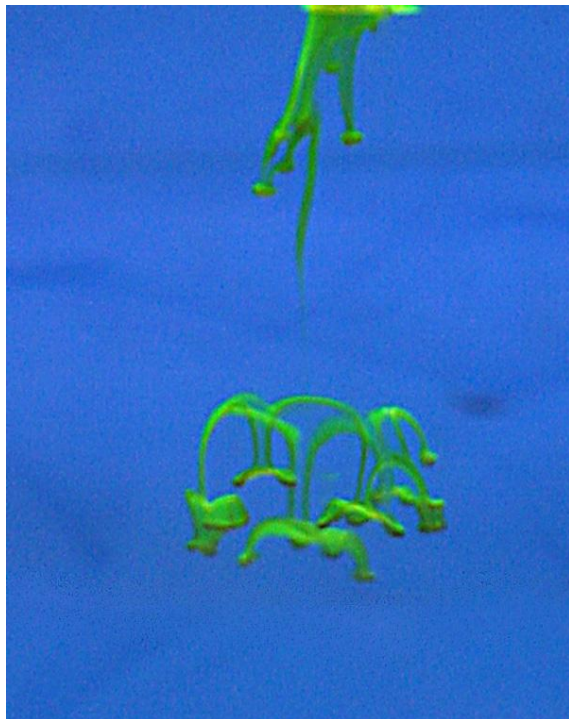
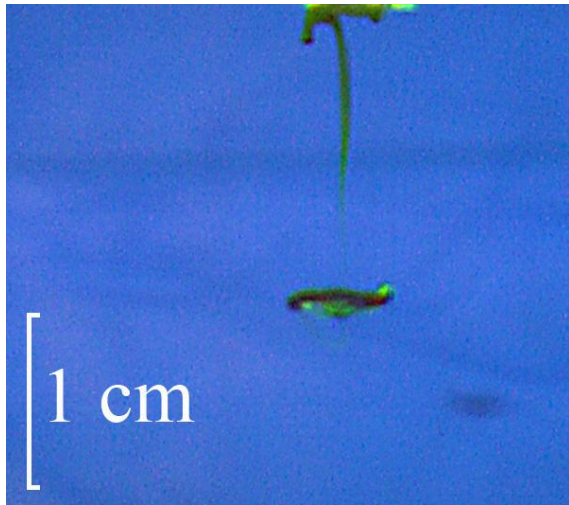
Formation of coloured envelopes on a high gradient interfaces inside the density wake.

Electrolytic precipitation –  
visualization of a flow past a horizontally moving sphere  
in a homogeneous and uniformly stratified fluid



$$D = 3 \text{ см}, \text{Re} = 350.$$

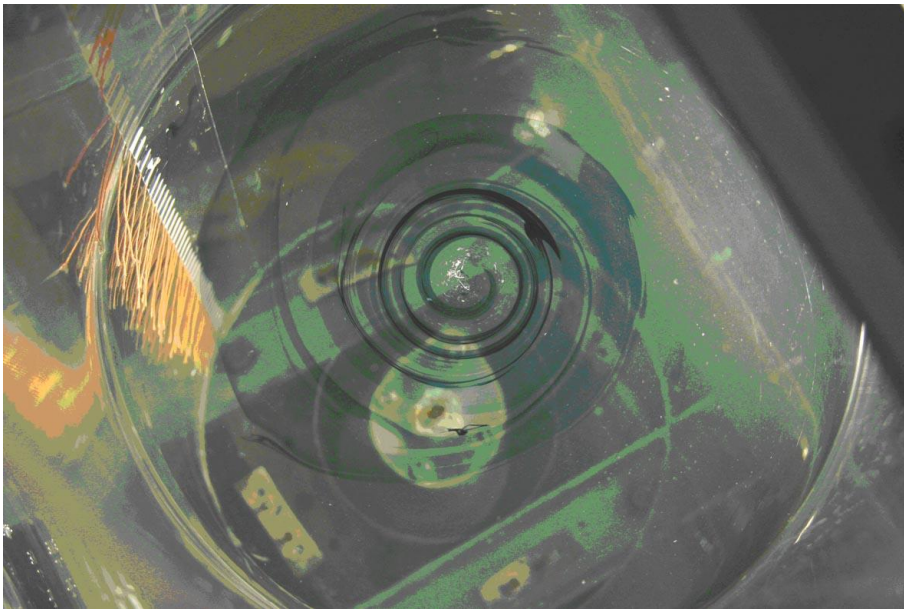
Cascade of vortex rings produced by a dye drop  
(diluted solution of Uranil in tap water) fallen on the surface of a fluid at rest



