Transition to chaos in double-diffusive and penetrative convection

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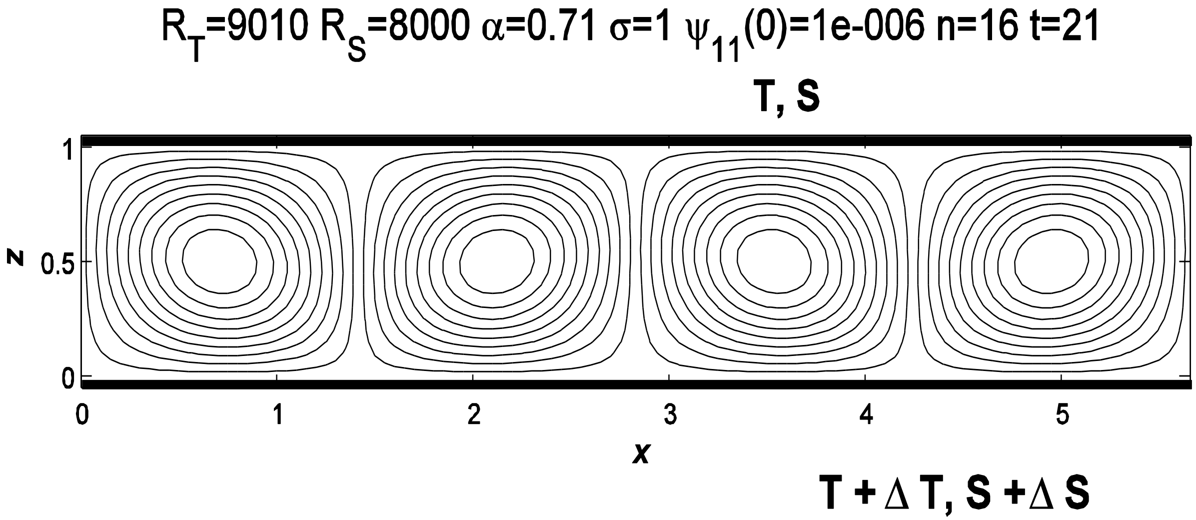
Облачные вычисления:
образование, исследования, разработки

Doubly diffusive convection

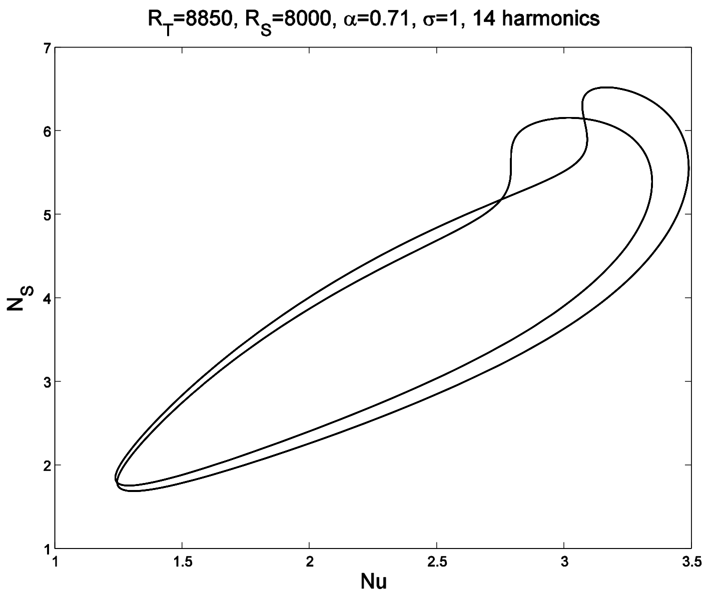


$$T = T_0 + (T_1 - T_0)(1 - z + \tau),$$

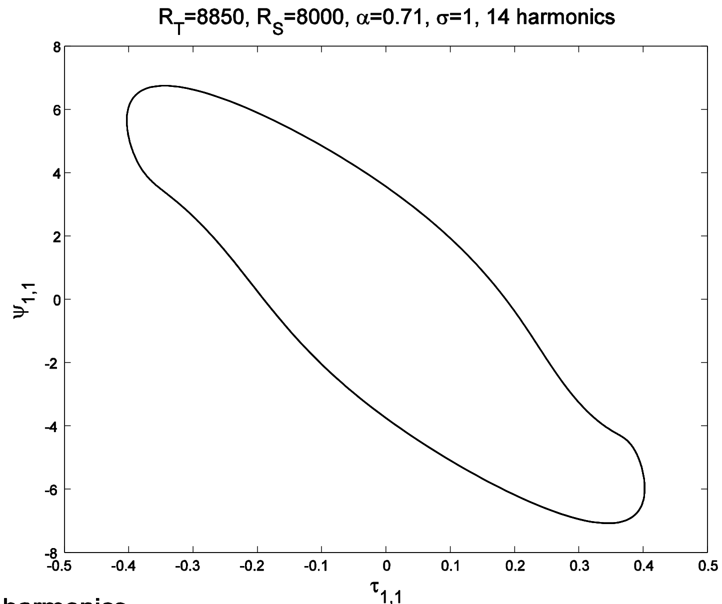
$$S = S_0 + (S_1 - S_0)(1 - z + s)$$

Boussinesq approximation

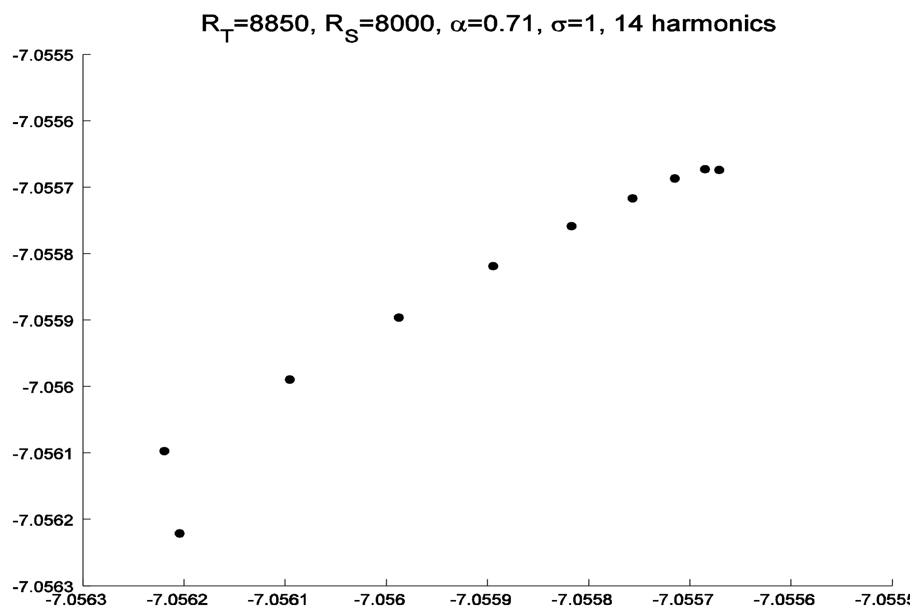
$$\rho = \rho_0(1 - \alpha T + \beta S)$$



R_T=8850, R_S=8000, α=0.71, σ=1, 14 harmonics
N_S
Nu
1
1.5
2
2.5
3
3.5
2
3
4
5
6
7



R_T=8850, R_S=8000, α=0.71, σ=1, 14 harmonics
$\psi_{1,1}$
$\tau_{1,1}$
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
-8
-6
-4
-2
0
2
4
6
8



R_T=8850, R_S=8000, α=0.71, σ=1, 14 harmonics
-7.0555
-7.0556
-7.0557
-7.0558
-7.0559
-7.056
-7.0561
-7.0562
-7.0563
-7.0563
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-7.056
-7.0559
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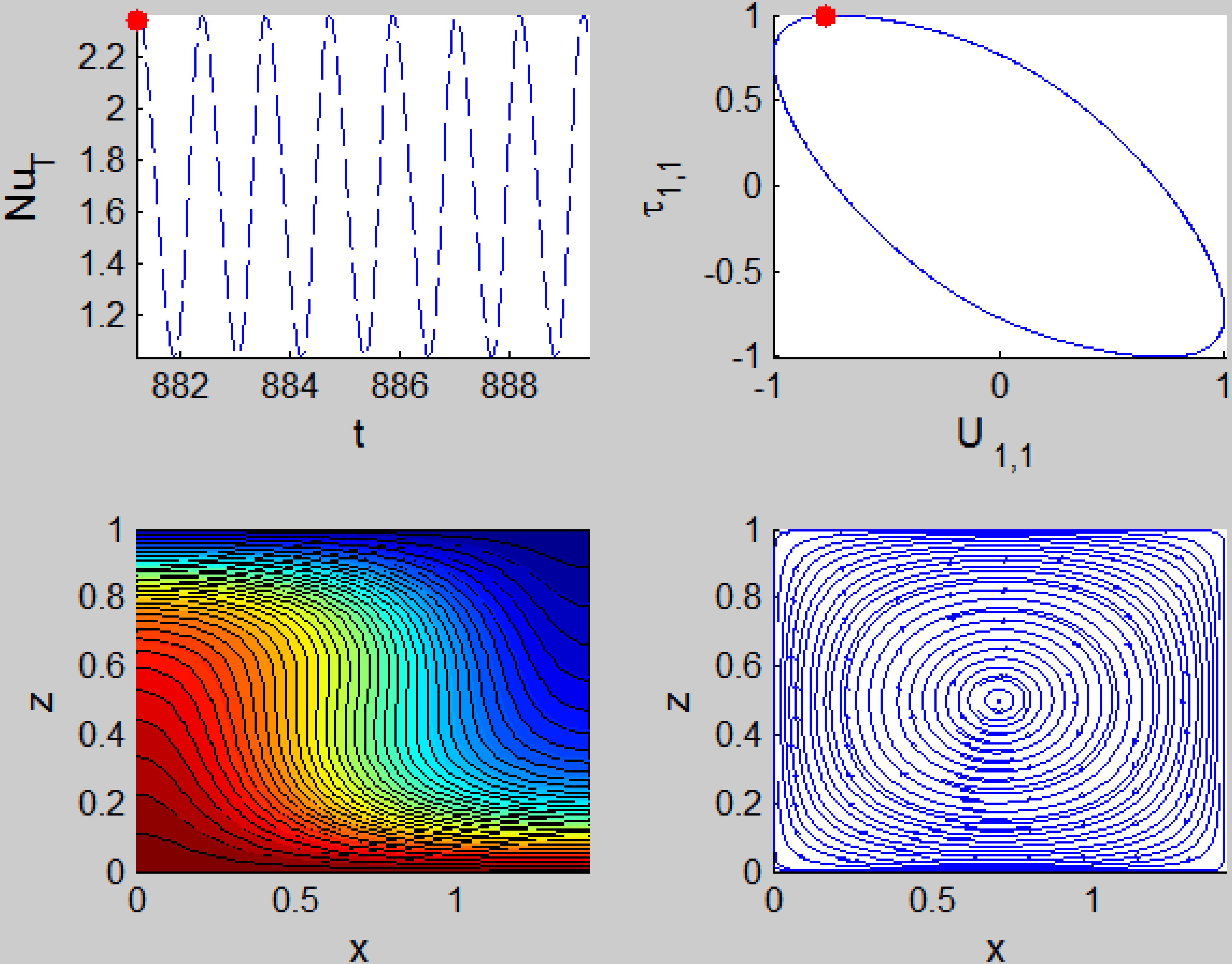
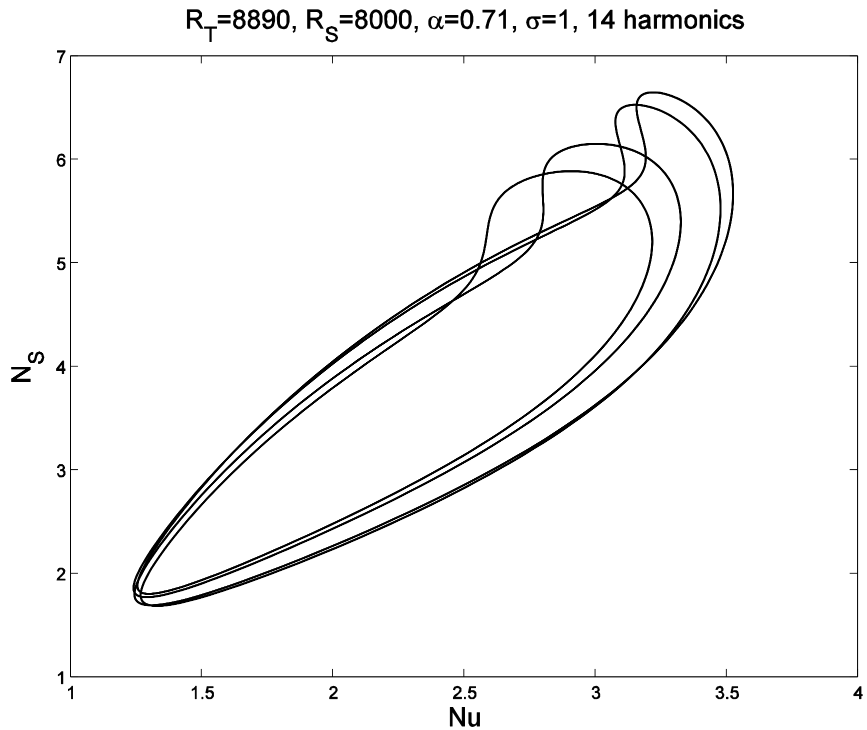
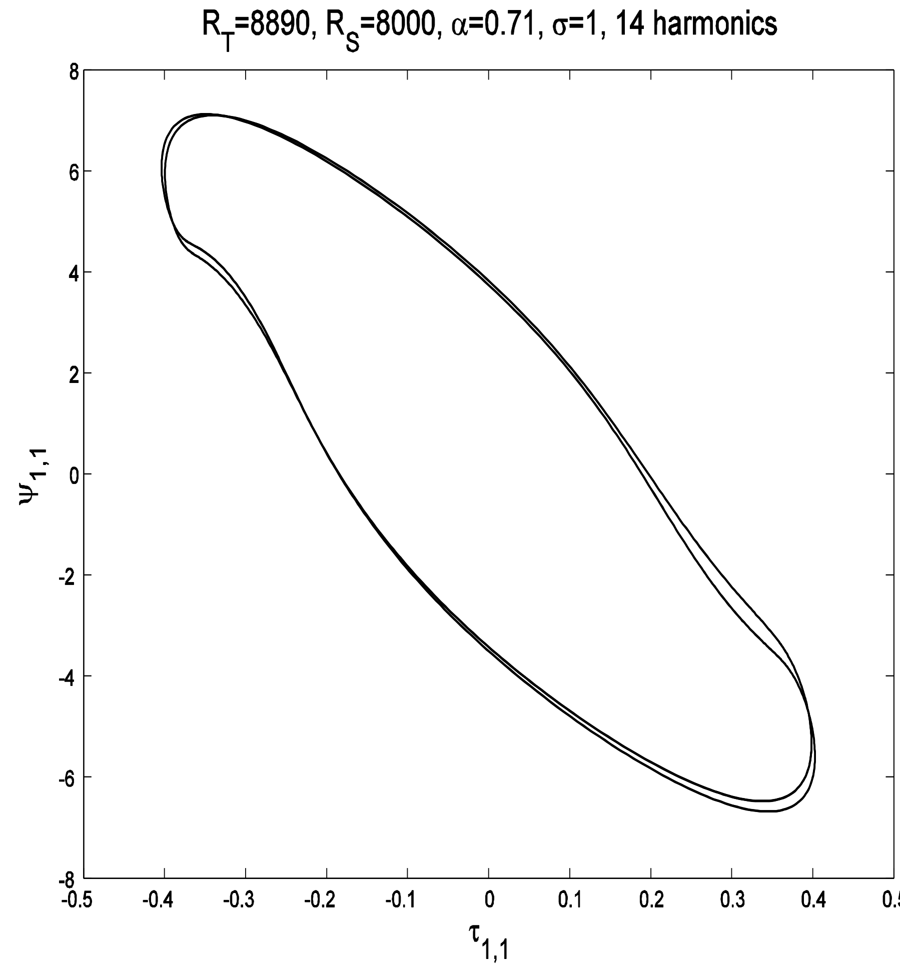
-7.0557
-7.0556
-7.0555