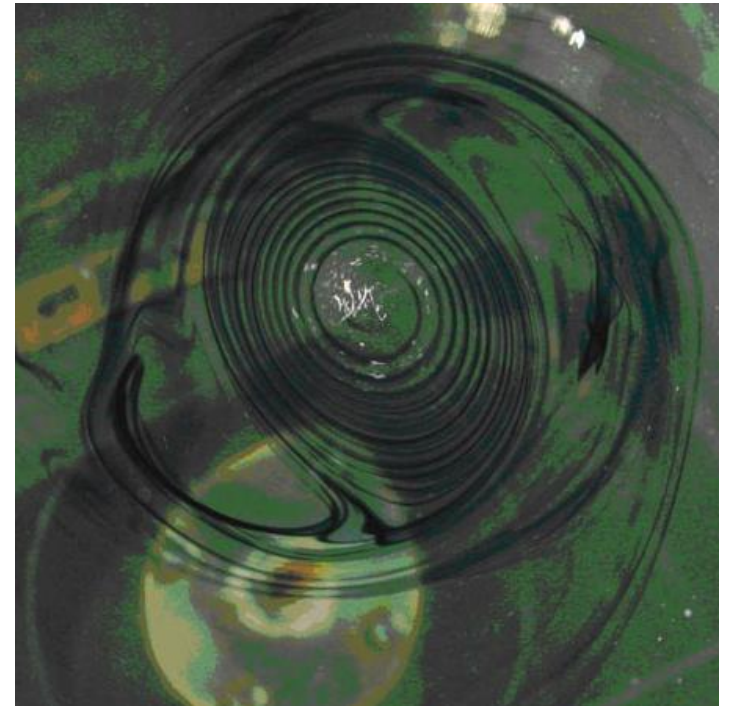
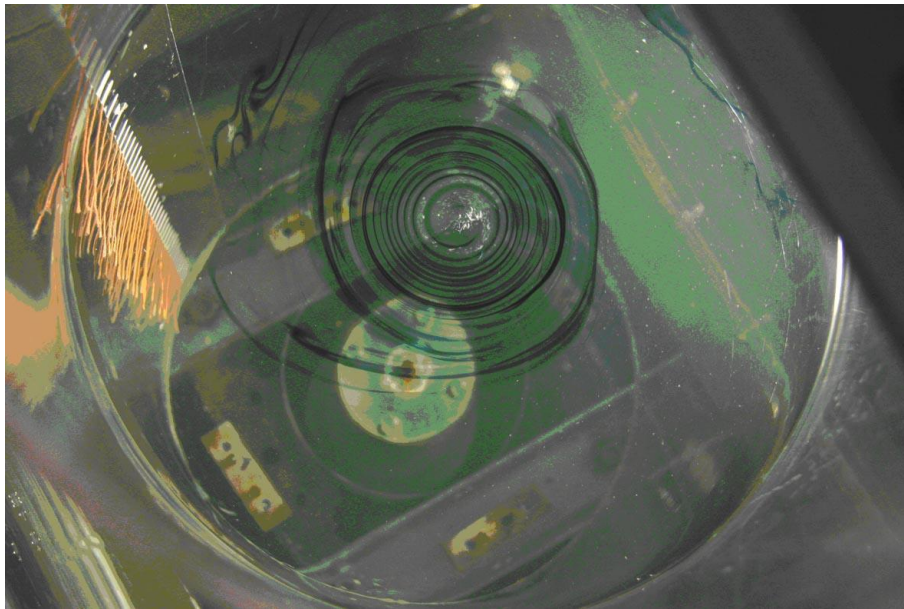


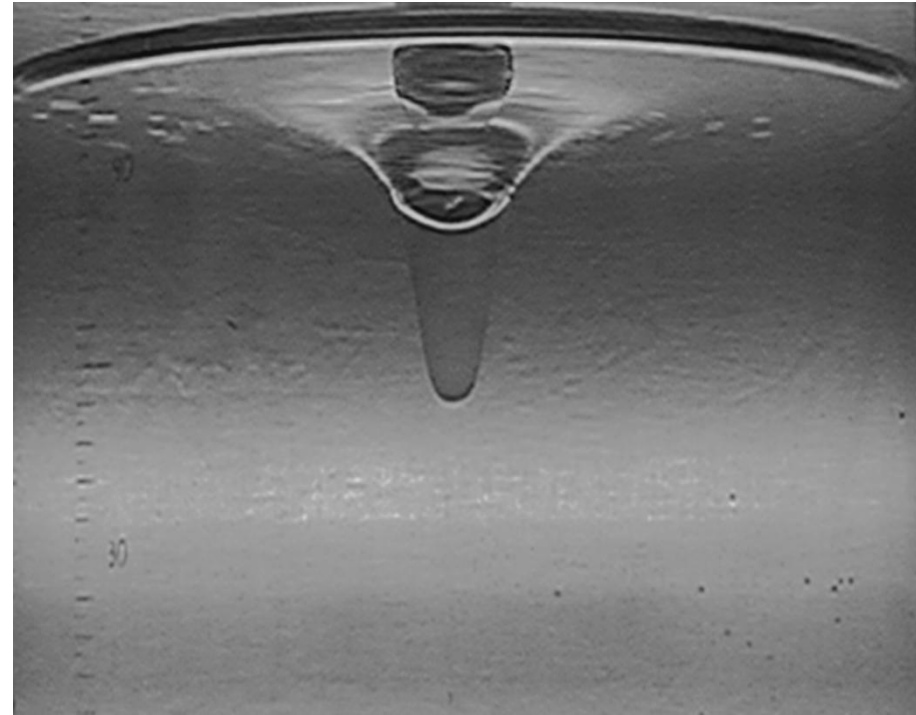
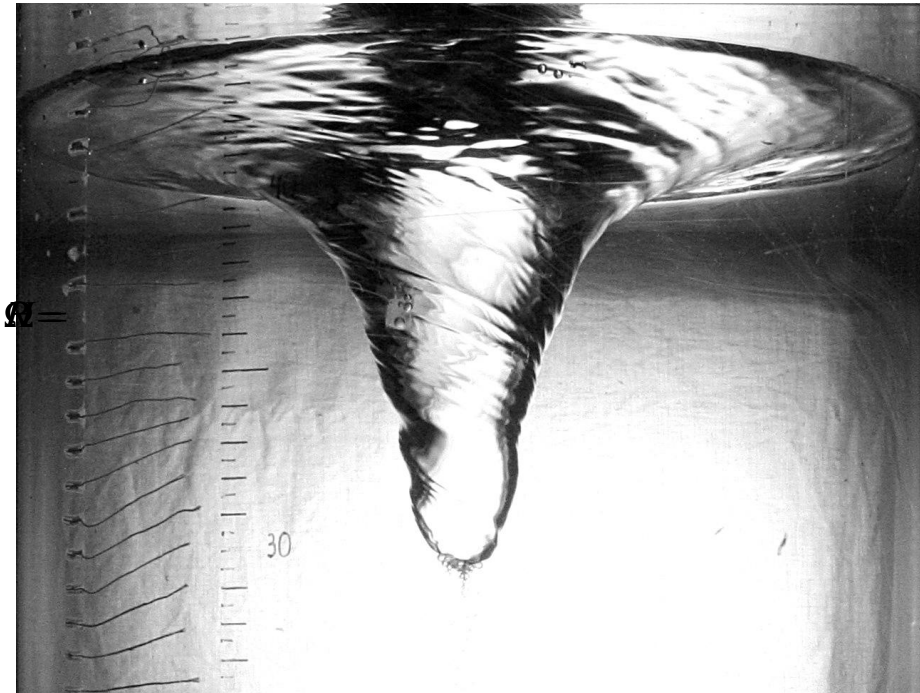
**EXPERIMENTAL STUDIES OF  
DYE TRANSPORT IN A ROTATING FLUID OR  
IN THE COMPOUND VORTEX CONTACTING  
WITH A FREE SURFACE –**

**G.I. Taylor problem reminded by TEXTBOOK  
of G.K. Batchelor**



Transformation of ink dye spot into spiral filament arms on the surface of fluid with compound vortex





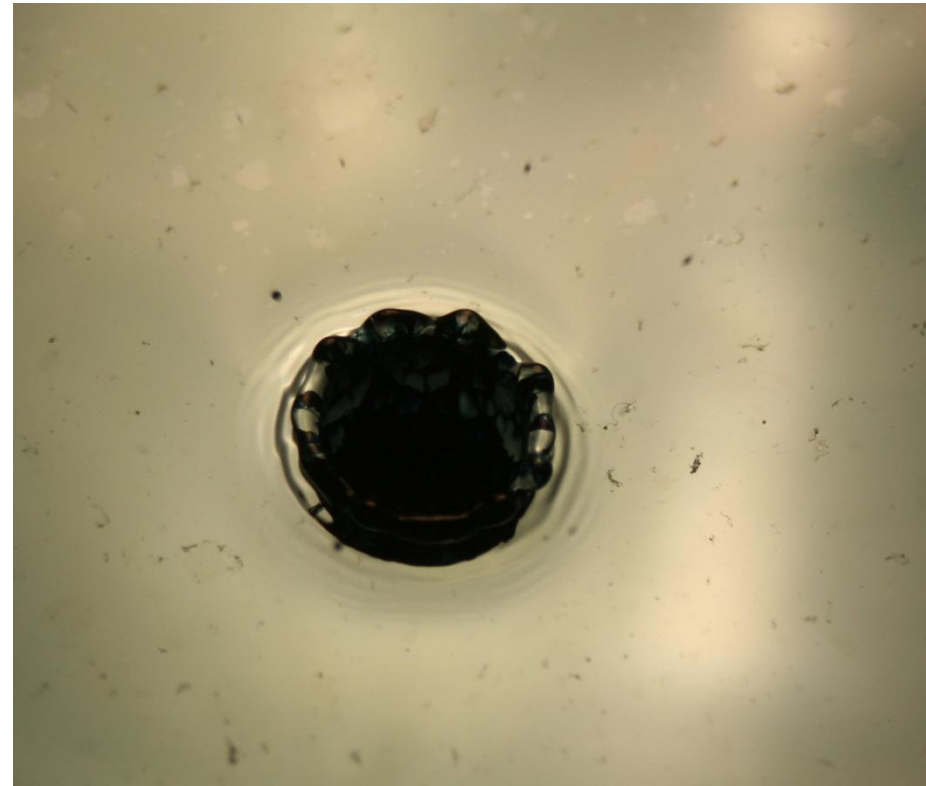
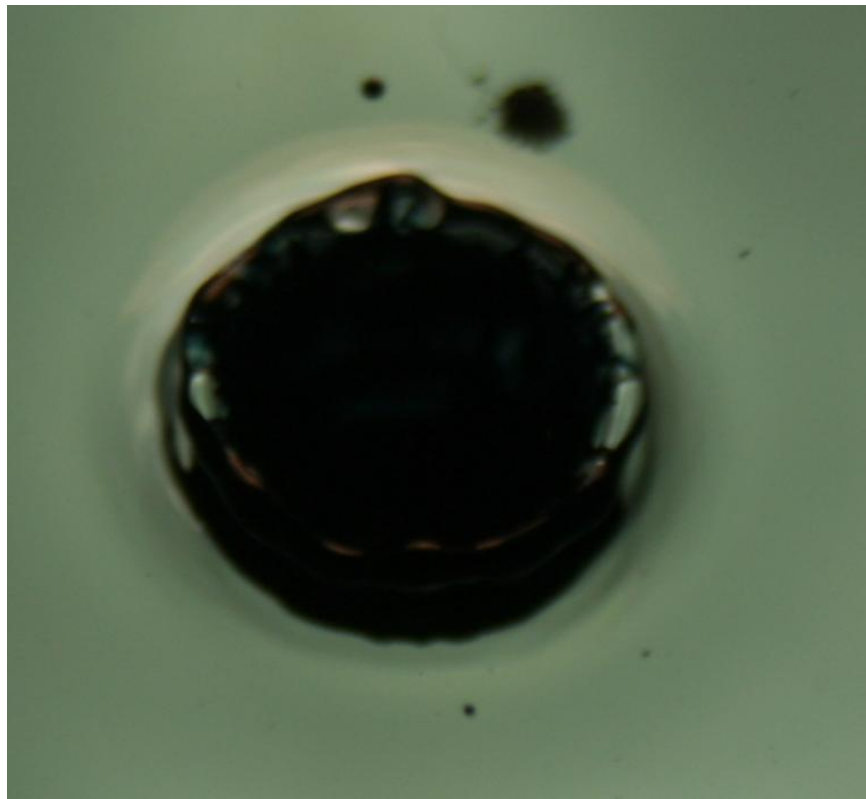
Форма каверны составного вихря в цилиндрическом контейнере, 40 см, 7.5 см: *а)* – чистая вода 750 об/мин; *б)* – с добавлением 30 мл касторового масла, 730 об/мин.