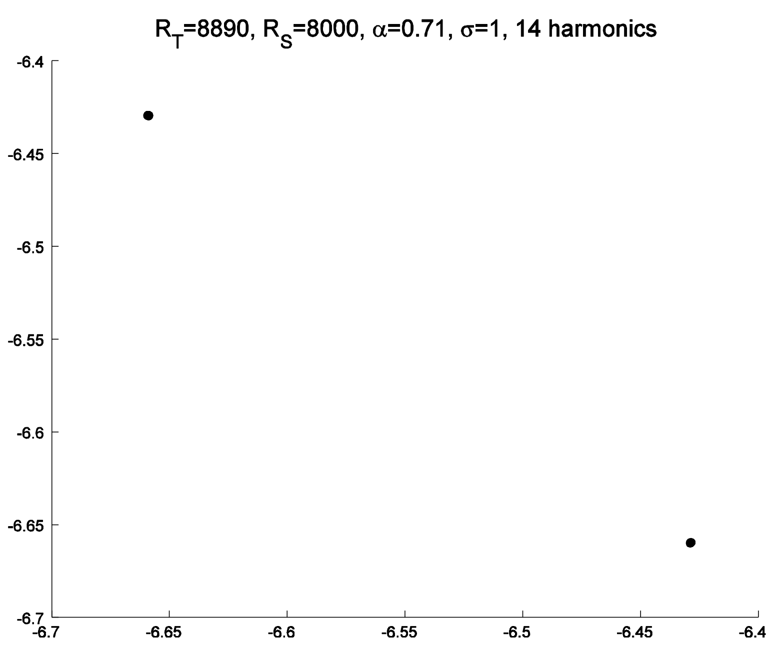
2.2
2
1.8
1.6
1.4
1.2
Nu_T
882
884
886
888
t
1
0.5
0
-0.5
-1
τ_1,1
-1
0
1
U_1,1
1
0.8
0.6
0.4
0.2
0
z
0
0.5
1
x
1
0.8
0.6
0.4
0.2
0
z
0
0.5
1
x



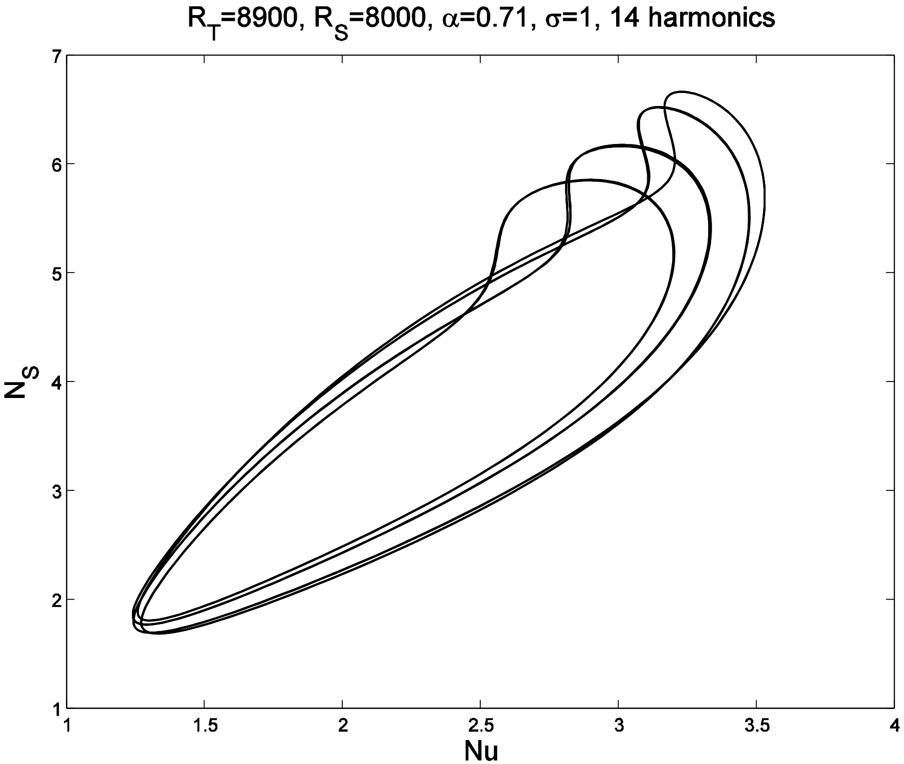
R_T=8890, R_S=8000, α=0.71, σ=1, 14 harmonics
N_S
Nu
1 1.5 2 2.5 3 3.5 4
1 2 3 4 5 6 7

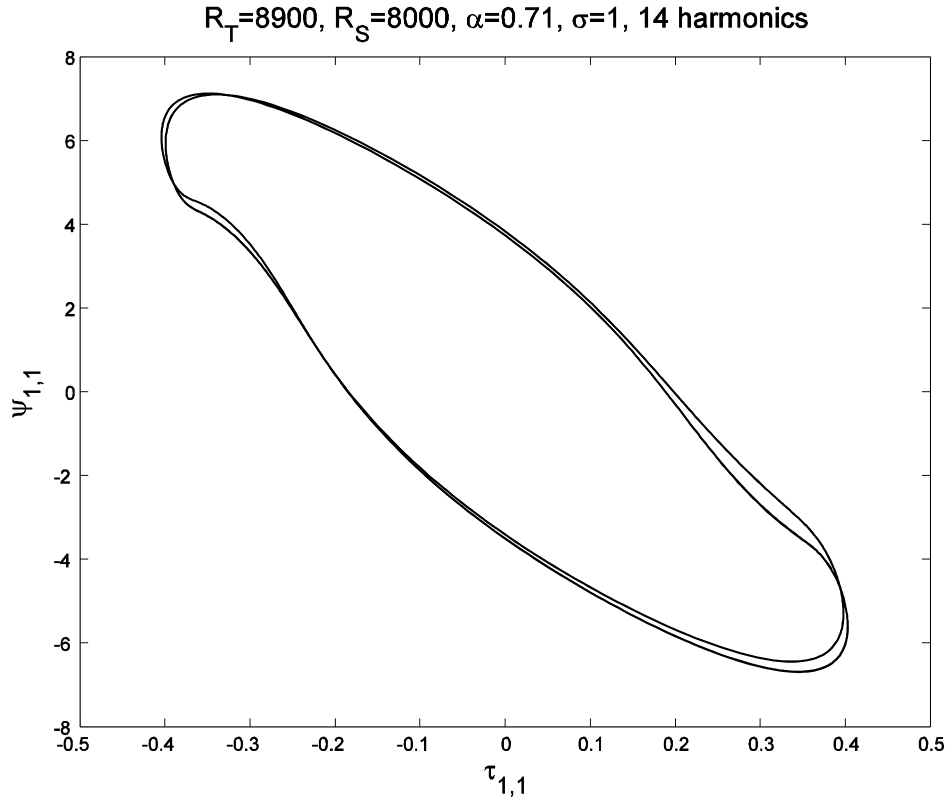


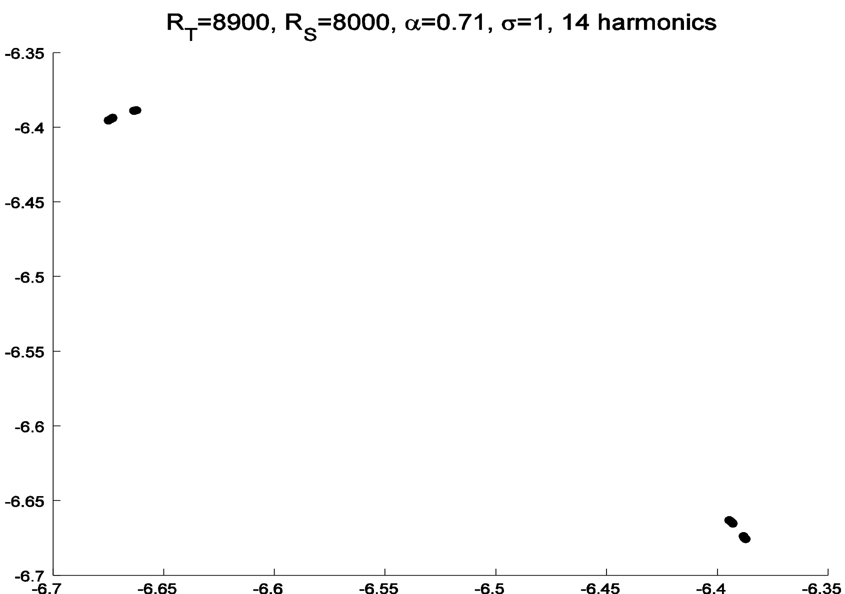
R_T=8890, R_S=8000, α=0.71, σ=1, 14 harmonics
ψ_1,1
τ_1,1
-0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 0.5
-8 -6 -4 -2 0 2 4 6 8

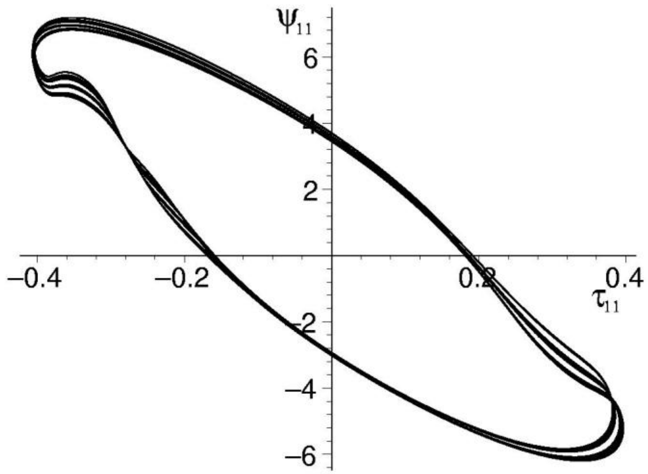


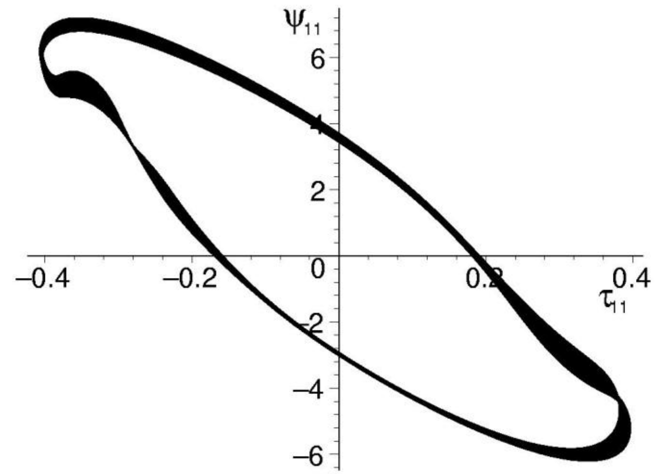
R_T=8890, R_S=8000, α=0.71, σ=1, 14 harmonics
-6.7 -6.65 -6.6 -6.55 -6.5 -6.45 -6.4
-6.4 -6.45 -6.5 -6.55 -6.6 -6.65 -6.7