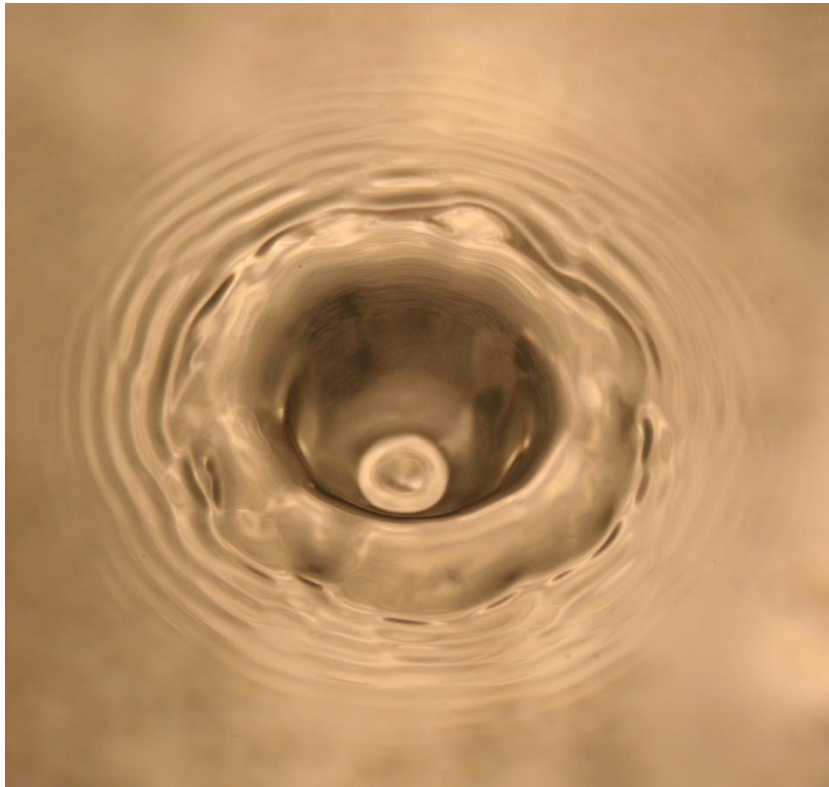


# Fluid mechanics needs in development

- Methods of measurements of real physical variables (momentum, pressure, density...) and checking of measurements accuracy in real experiment *ex temporo, in situ* – supernumerary(excess) method;
- Methods of identifications of all flow components – both large and small scales;
- Theory of continual system evolution taking into account direct interaction of **all** flow components – both large and small scales (redics and sidics).

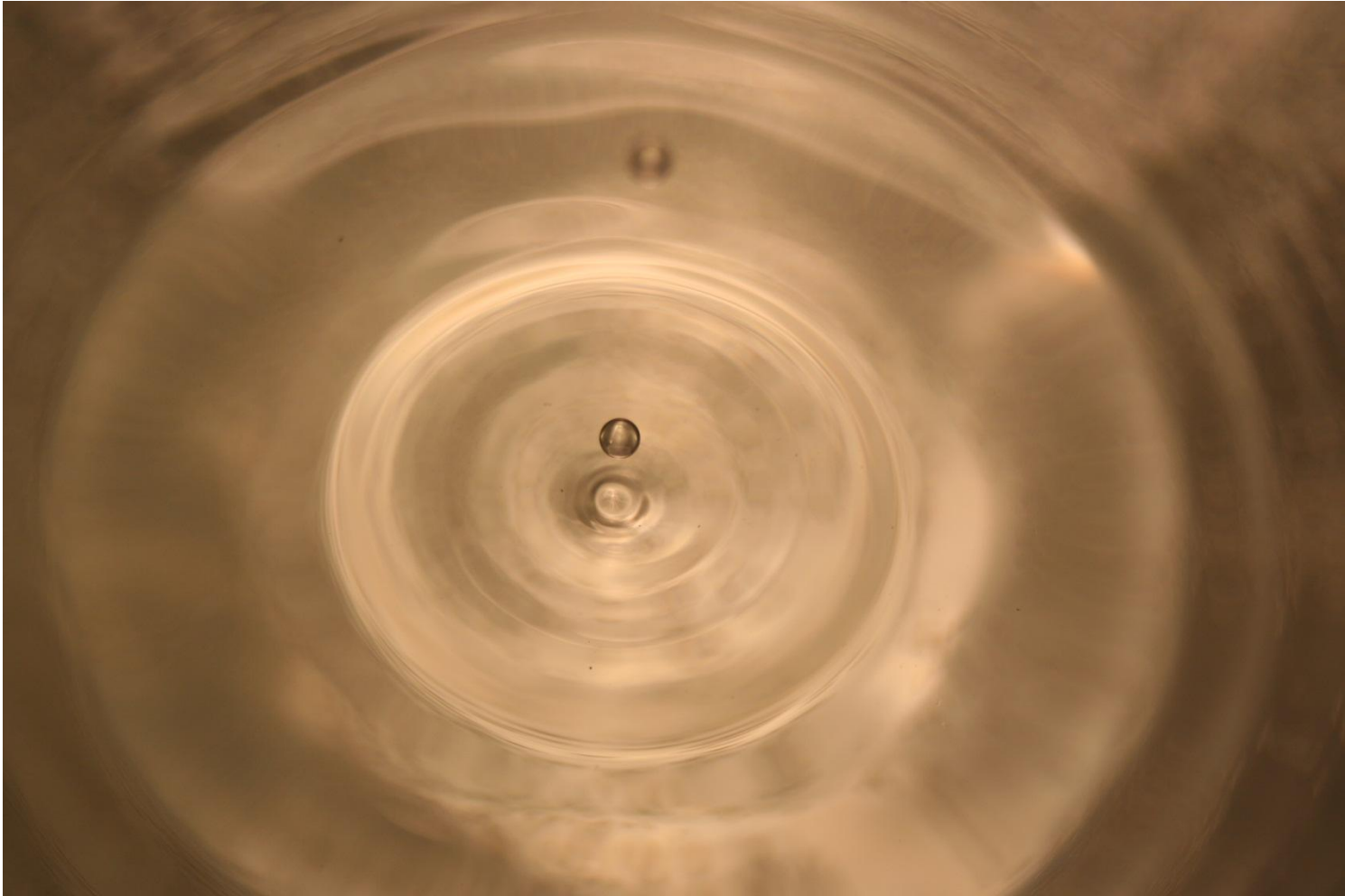


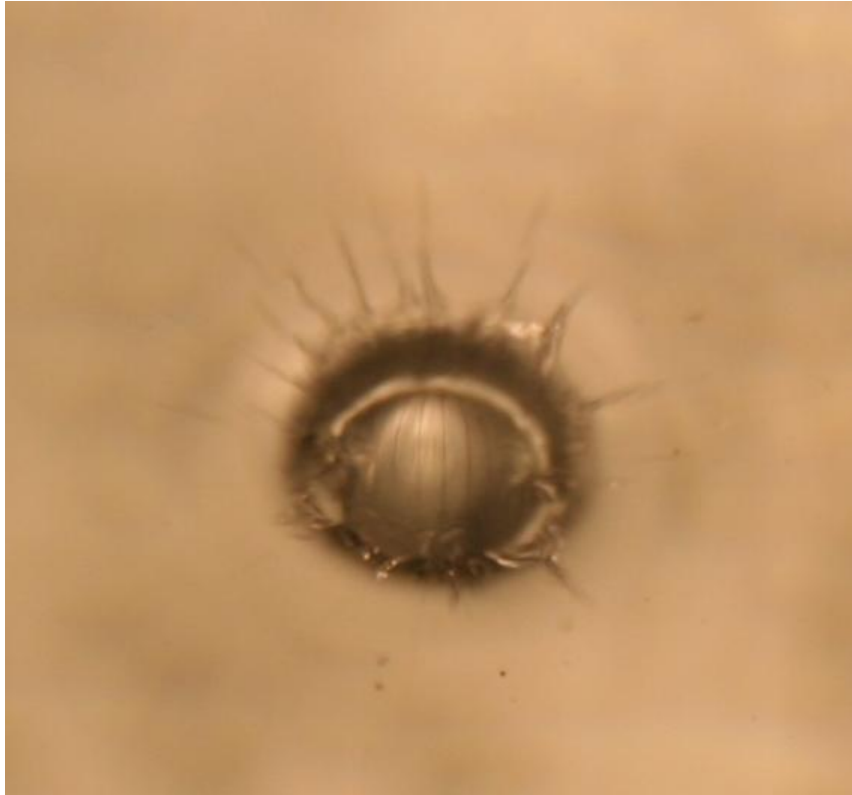




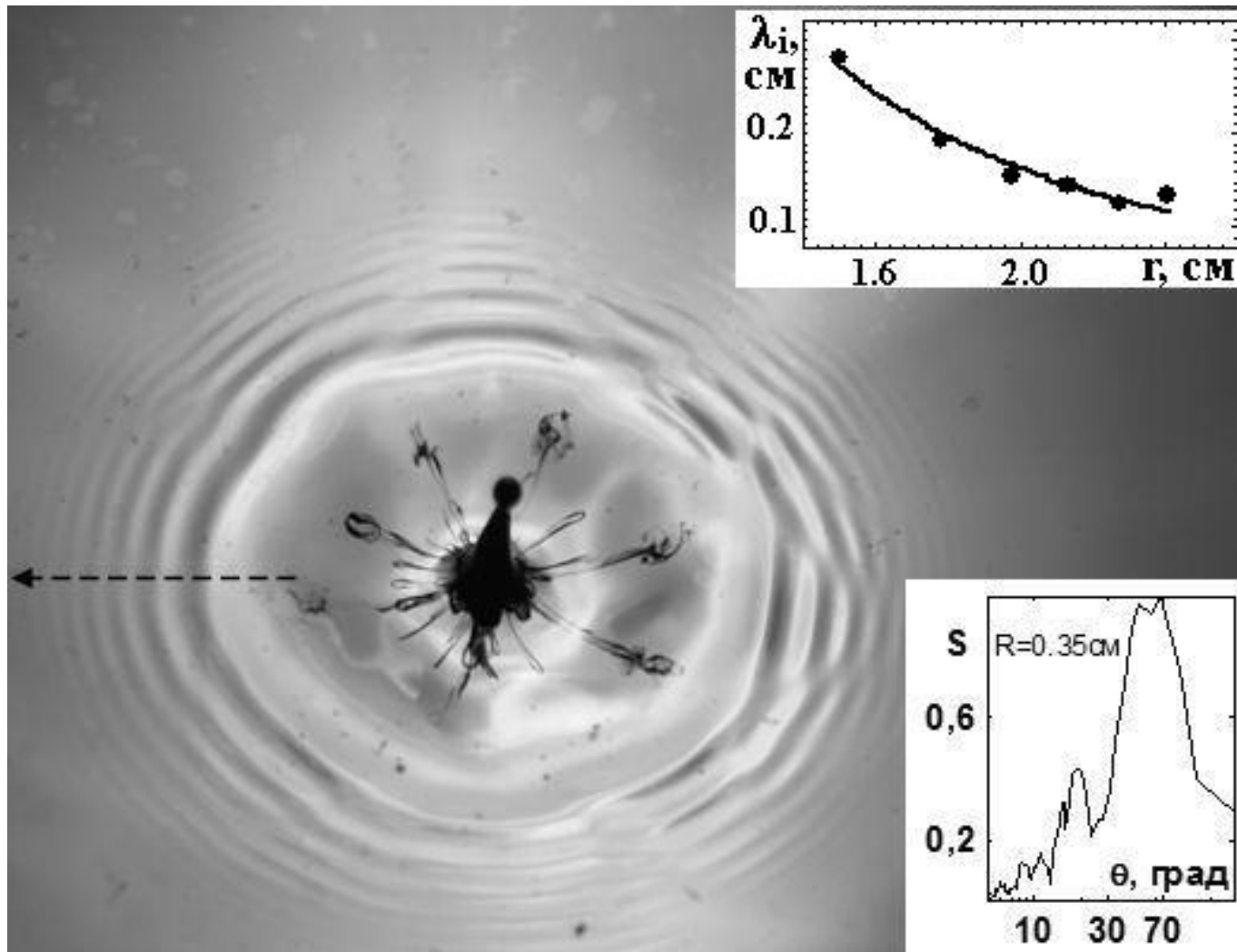


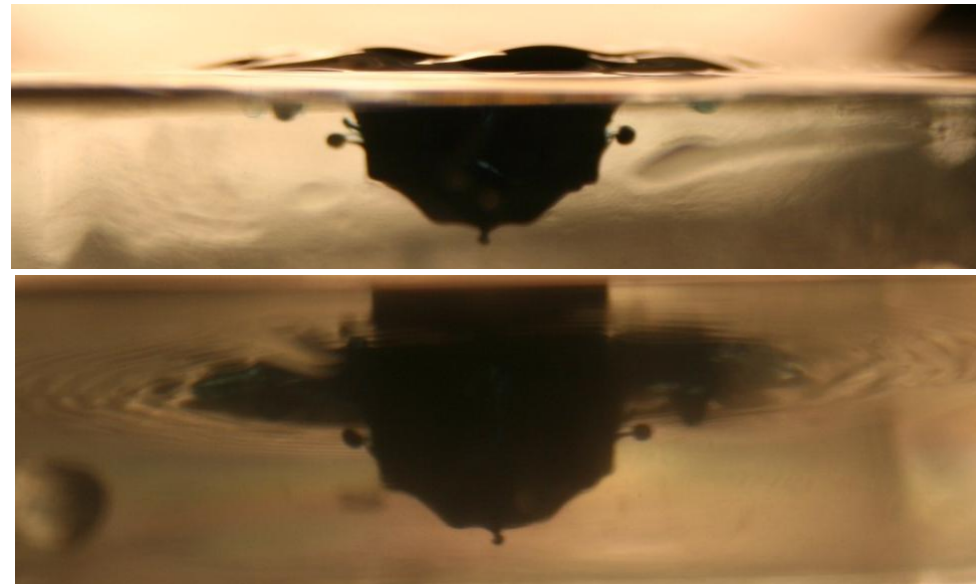