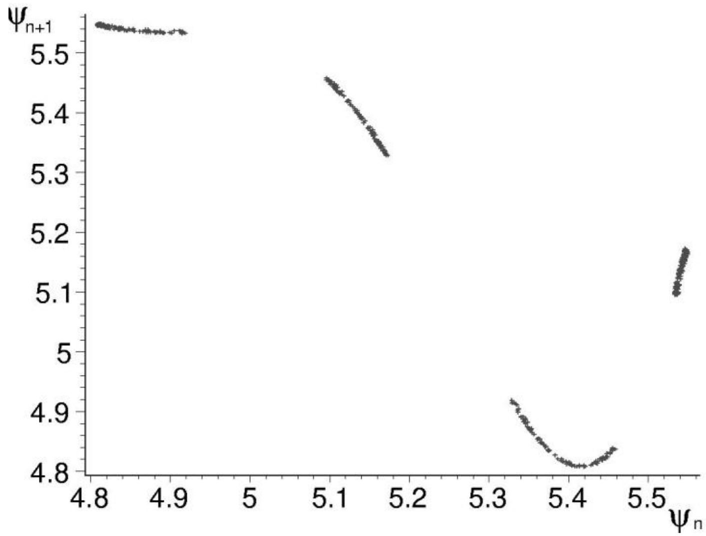

R_T=8900, R_S=8000, α=0.71, σ=1, 14 harmonics
N_S
Nu

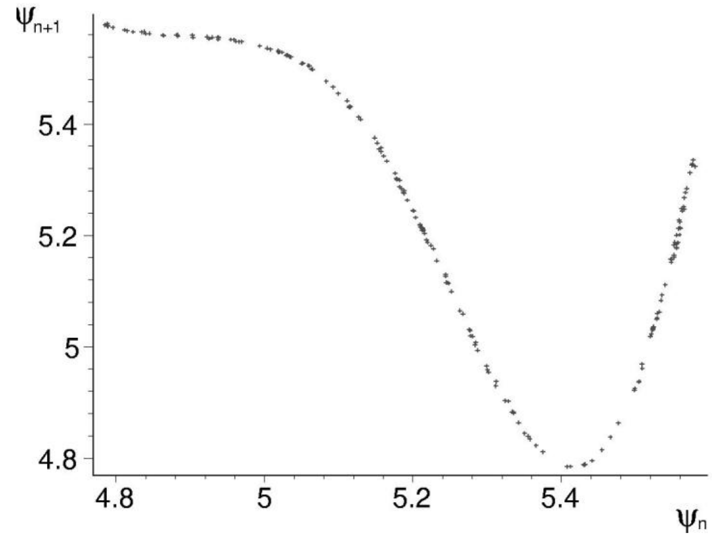

R_T=8900, R_S=8000, α=0.71, σ=1, 14 harmonics
$\psi_{1,1}$
$\tau_{1,1}$

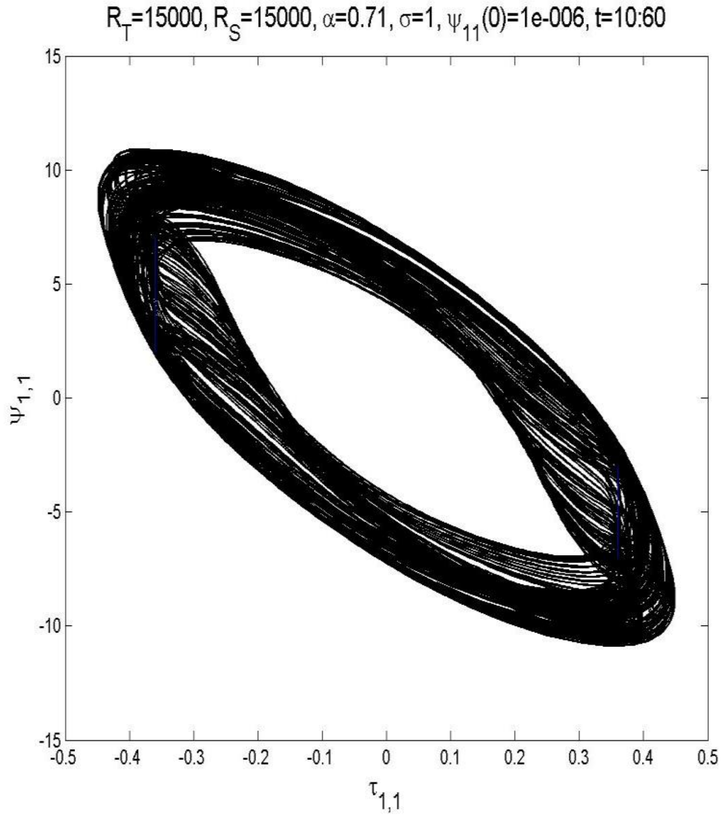

R_T=8900, R_S=8000, α=0.71, σ=1, 14 harmonics


ψ11
6
4
2
−0.4
−0.2
0.2
0.4
τ11
−2
−4
−6

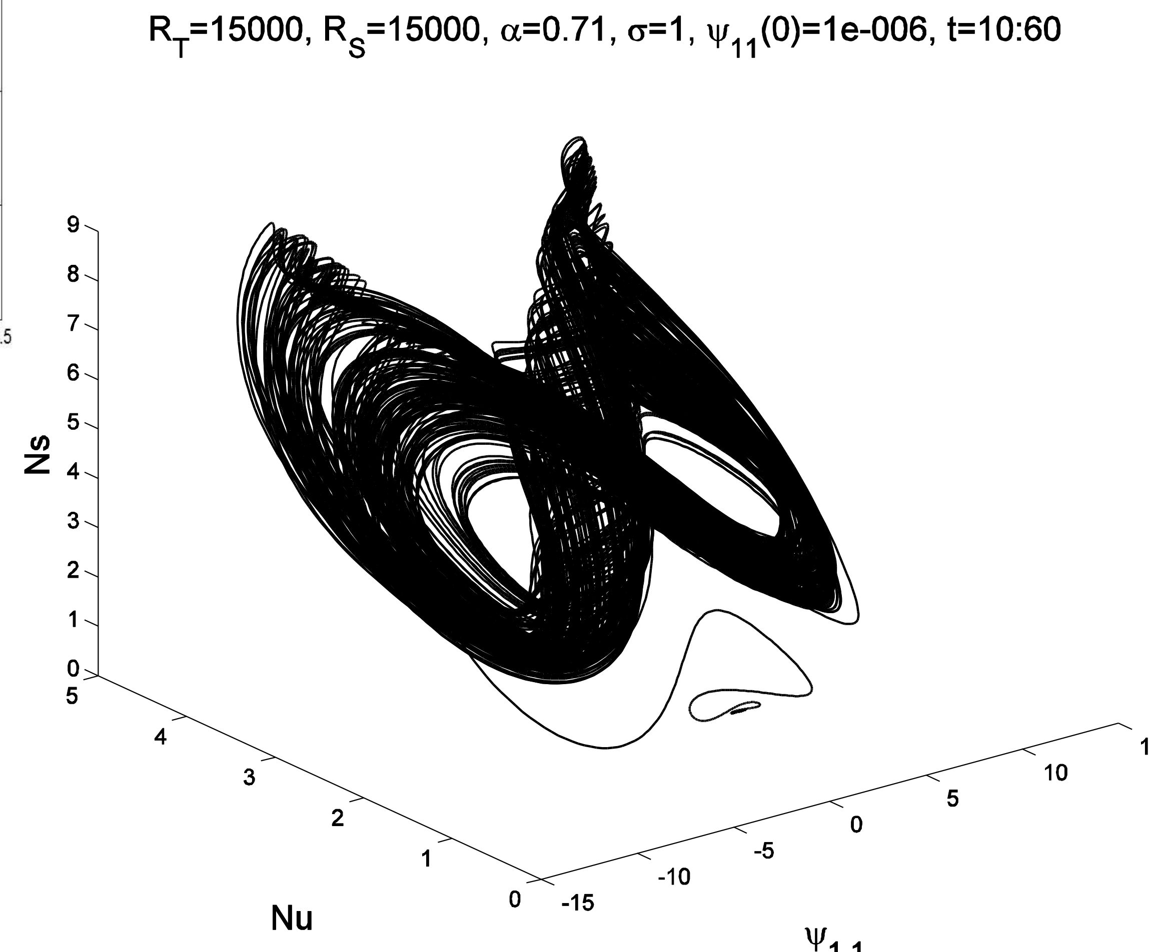

ψ11
6
4
2
0
−0.4
−0.2
0.2
0.4
τ11
−2
−4
−6


ψn+1
5.5
5.4
5.3
5.2
5.1
5
4.9
4.8
4.8
4.9
5
5.1
5.2
5.3
5.4
5.5
ψn


ψn+1
5.4
5.2
5
4.8
4.8
5
5.2
5.4
ψn



R_T=15000, R_S=15000, α=0.71, σ=1, ψ_11(0)=1e-006, t=10:60
ψ_1,1
τ_1,1
15
10
5
0
-5
-10
-15
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5



R_T=15000, R_S=15000, α=0.71, σ=1, ψ_11(0)=1e-006, t=10:60
Ns
Nu
ψ_1,1
9
8
7
6
5
4
3
2
1
0
5
4
3
2
1
0
-15
-10
-5
0
5
10
15

Classical, “normal” fluids

Equation of state

$$\rho = \rho_0(1 - \alpha(T - T_0))$$

In this sense water is “abnormal” fluid.

The interaction of the convective stable and unstable layers of a fluid often occurs in the geophysical and technical applications. It is known that the density of water has a maximum at the temperature close to 4°C. Any convective motions inside the layer result in the interaction of the stable and unstable parts of the layer so the convection differs from classical Rayleigh-Benard convection.

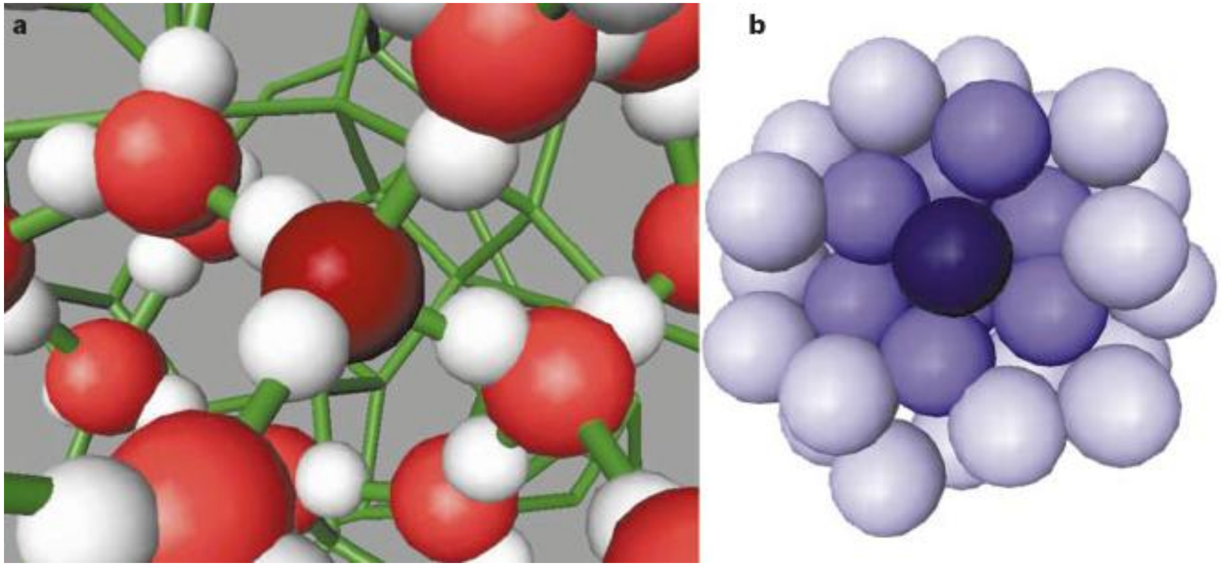
We considered the flat horizontal layer of water with constant temperatures and stress-free conditions at the boundaries. The evolution of the hydrodynamic regimes and the transition to the chaotic motions was investigated numerically by means of pseudospectral method. The horizontal scale of the periodicity cell was chosen with particular attention.

The problem was studied for the case when the point of density maximum in the conductive state is in the middle of the layer. The existence of the different areas of hysteresis was detected. It can be illustrated with the help of the average heat fluxes.

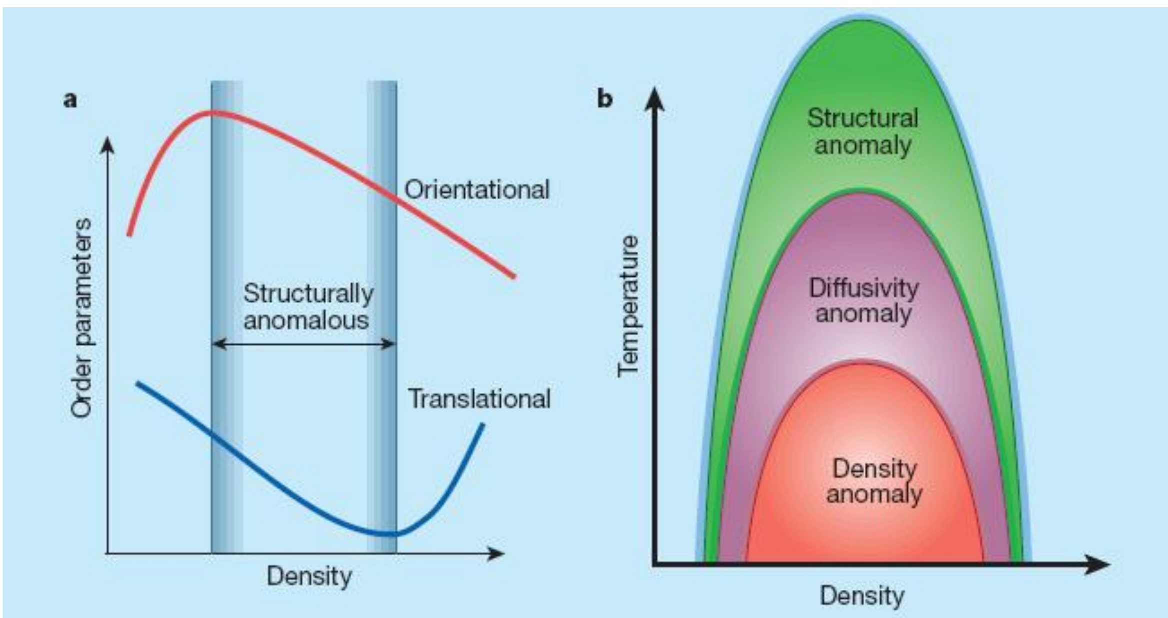
The scenario for the onset of turbulence is the following. At first, steady motions lose the stability and the periodic motion sets up. Then the bifurcation of the period-doubling occurs. After this mode, the quasiperiodic motion is observed for which the attractor in the phase space is a torus. Then the stochastic regime evolves. This chaotic motion shows a strongly pronounced intermittency with random turbulent bursts and the enhancement of the motion while there is a base motion with nearly constant amplitude at the background.

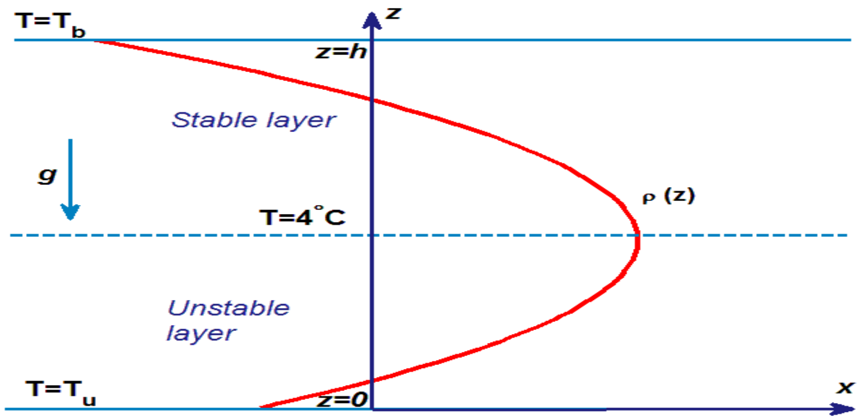
Errington, J. R. & Debenedetti, P. G. *Nature* 409, 318–321 (2001).

Water



Fluid with atomic structure





Boundary conditions:

$$z = 0: \qquad w = 0, \quad \frac{\partial u}{\partial z} = 0, \quad T = T_b$$

$$z = h: \qquad w = 0, \quad \frac{\partial u}{\partial z} = 0, \quad T = T_u$$

$$\min(T_b, T_u) < 4^\circ < \max(T_b, T_u)$$

For water with temperature 0° - 14° for atmospheric pressure experimental data are well described by quadratic approximation with the maximum at about 4° [Veronis 1963]*

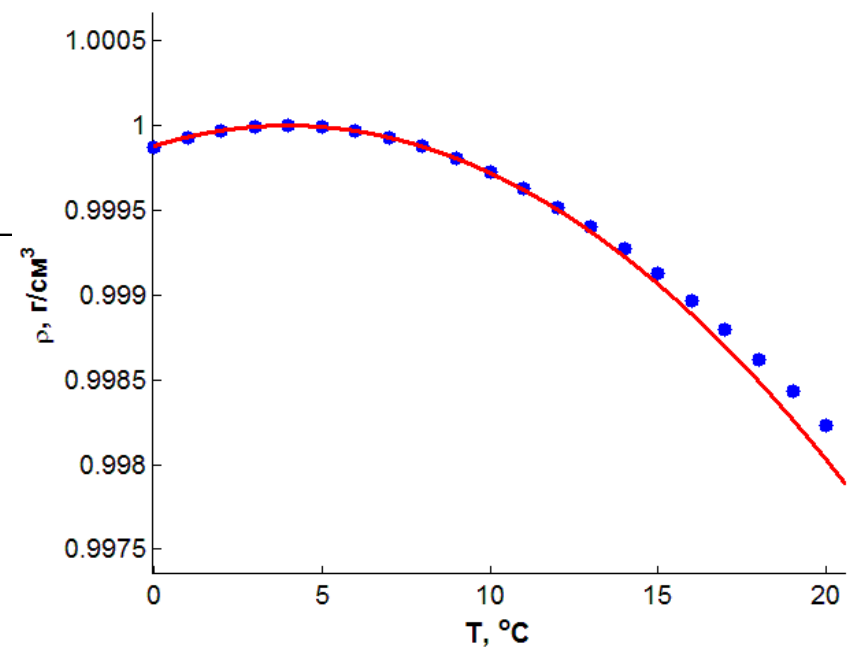
$$\rho = \rho_4 \left(1 - \alpha_4 (T - T_4)^2\right),$$

where

$$\rho_4 = \rho|_{T=4^\circ}, \; T_4 = 4^\circ, \; \alpha_4 = 7.68 \cdot 10^{-6} \, (^\circ\mathrm{C})^{-2}$$

*** B in a more general case temperature maximum is expressed through the pressure (pressure measured in atm.):**

$$T = 3.98^\circ\mathrm{C} - 0.0225(p - 1)$$



General dependency of water density on temperature and pressure:

$$\rho(T,\, p) = \rho_m(p)(1 - \varphi(p)(T - T_m(p))^2)$$

where

$$\rho_m = 999.972 + 4.916021 \cdot 10^{-2} p$$

$$\varphi(p) = 8.572628 \cdot 10^{-6} - 7.061491 \cdot 10^{-9} p$$

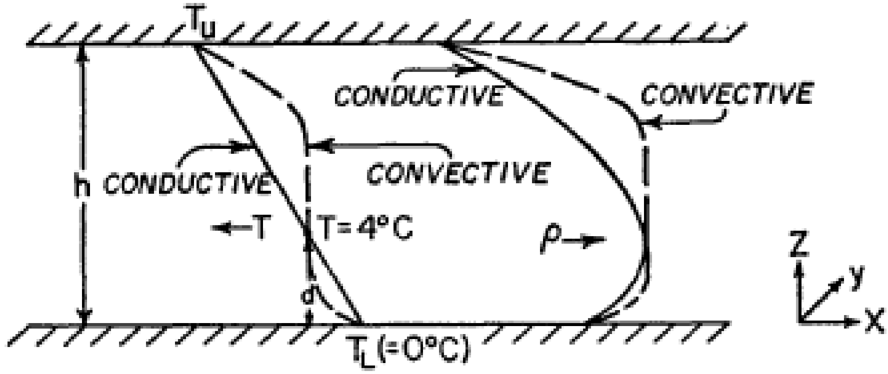
$$T_m(p) = 3.985694 - 0.020617 p$$

here p=0 corresponds to atmospheric pressure.

***Veronis*, G.**

***Penetrative Convection*.**

Astrophysical Journal, V. 137, P. 641-663, 1963.

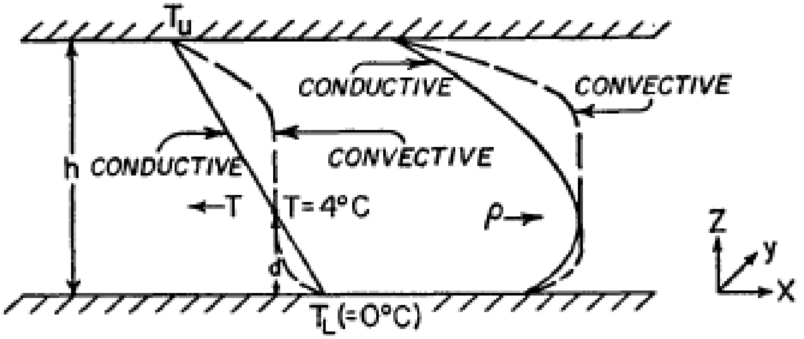


Mathematical formulation

Veronis G. // Astrophys. J. 1963, V. 137, 641-663:

$$\rho = \rho_4\,(1 - \alpha_4 (T - T_4)^2),$$

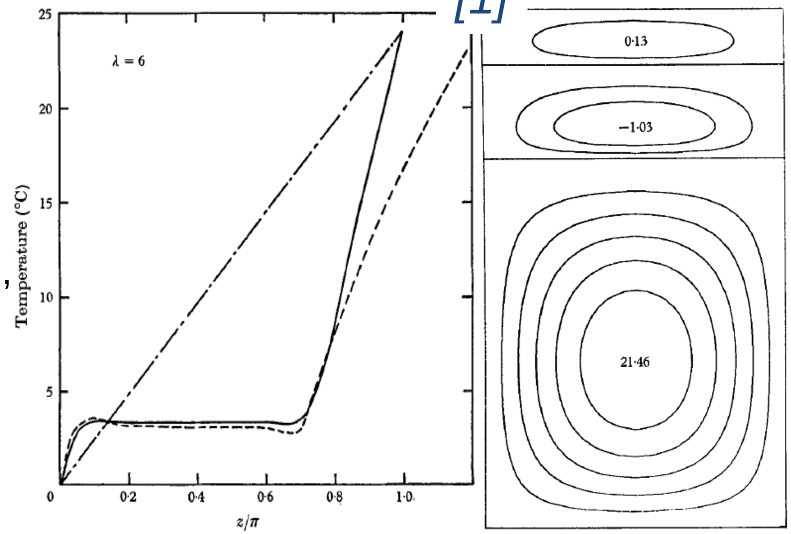
$$\rho_4 = \rho|_{T=4^\circ},\ T_4 = 4^\circ,\ \alpha_4 = 7.68 \cdot 10^{-6}\,(^\circ\mathrm{C})^{-2}$$



Steady and periodic regimes

[1] Musman S. // J. Fluid Mech. 1968, V. 31, 343-360.
[2] Блохина Н.С., Блохин А.С. // ДАН СССР. 1970, Т.193, №4, 805-807
[3] Moore D.R., Weiss N.O. // J. Fluid Mech. 1973, V. 61, 553-581
[4] Блохин А.С. Блохина Н.С., Макаева О.С., Старцева З.П. // Водные ресурсы, 1974. №4, 154-169
[5] Blake K.R., Poulikakos D., Bejan A. // Phys. Fluids. 1984, V. 27, 2608-2616
[6] Надолин К.А. // МЖГ. 1989, №1, 43-49
[7] Tong W., Koster N.J. // Wärme- und Stofftübertragung. 1993, V. 29, 37-49

[1]



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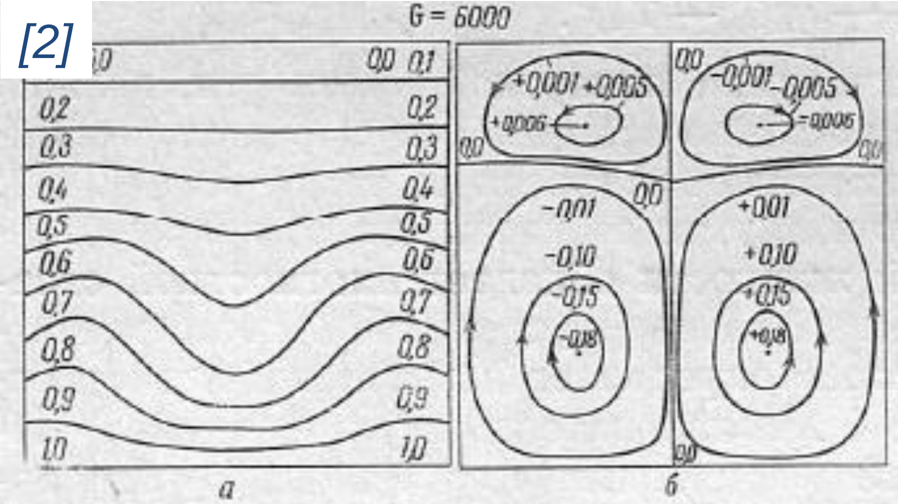

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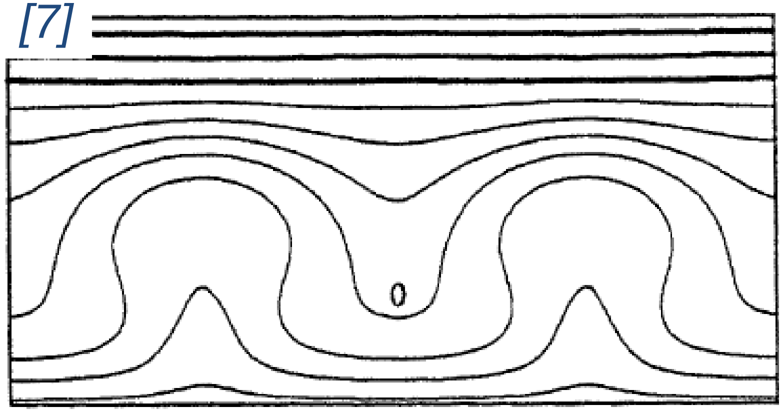


Some references

1. *Veronis G.* Penetrative convection. // Astrophys. J. 1963, V. 137, P. 641-663.
2. *Musman S.* Penetrative convection. // J. Fluid Mech. 1968, V. 31, P. 343-360.
3. *Moore D.R., Weiss N.O.* Nonlinear penetrative convection. // J. Fluid Mech. 1973, V. 61, P. 553-581.
4. *Blake K.R., Poulikakos D., Bejan A.* Natural convection near 4° in a horizontal water layer heated from below. // Phys. Fluids. 1984, V. 27, P. 2608-2616.
5. *Walden R. W., Ahlers G.* Non-Boussinesq and penetrative convection in a cylindrical cell. // J. Fluid Mech. 1981, V. 109, P. 89-114.
6. *Matthews P.C.* A model for the onset of penetrative convection. // J. Fluid Mech. 1988, V. 188, P. 571-583.
7. *Nadolin K.A.* Convection in a horizontal layer with inversion of specific volume. // Fluid dynamics. 1989, №1, C.43-49.
8. *Gershuni G.Z., Zhuhovitsky E.M., Nepomniashi A.A.* Stability of convective structures. // Moscow, Nauka, 1989. 320 c.
9. *Nadolin K.A.* OAbout penetrative convection in the approximation of isothermally incompressible fluid. // Fluid dynamics. 1996, №2, C.40-52.
10. *Tong W., Koster N.J.* Penetrative convection in sublayer of water including density inversion. // Wärme- und Stofftübertragung. 1993, V. 29, P. 37-49.
11. *Large E.D.* An Experimental Investigation of Penetrative Convection in Water Near 4°C. Diss., The Ohio State University, 2010

Initial set of equations:

$$\rho\frac{d\mathbf{v}}{dt} = -\nabla p - g\rho\mathbf{e}_z + \mu\Delta\mathbf{v} + \left(\frac{\mu}{3} + \zeta\right)\nabla(\operatorname{div}\mathbf{v})$$

$$\frac{\partial\rho}{\partial t} + \mathbf{v}\cdot\nabla\rho + \rho\operatorname{div}\mathbf{v} = 0$$

$$\rho C_v\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)T + p\operatorname{div}\mathbf{v} = \varkappa\Delta T + \mu\nabla\cdot(\mathbf{v}\cdot\nabla\mathbf{v}) - \frac{2}{3}\mu(\operatorname{div}\mathbf{v})^2 + Q$$

$$\rho = \rho_4(1 - \alpha_4(T - T_4)^2)$$

Boussinesq approximation

Solution = static solution + perturbations:

$$f = f_m + f_0(z) + \tilde{f}$$

(f – one of the functions ρ, T or p)

Basic assumptions:

$$\frac{\triangle\rho_0}{\rho_m} \equiv \varepsilon \ll 1$$

$$\left|\frac{\tilde{\rho}}{\rho_m}\right| \leq O(\varepsilon)$$

where $\triangle\rho_0 = \max_z \rho_0 - \min_z \rho_0 > 0$ **- maximum functions deviation of** $\rho_0(z)$ **In the layer.**

In heat transfer equation dissipation is neglected

With this assumptions:

$$\frac{d\mathbf{v}}{dt} = -\nabla p + \nu \triangle \mathbf{v} + (\rho - \rho_4) g \mathbf{e}_z$$

$$\frac{dT}{dt} = \kappa \triangle T$$

$$\operatorname{div} \mathbf{v} = 0$$

Equations = static state + perturbations:

$$T_{total} = T_b - T_0(z) + T = T_b - \frac{T_b - T_u}{h} z + T$$

$$p_{total} = p_0(z) + p$$

$$\mathbf{v}_{total} = \mathbf{v} = \{u, v, w\}$$

$$\rho_{total} = \rho_4 + \rho_0(z) + \rho = \rho_4(1 + \alpha_4 T_0^2) + \rho_4 \alpha_4 T(T - 2T_0 + 2(T_b - T_4))$$

$$T_{total} = T_b - T_0(z) + T = T_b - \frac{T_b - T_u}{h} z + T$$

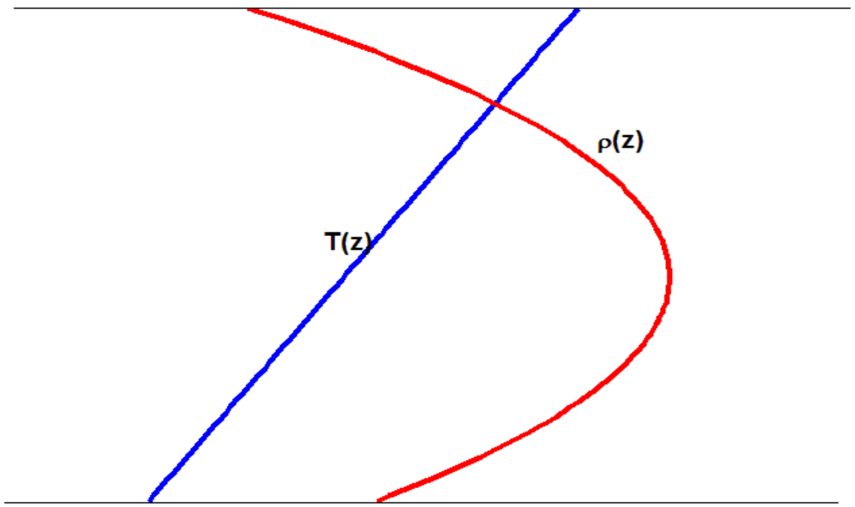
$$p_{total} = p_0(z) + p$$

$$\mathbf{v}_{total} = \mathbf{v} = \{u, v, w\}$$

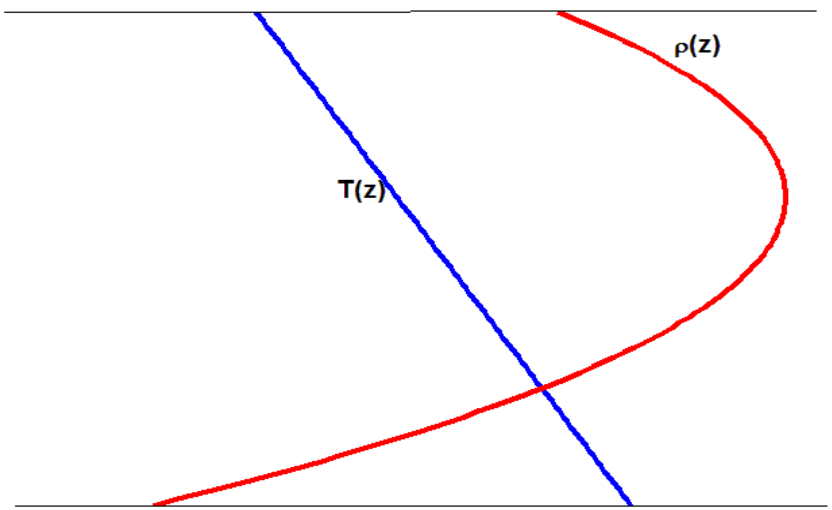
$$\rho_{total} = \rho_4 + \rho_0(z) + \rho = \rho_4(1 + \alpha_4 T_0^2) + \rho_4 \alpha_4 T(T - 2T_0 + 2(T_b - T_4))$$