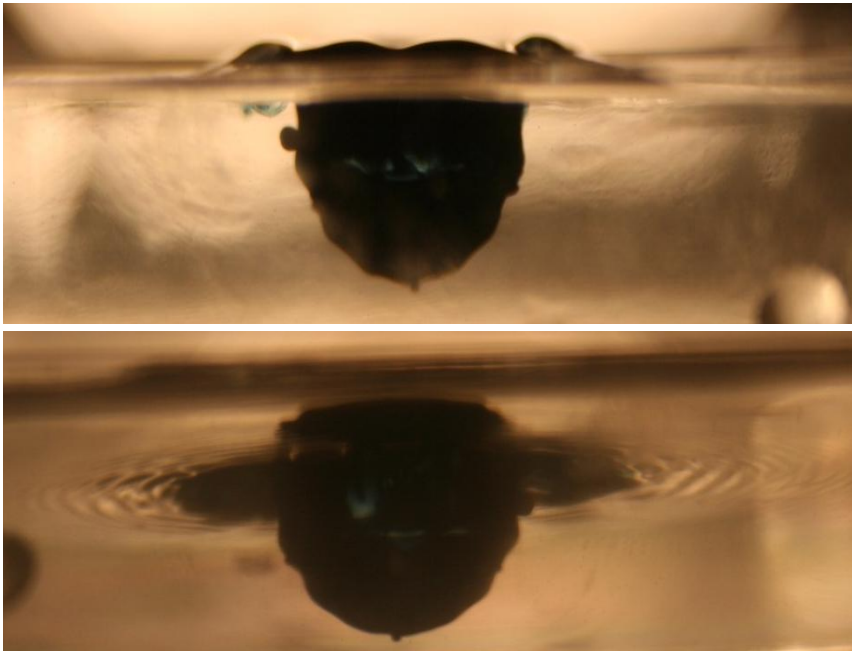




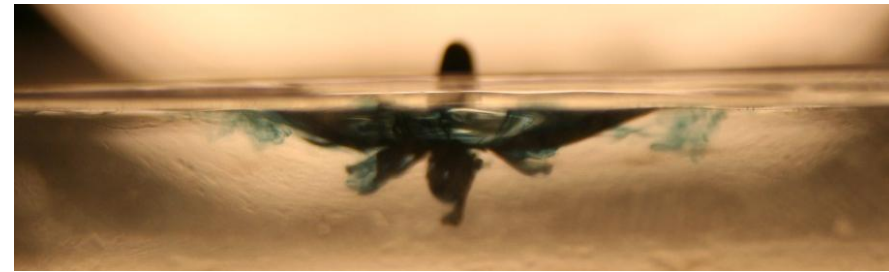
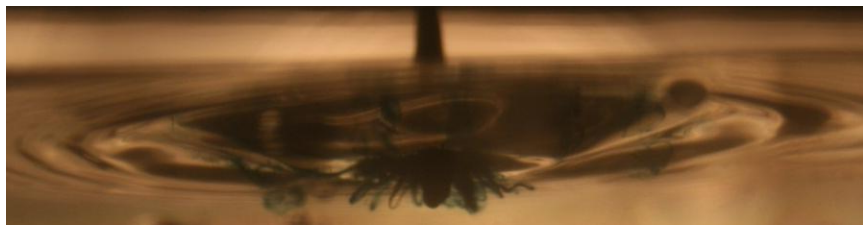
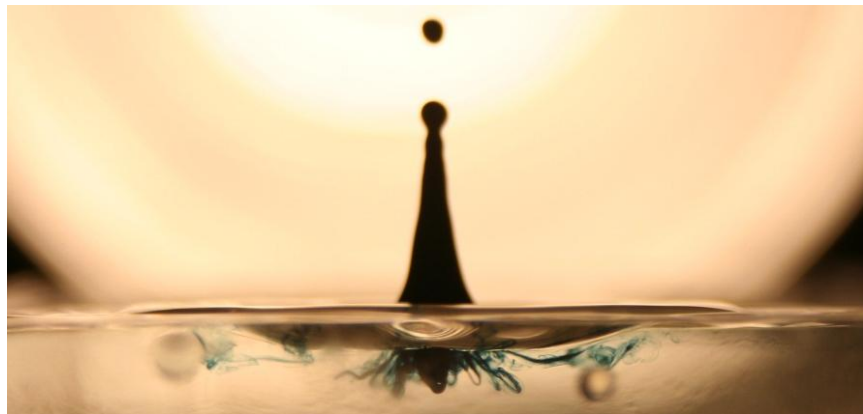