Static distributions of temperature and density

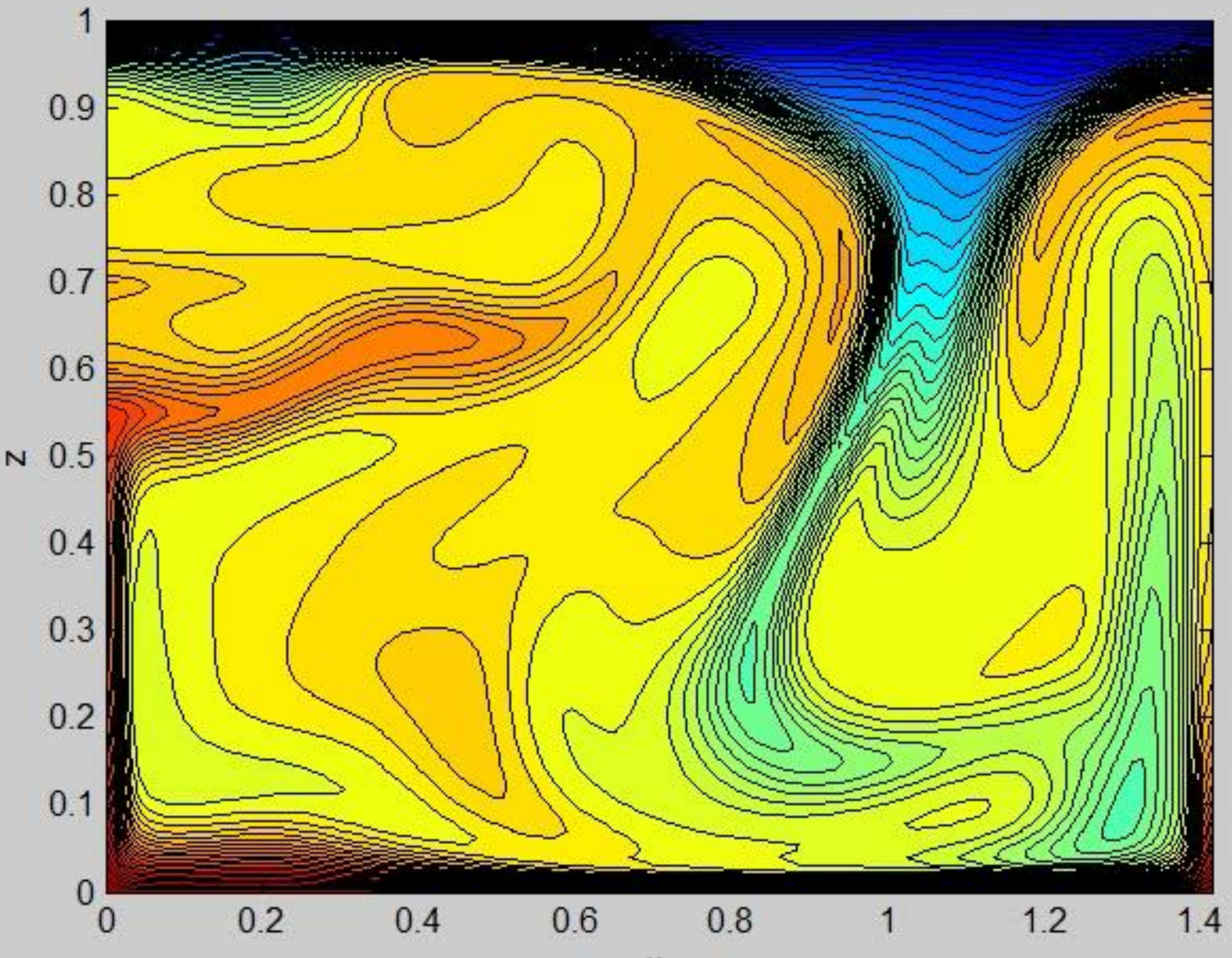
In the case $T_b<T_u$



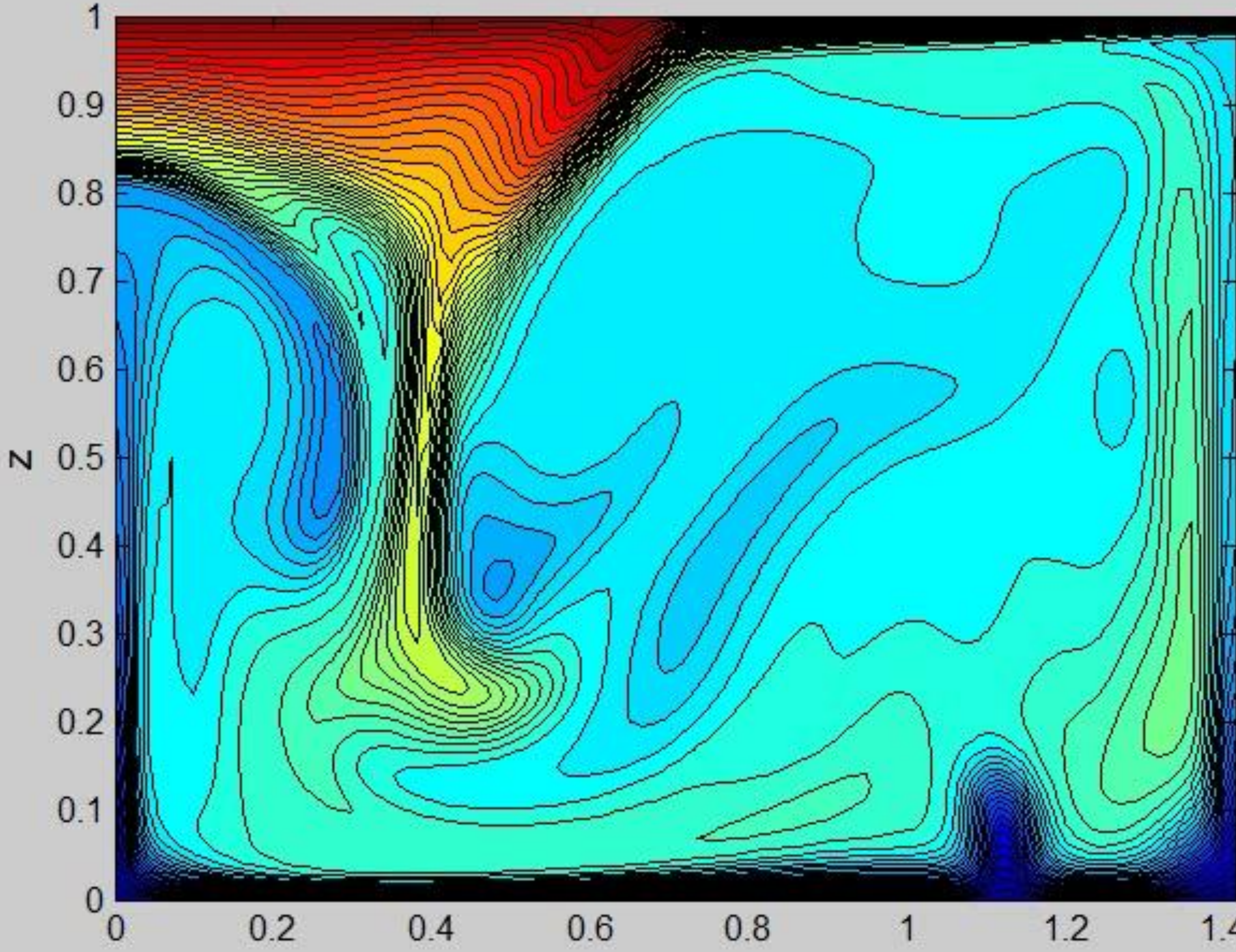
In the case $T_b>T_u$



Point of density maximum can be anywhere inside the layer and is determined by temperature on boundaries.



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
z
0
0.2
0.4
0.6
0.8
1
1.2
1.4


z
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4

Nondimensional system of equations for perturbations :

$$\frac{\partial \mathbf{v}}{\partial t} - \zeta = -\nabla q + \sigma \, \triangle \mathbf{v} + \sigma R T (T + 2\lambda - 2z) \mathbf{e}_z$$

$$\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) \, T = w + \triangle T$$

$$\operatorname{div} \mathbf{v} = 0$$

Notations:

$$q = \sigma p + \frac{1}{2} |\mathbf{v}|^2, \; \zeta = \mathbf{v} \times \operatorname{rot} \mathbf{v}$$

$$\zeta_1 = -w \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$

$$\zeta_3 = u \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$

Parameters:

$$\sigma = \frac{\nu}{\kappa}$$

Prandtl number
(for water at 4°C σ=11.5968)

$$R = \frac{g \alpha_4 h^3 (T_b - T_u)^2}{\nu \kappa}$$

Rayleigh number

$$\lambda = \frac{T_b - T_4}{T_b - T_u}$$

location of density maximum in conductive state

With stress free boundary conditions on vertical boundaries, trigonometric decomposition gives:

$$u = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{u}_{mn} \sin(\pi m \alpha x) \cos(\pi n z), \; w = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{w}_{mn} \cos(\pi m \alpha x) \sin(\pi n z),$$

$$T = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{T}_{mn} \cos(\pi m \alpha x) \sin(\pi n z)$$

then

$$q = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{q}_{mn} \cos(\pi m \alpha x) \cos(\pi n z),$$

$$\zeta_1 = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{\zeta}_{1\;mn} \sin(\pi m \alpha x) \cos(\pi n z), \; \zeta_3 = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{\zeta}_{3\;mn} \cos(\pi m \alpha x) \sin(\pi n z),$$

$$(\mathbf{v} \cdot \nabla)\, T = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{a}_{mn} \cos(\pi m \alpha x) \sin(\pi n z),$$

$$T(T + 2\lambda - 2z) \stackrel{def}{=} T_{nonlin} = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{b}_{mn} \cos(\pi m \alpha x) \sin(\pi n z)$$

Pseudospectral method

Let the Fourier coefficients are known for functions f and g: $\hat{f}_{mn}$ и $\hat{g}_{mn}$

I.e. it is nessesary to find Fourier of their multiplication: $a_{mn} \equiv \widehat{fg}_{mn} = ?$

Algorithm:

$$\hat{f}_{mn} \xrightarrow{IFFT} f_{kl} \equiv f(x_k, z_l)$$

$$\hat{g}_{mn} \xrightarrow{IFFT} g_{kl} \equiv g(x_k, z_l)$$

⇓

$$\left.\begin{array}{l} f_{kl} \equiv f(x_k, z_l) \\ g_{kl} \equiv g(x_k, z_l) \end{array}\right\} \longrightarrow (fg)_{kl} \equiv f_{kl} \cdot g_{kl}$$

⇓

$$(fg)_{kl} \xrightarrow{FFT} a_{mn} \equiv \widehat{fg}_{mn}$$

System for Fourier coefficients

$$\frac{d\hat{u}_{mn}}{dt} = \hat{\zeta}_{1\,mn} - \frac{B_m^X}{C_{mn}} \left(A_m^X \hat{\zeta}_{1\,mn} + A_n^Z \hat{\zeta}_{3\,mn} + \sigma R\, A_n^Z \hat{b}_{mn} \right) + \sigma C_{mn} \hat{u}_{mn}$$

$$\frac{d\hat{w}_{mn}}{dt} = \hat{\zeta}_{3\,mn} - \frac{B_n^Z}{C_{mn}} \left(A_m^X \hat{\zeta}_{1\,mn} + A_n^Z \hat{\zeta}_{3\,mn} + \sigma R\, A_n^Z \hat{b}_{mn} \right) + \sigma C_{mn} \hat{w}_{mn} + \sigma R \hat{b}_{mn}$$

$$\frac{d\hat{T}_{mn}}{dt} = -\hat{a}_{mn} + \hat{w}_{mn} + C_{mn} \hat{T}_{mn}$$

Nonlinear terms:

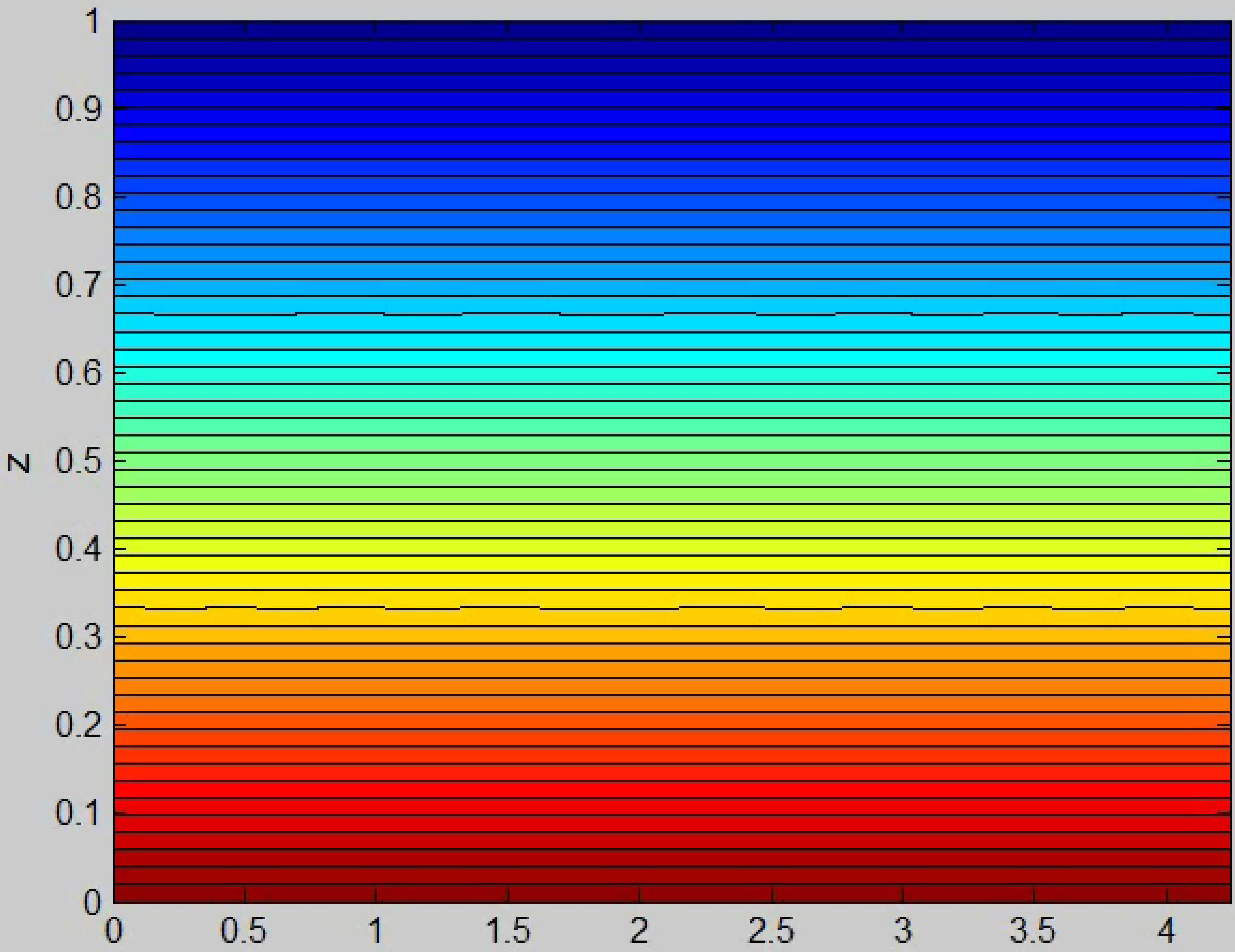
$$(\mathrm{v} \cdot \nabla)\, T = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{a}_{mn} \cos(\pi m \alpha x) \sin(\pi n z),$$

$$T(T + 2\lambda - 2z) \overset{def}{=} T_{nonlin} = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{b}_{mn} \cos(\pi m \alpha x) \sin(\pi n z)$$

$$\zeta_1 = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{\zeta}_{1\,mn} \sin(\pi m \alpha x) \cos(\pi n z), \quad \zeta_3 = \sum_{m=0}^{M} \sum_{n=0}^{N} \hat{\zeta}_{3\,mn} \cos(\pi m \alpha x) \sin(\pi n z),$$

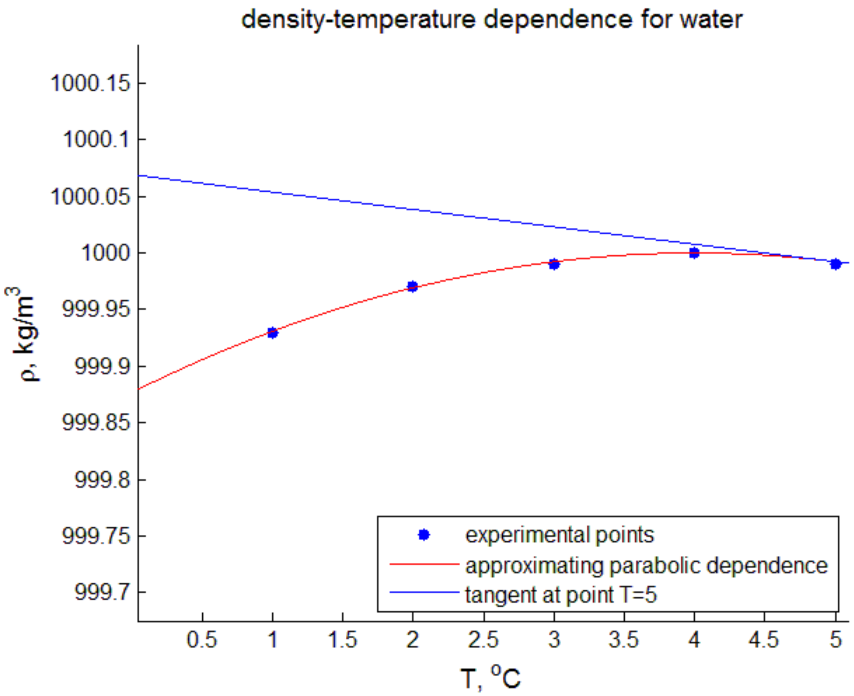
Notations for coefficients corresponding to spacial differentiation in spectral space:

$$A_m^{\mathrm{X}} = \pi m \alpha, \; B_m^{\mathrm{X}} = -\pi m \alpha,$$
$$A_n^{\mathrm{Z}} = \pi n, \; B_n^{\mathrm{Z}} = -\pi n,$$
$$C_{mn} = -\pi^2 (m^2 \alpha^2 + n^2)$$



z
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.5
1
1.5
2
2.5
3
3.5
4

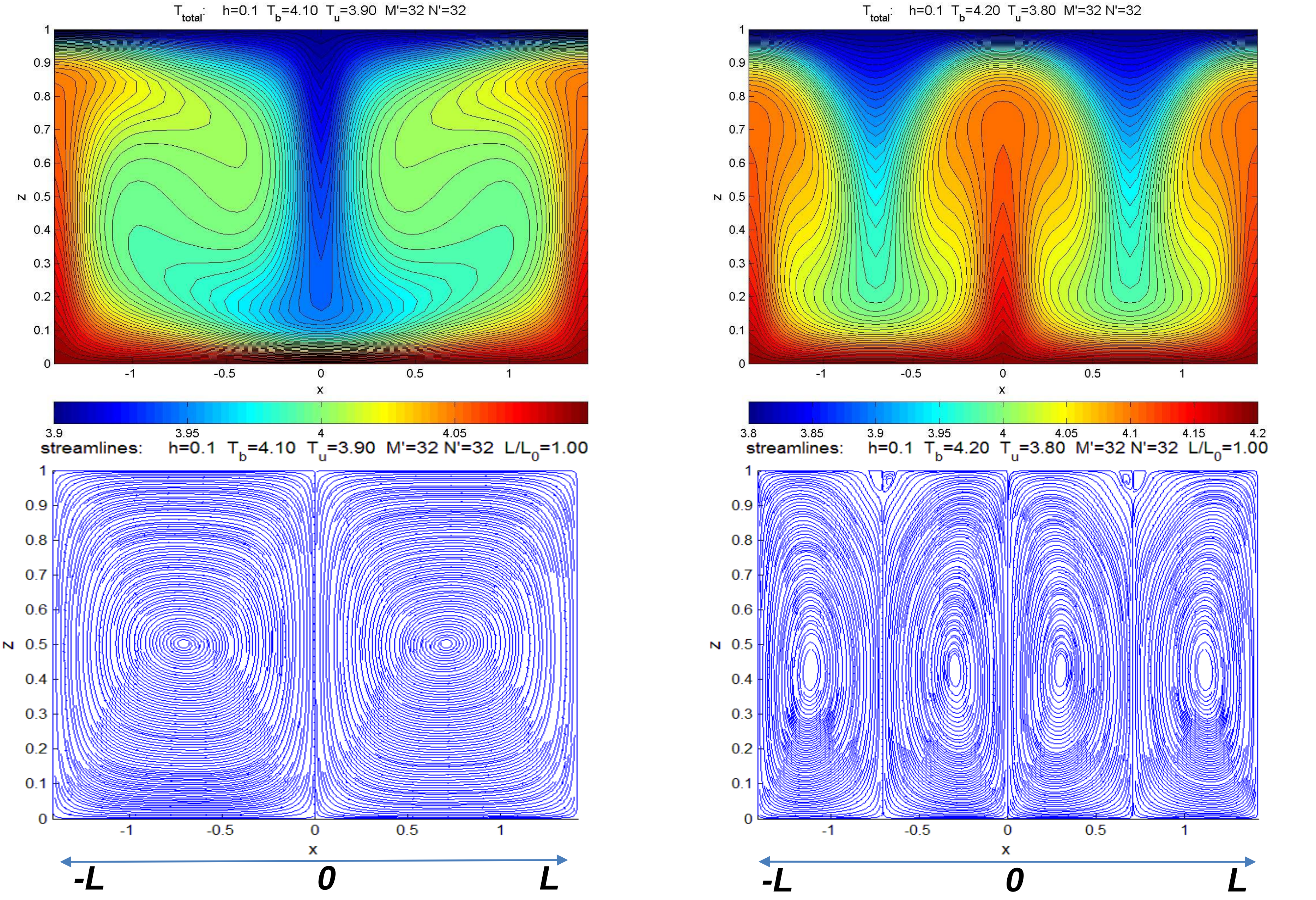
Comparison of results of penetrative convection and classical RB convection in the case of equal heights of stable and unstable layers



To visualise the difference, conductive temperature gradient is taken to be equal to gradient on one of the boundaries for penetrative convection

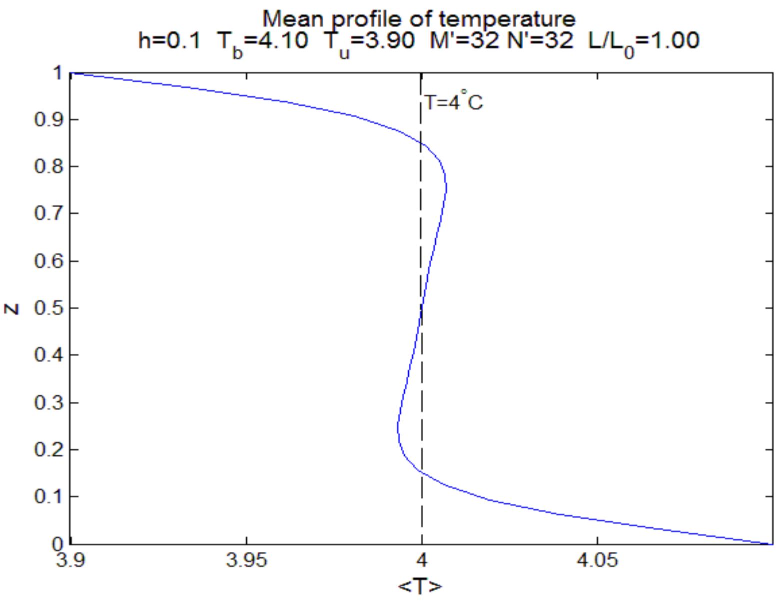
Classical RB (lhs) and penetrative (rhs) convection, steady mode (L/L_0=1)

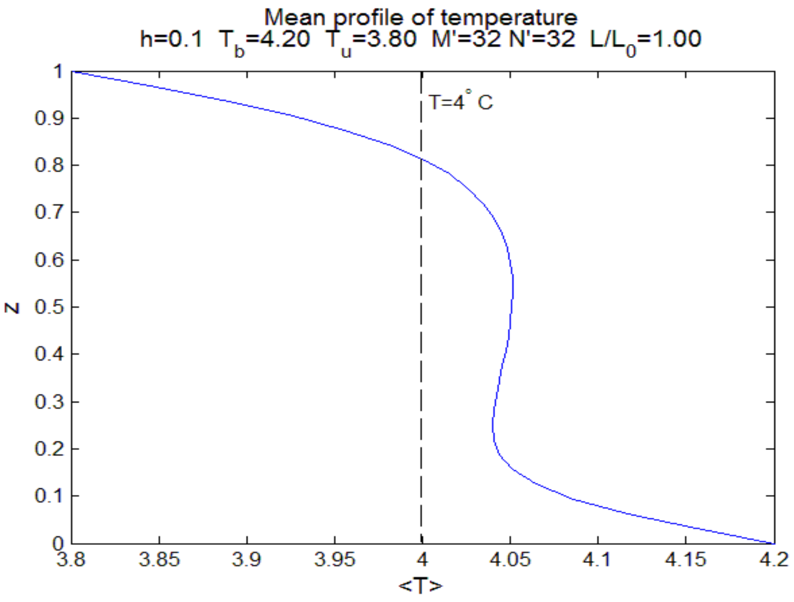
Isotherms and streamlines



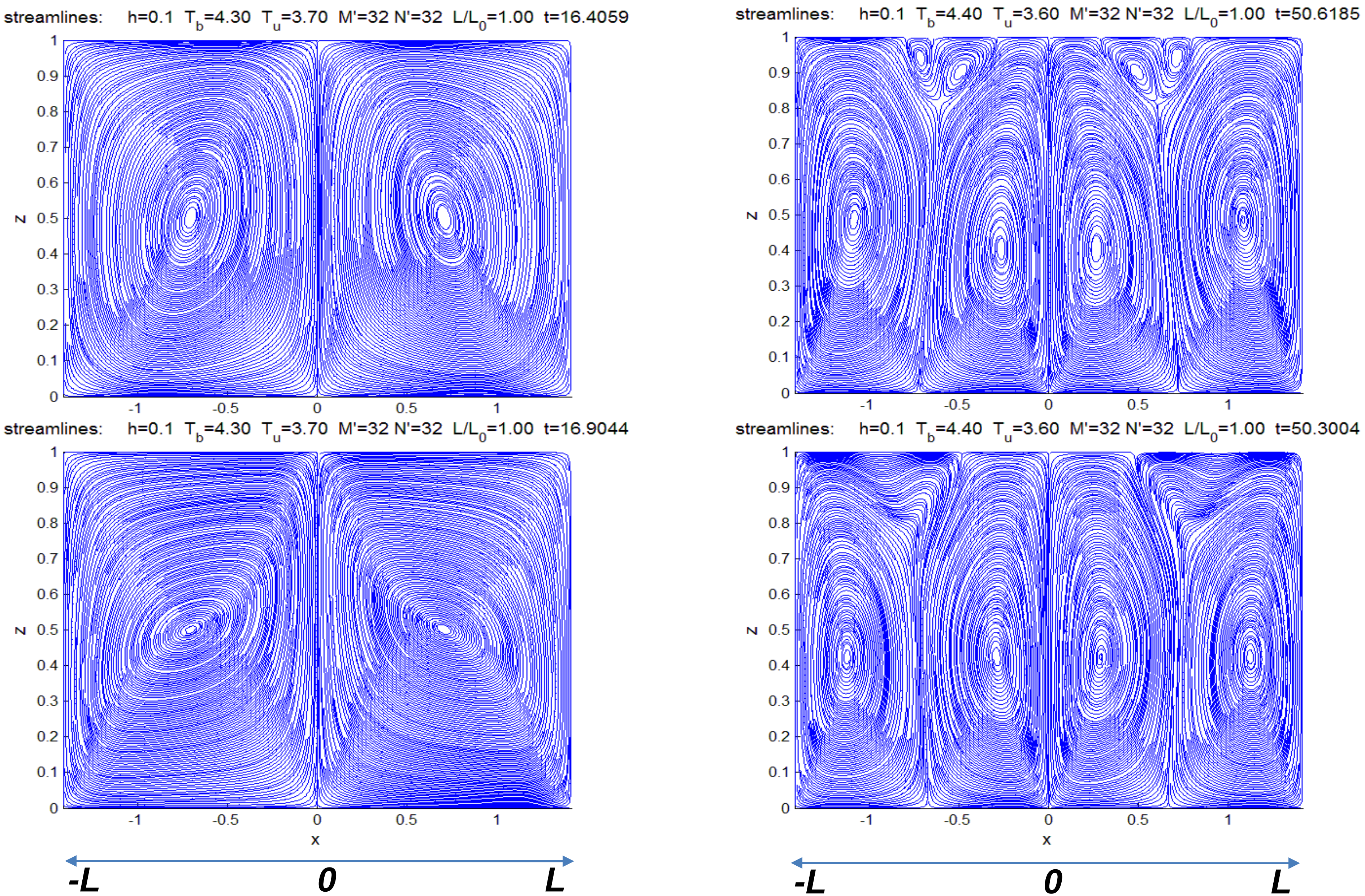
Classical RB (lhs) and penetrative (rhs) convection: steady mode (L/L_0=1)

Mean temperature profile



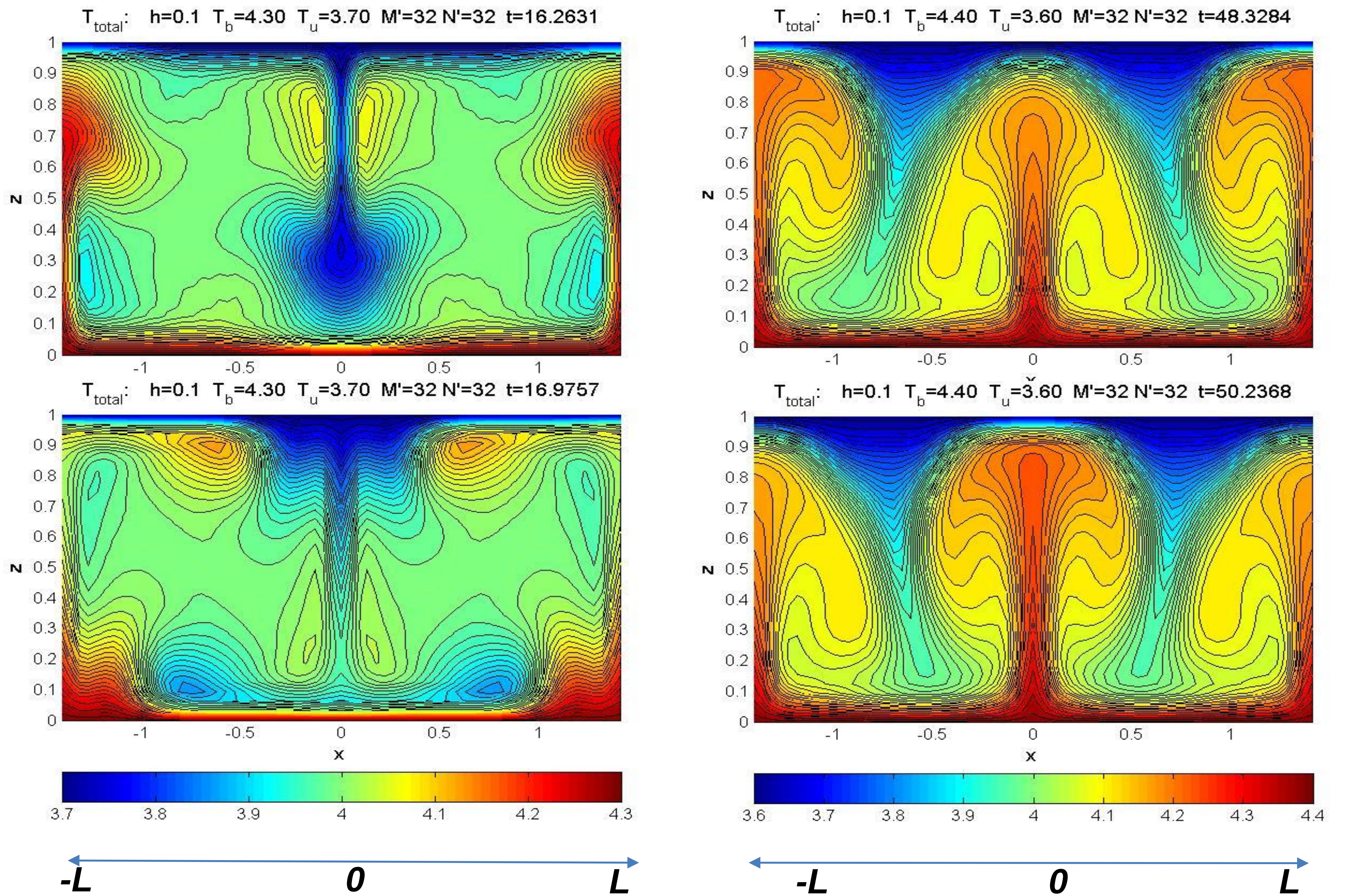


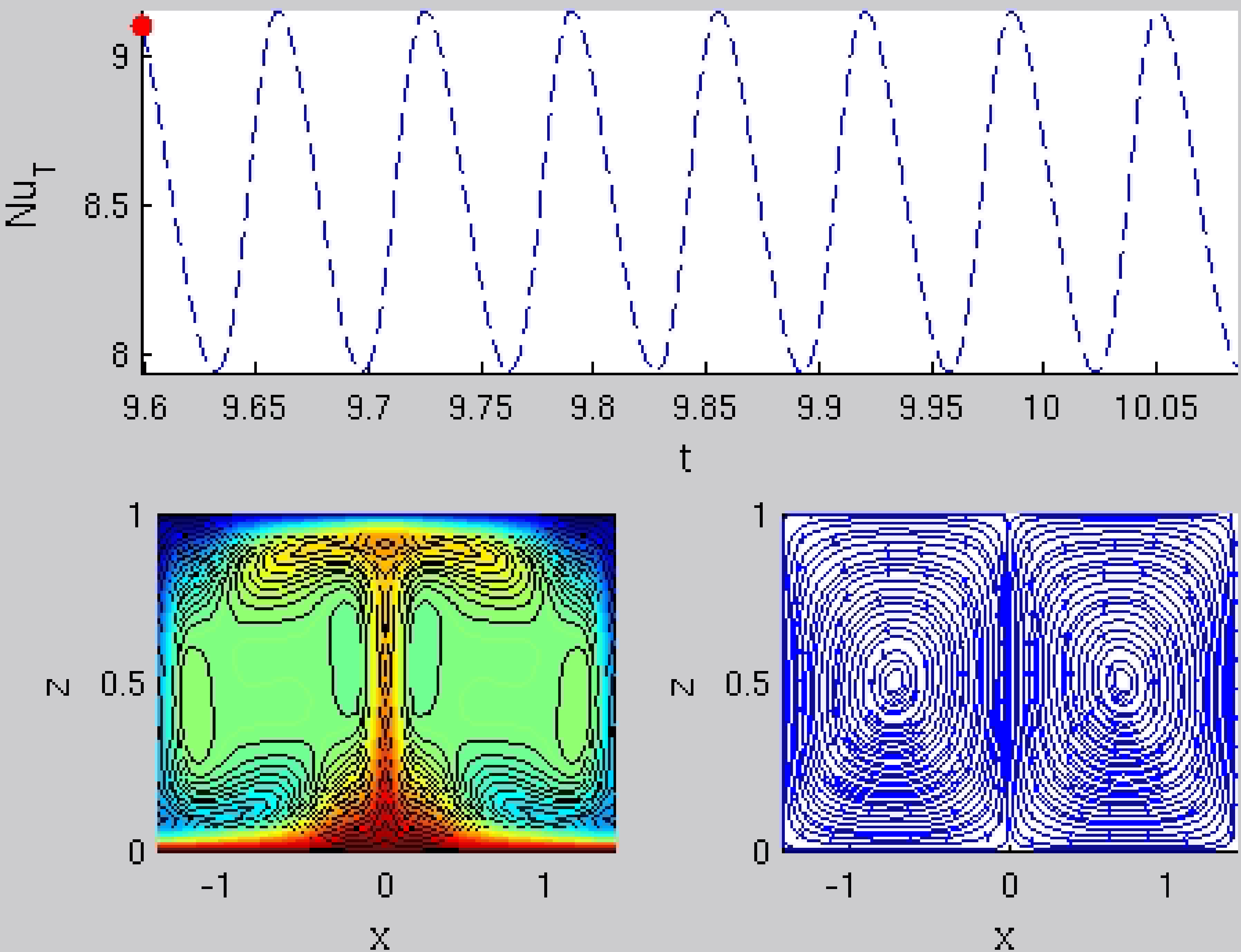
Classical RB (lhs) and penetrative (rhs) convection, time-periodical mode (L/L0=1). *Streamlines*