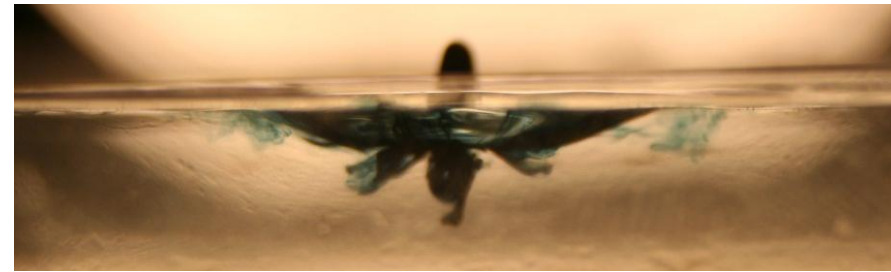
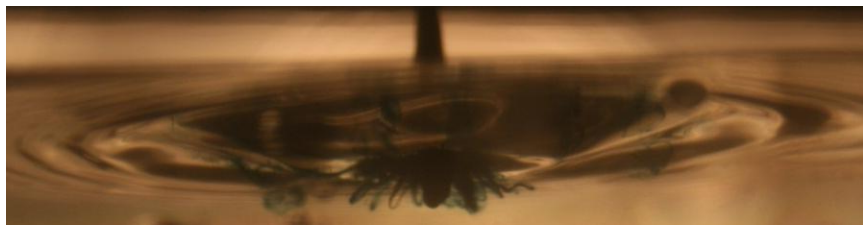
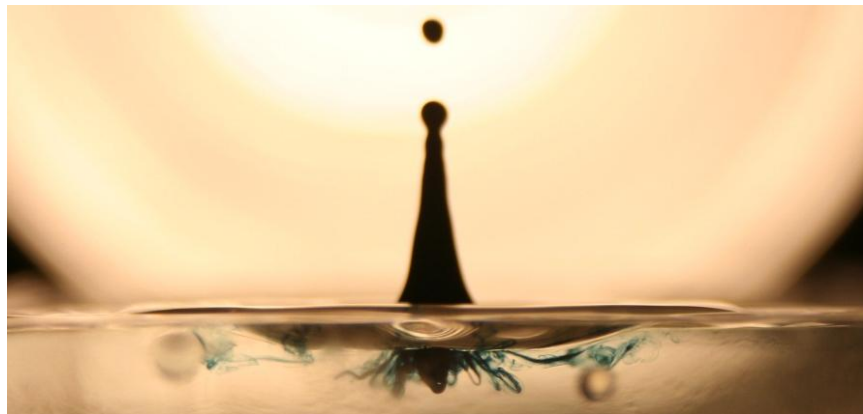




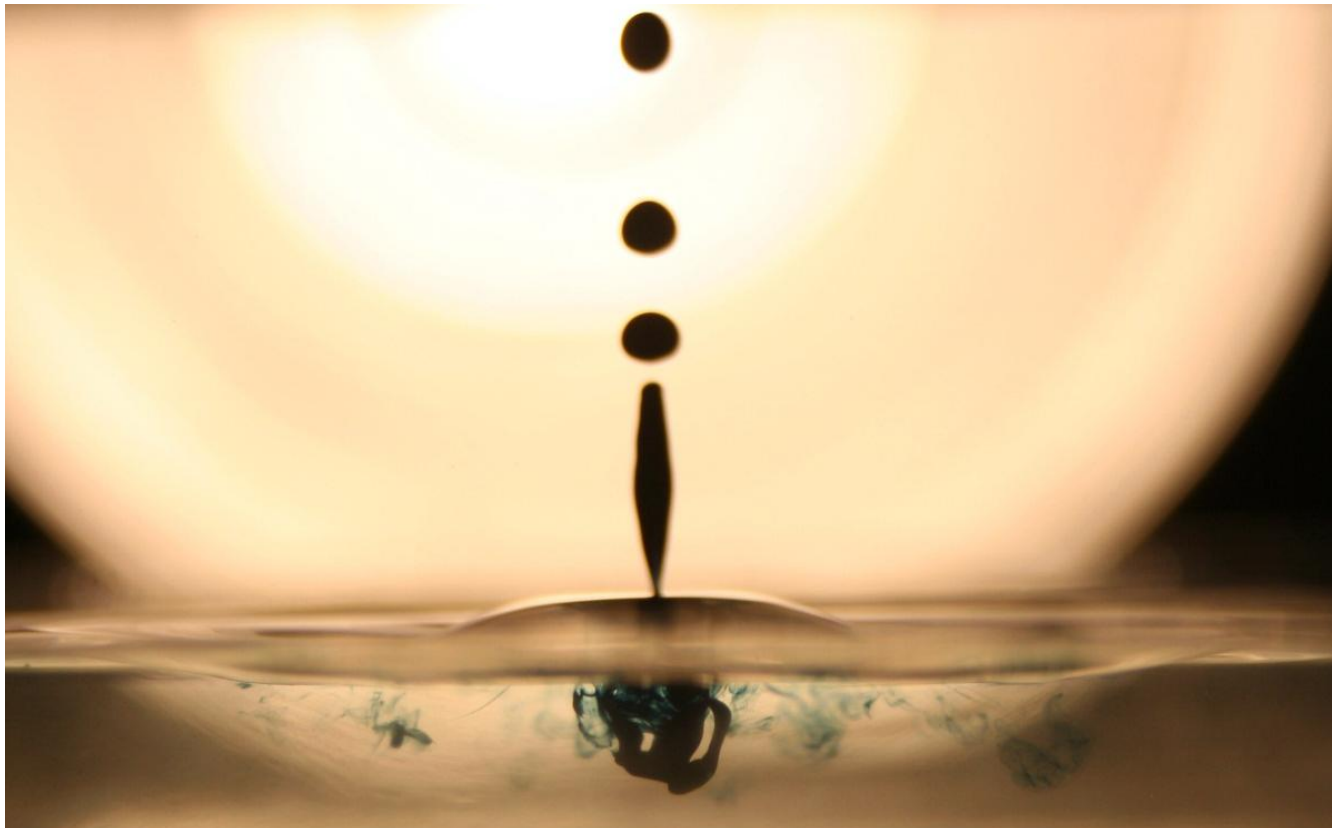
Formation of cavern with percolation jets with the head droplet (leading vortex ring?)











Flash and wakes of thin jets inside water layer produced by a drop of a ink solution





## Лентикулярисы Фудзиямы



The first part of the growth of a physical science consists in the discovery of a system of quantities on which its phenomena may be conceived to depend.

The next stage is the discovery of the mathematical form of the relations between these quantities.

After this, the science may be treated as a mathematical science, and the verification of the laws is effected by a theoretical investigation of the conditions under which certain quantities can be most accurately measured, followed by an experimental realization of these conditions...

...through the progress of science... we have become acquainted with so large a number of physical quantities that a classification of them is desirable.

Maxwell J.C. Mathematical classification of physical quantities // Proc. L. Math. Soc. 1869. S.1-3. P. 224 – 233.