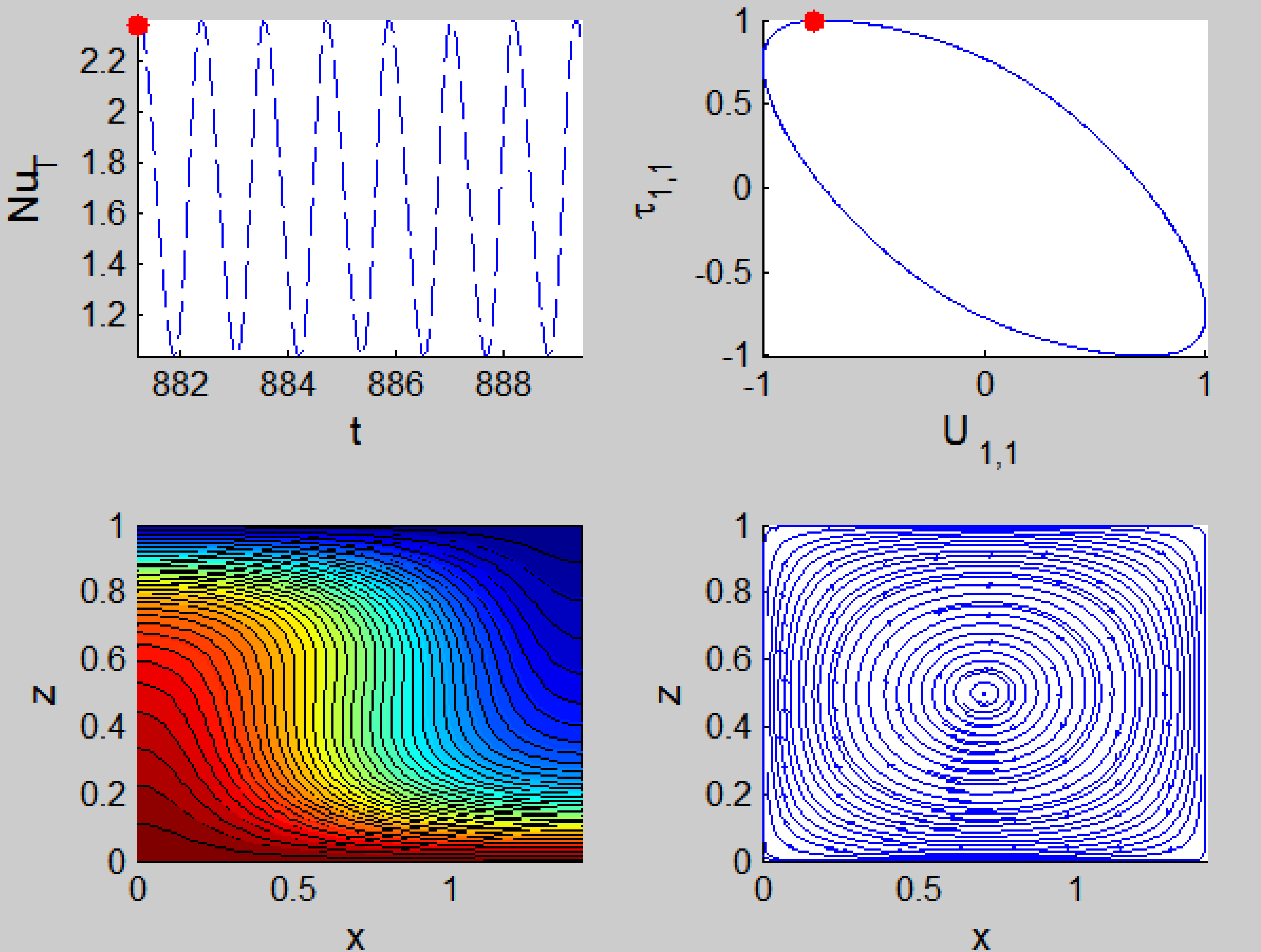
Classical RB (lhs) and penetrative (rhs) convection, time-periodical mode (L/L0=1)

Temperature distribution

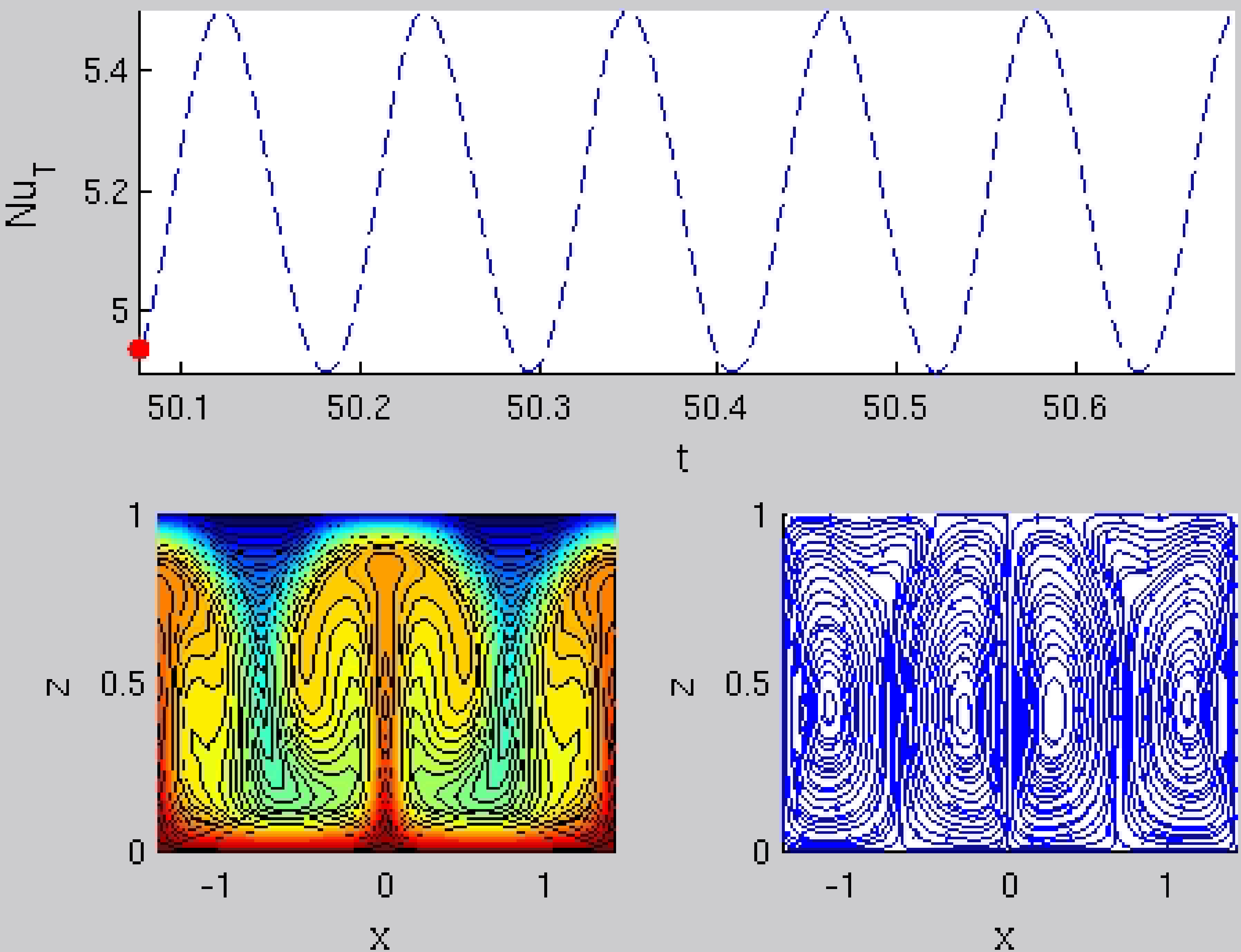




Nu_T
9
8.5
8
9.6
9.65
9.7
9.75
9.8
9.85
9.9
9.95
10
10.05
t
z
1
0.5
0
-1
0
1
x
z
1
0.5
0
-1
0
1
x



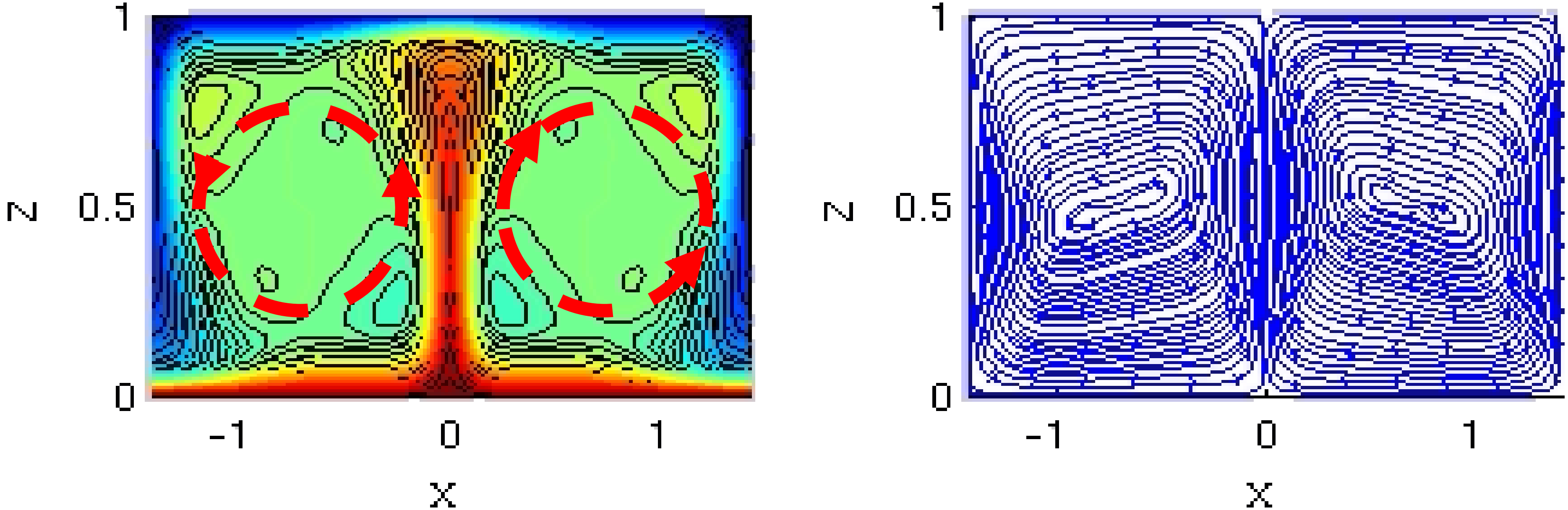
Nu_T
t
882
884
886
888
2.2
2
1.8
1.6
1.4
1.2
τ_1,1
U_1,1
1
0.5
0
-0.5
-1
z
x
0.8
0.6
0.4
0.2



5.4
5.2
5
Nu_T
50.1
50.2
50.3
50.4
50.5
50.6
t
1
0.5
0
z
-1
0
1
x
1
0.5
0
z
-1
0
1
x

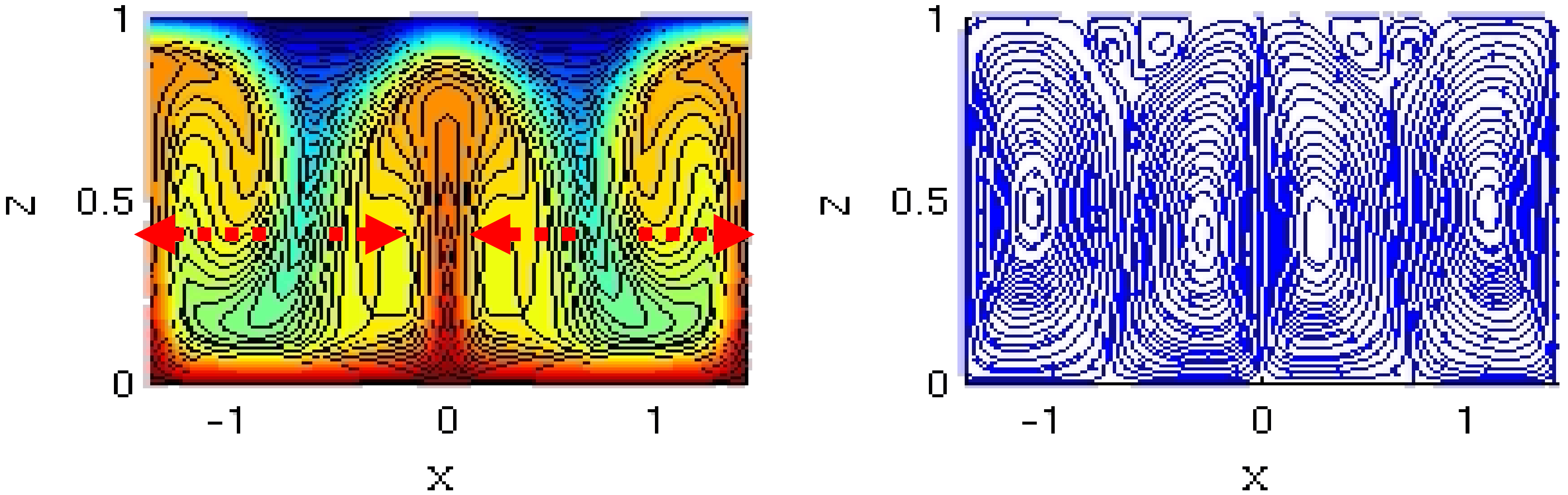
Rayleigh-Benard convection (linear dependancy ρ(T))

Periodic motion



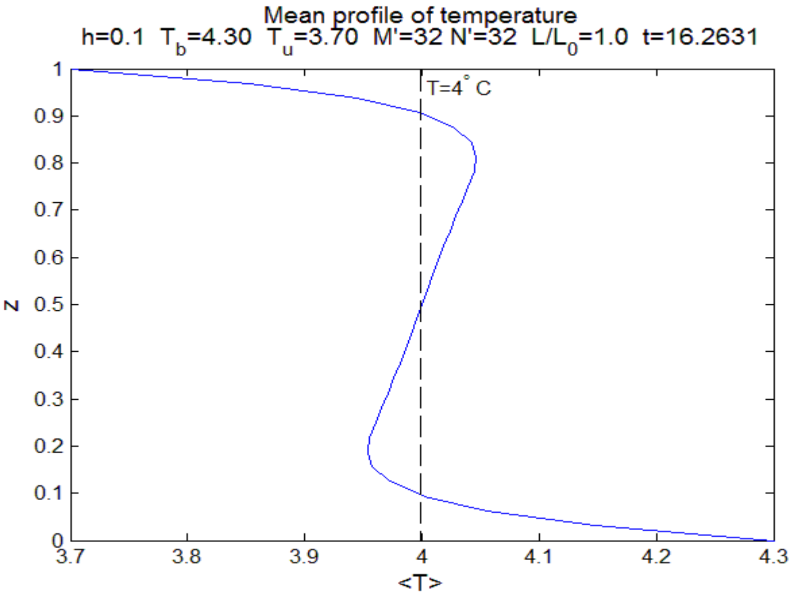
Penetrative convection (quadratic ρ(T))