## Natural oil spills from bottom deposits in the Gulf of Mexico



<http://petesplace-peter.blogspot.com/2009/02/oil-seeps-in-gulf-of-mexico.html>



Density marker  
for measurement of  
buoyancy period and  
horizontal component  
of velocity

$$\frac{\eta(r,t)}{\eta_m} = \left(\frac{R}{r}\right)^2 \left(\frac{\gamma}{t}\right) \cdot \exp\left(-\frac{\gamma}{t}\right) \cdot \cos Nt \left(1 - \frac{7/4}{(Nt)^2} \left(1 - \frac{\gamma}{t}\right)\right)$$

$$\eta_m = V_0 / N, \quad \gamma = r^2 / 2\nu \quad (r - \text{distance from the marker centre}), \quad T_b = 6 \text{ c}, \quad N = 1,05 \text{ c}^{-1},$$

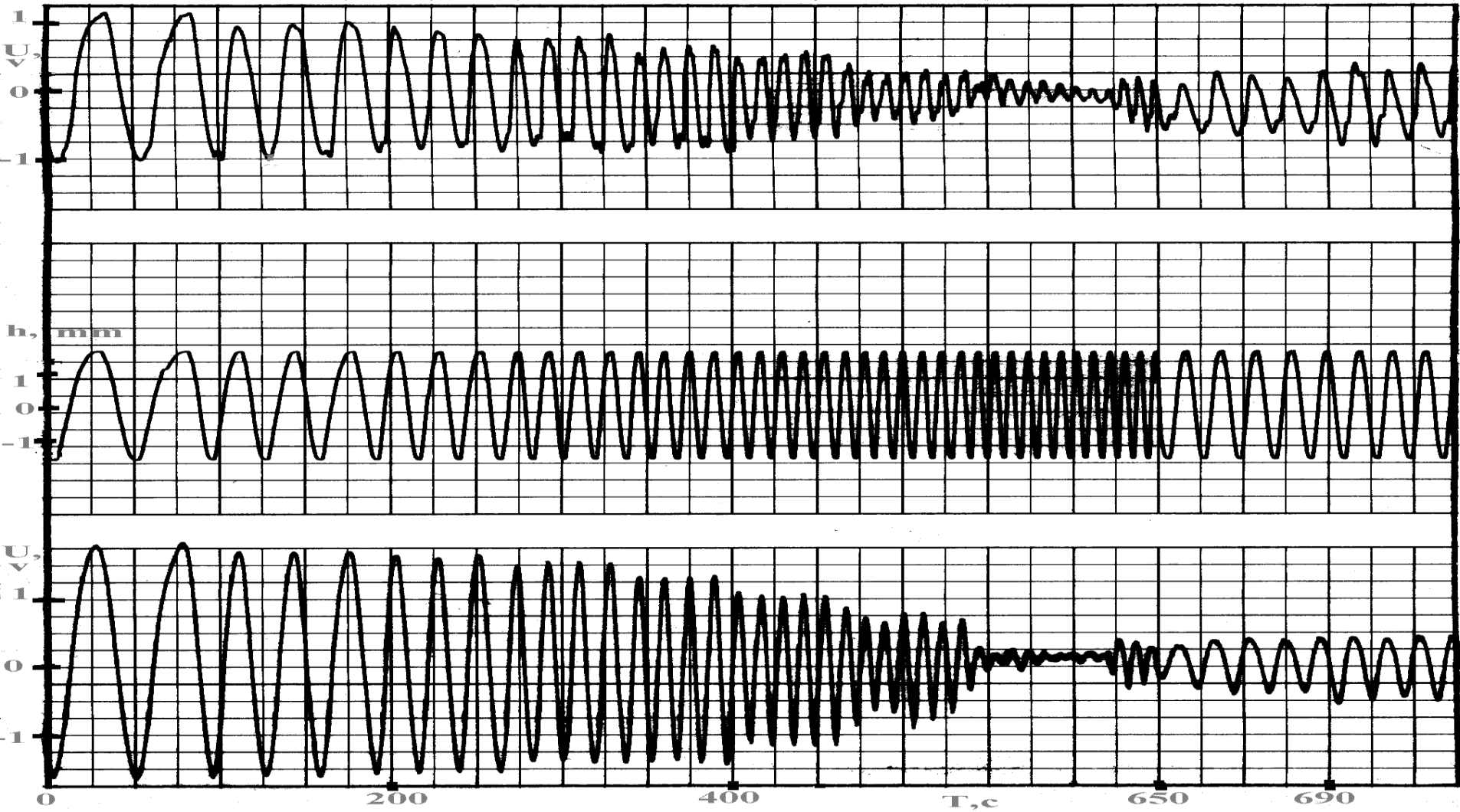
$$\nu = 0,01 \text{ cm}^2 / \text{c}. \quad R = 0,05 \text{ cm}, \quad U_0 = 7 \text{ cm} / \text{c}, \quad r = 0,2 ; 0,6 ; 1 \text{ cm},$$

$$\text{Re} = V_0 (2R) / \nu = 70, \quad \text{Fr} = 0,5 (V_0 / NR) = 70.$$



Pattern of periodic internal waves wedges contacting with conductivity probes in a fluid with variable buoyancy frequency





Response of point conductivity (above) and velocity of sound (below) on harmonic oscillations vertical oscillations with constant amplitude and variable frequency (in center)

## Continuity equation: Jean le Rond D'Alembert:

*Réflexions sur la cause générale des vents, Paris 1744, tide*

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \cdot \mathbf{v}) = 0 \quad dp = c_s^2 d\rho \quad \text{or} \quad \operatorname{div} \mathbf{v} = 0$$

## Euler L. Principes généraux du mouvement des fluids //

Mémoires de l'Académie royale des sciences et belles lettres. Berlin. 1757. V. 11 (papers of 1755 year). P. 274-315. = Opera omnia. Ser.II. V.12. P. 54-91.. (+ 1750, 1751, 1752)

$$\rho(p) \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} \right) = -\nabla p + \rho \mathbf{g}; \quad \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \cdot \mathbf{v}) = 0 \quad \operatorname{div} \mathbf{v} = 0$$

“Nevertheless everything that forms the theory of fluids is contained in the two above mentioned equations (§ 34), so for continuation of the study we need not in new principles of mechanics but only in instruments of analysis which is is not yet sufficiently developed for this purpose”.

## Euler-D'Alembert paradox