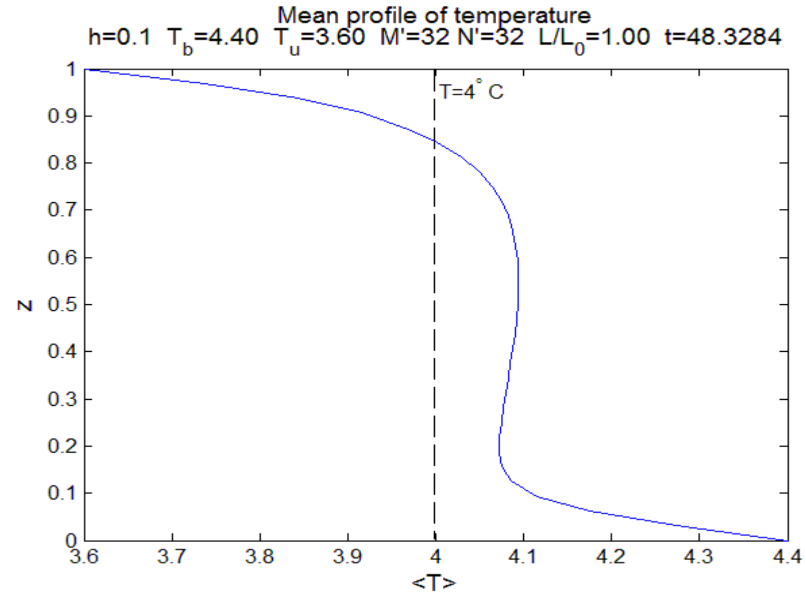
Periodic motion

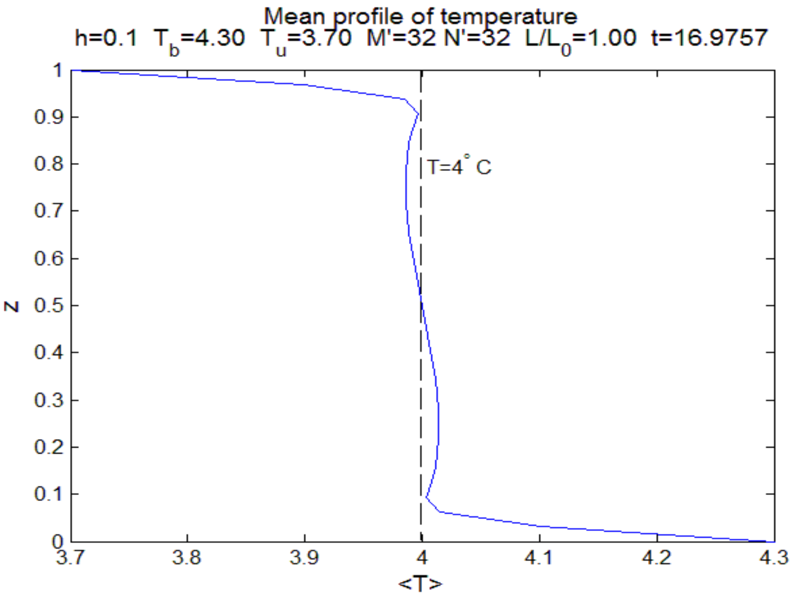


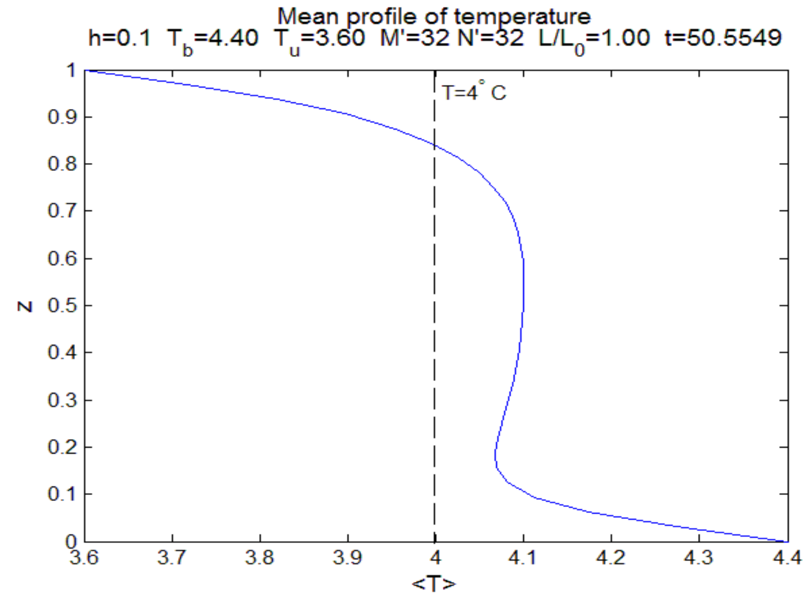
Classical RB (lhs) and penetrative (rhs) convection, time-periodical mode (L/L0=1)

Mean temperature profiles





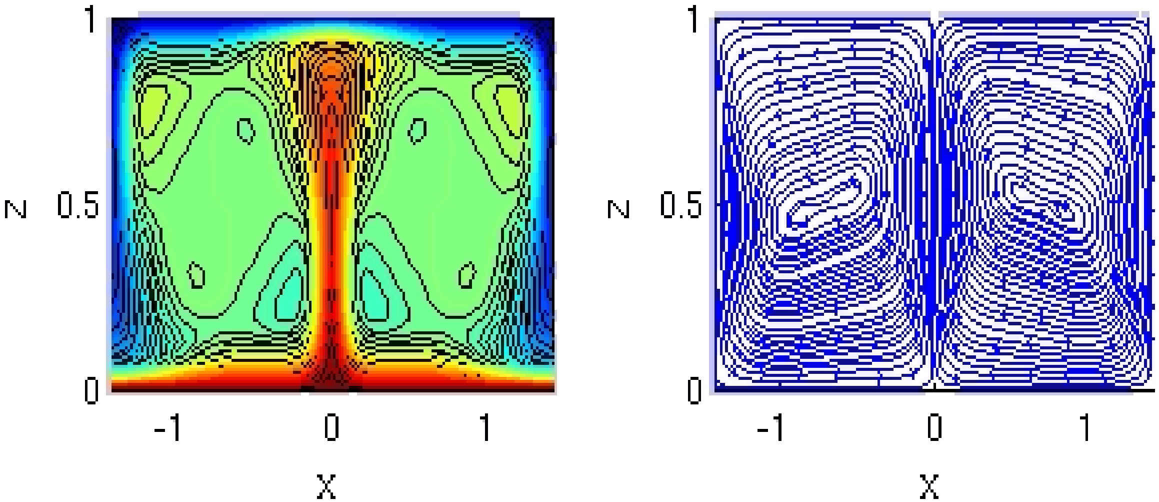




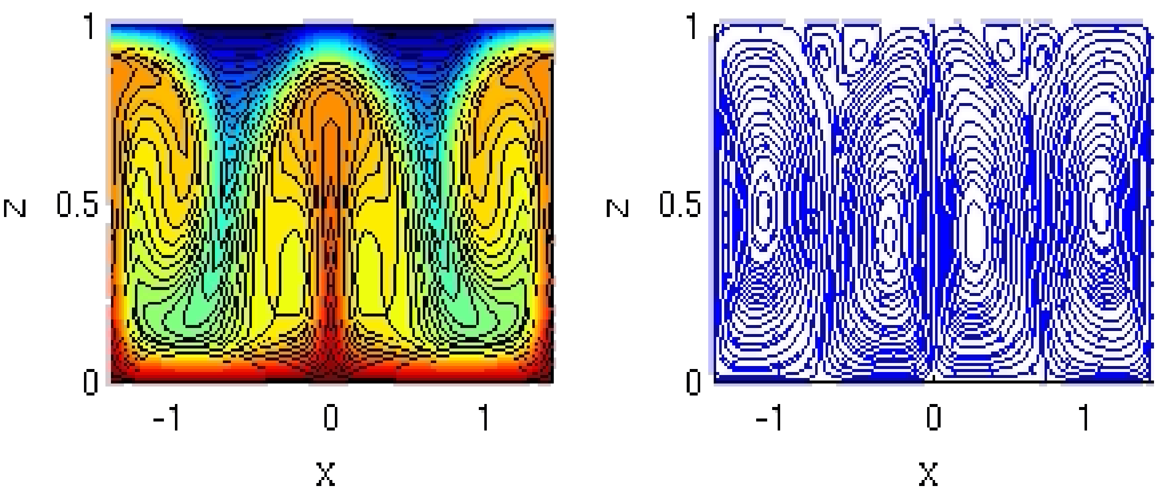
Differences from Rayleigh-Benard convection (linear ρ(T) dependence)

The point of density maximum is in the middle plane of the layer for penetrative convection (heating from below).

Rayleigh-Benard convection (isotherms, streamlines)



Penetrative convection (isotherms, streamlines)



RB:

-- motions arise only if there is heating from below

-- average temperature in the periodicity cell equals 4° C

-- periodic mode is characterized by the circular motions in the half of the cell and existence of small vortices in the middle of larger structures

Penetrative convection:

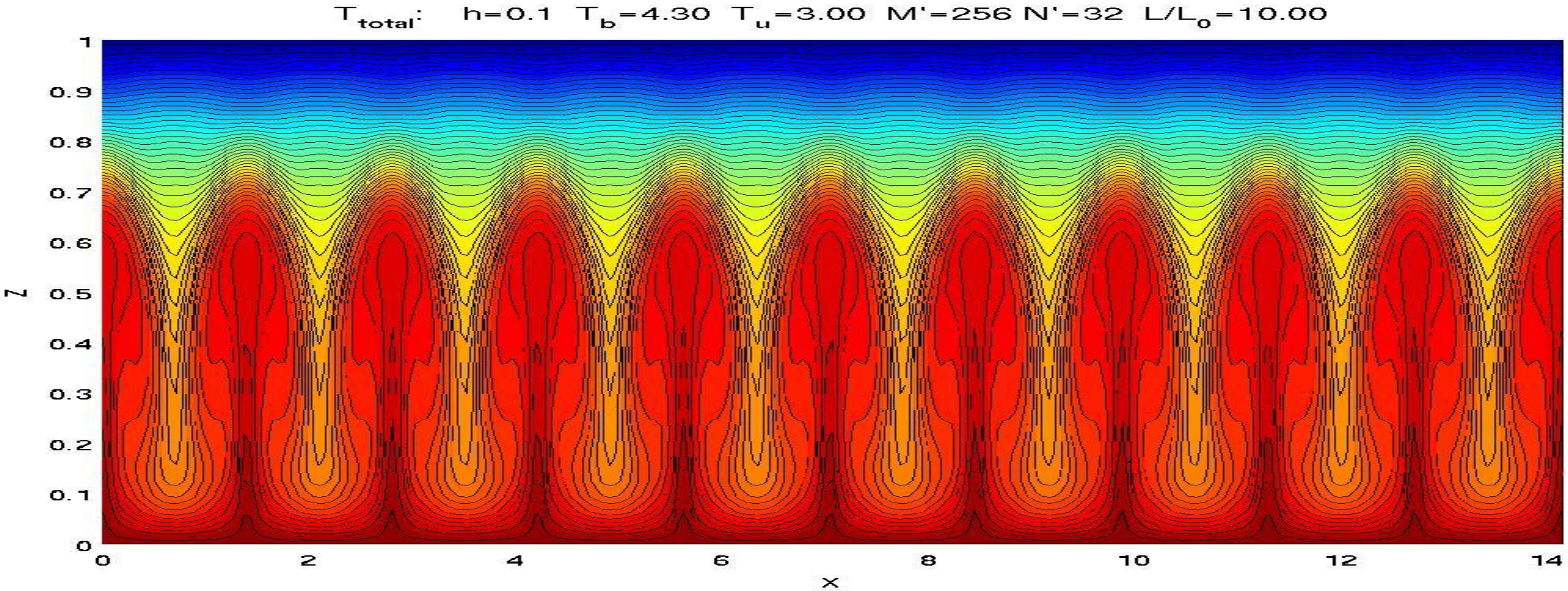
-- motions can arise either in case of heating from below or in case of cooling from below

-- average temperature is close to the temperature at the lower boundary (in this case > 4°C)

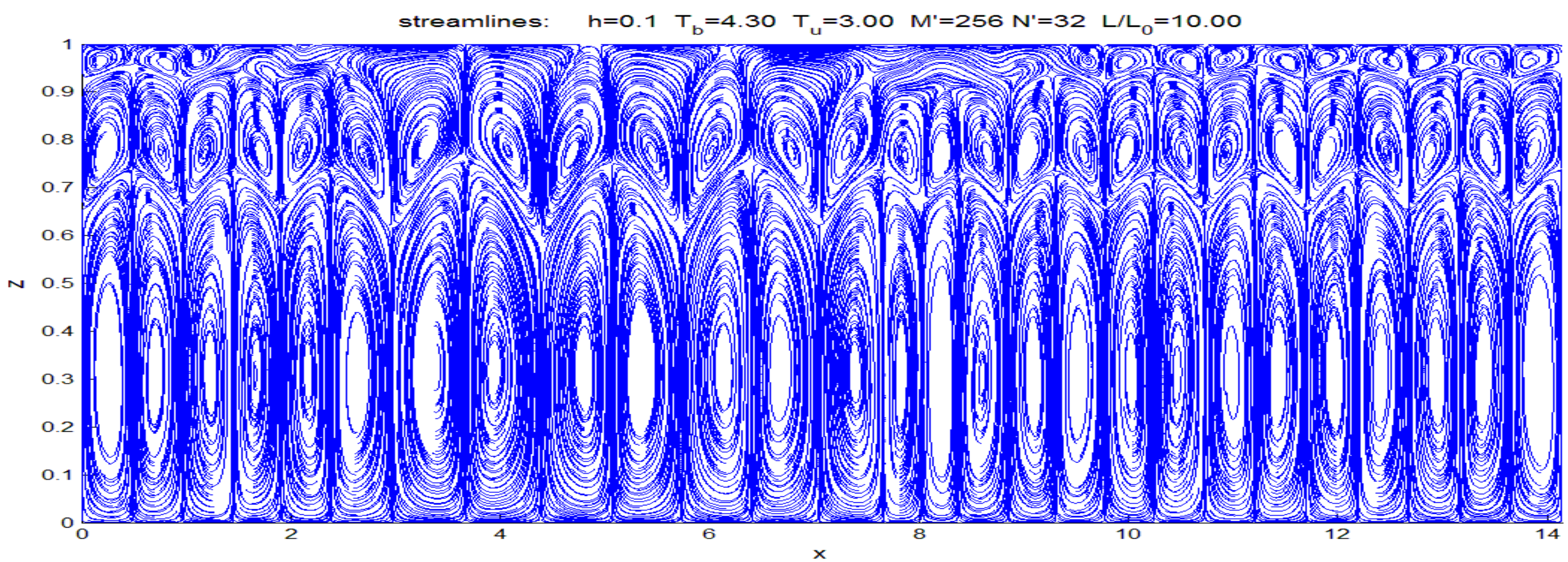
-- existence of the vortices near the upper boundary

-- typical horizontal scale is 2 times less than for RB convection

-- periodic mode is characterized by oscillations of the temperature “tails” and formation/destruction of vortices in the upper part of the layer



T_total: h=0.1 T_b=4.30 T_u=3.00 M'=256 N'=32 L/L_0=10.00
z
x



streamlines: h=0.1 T_b=4.30 T_u=3.00 M'=256 N'=32 L/L_0=10.00
z
x

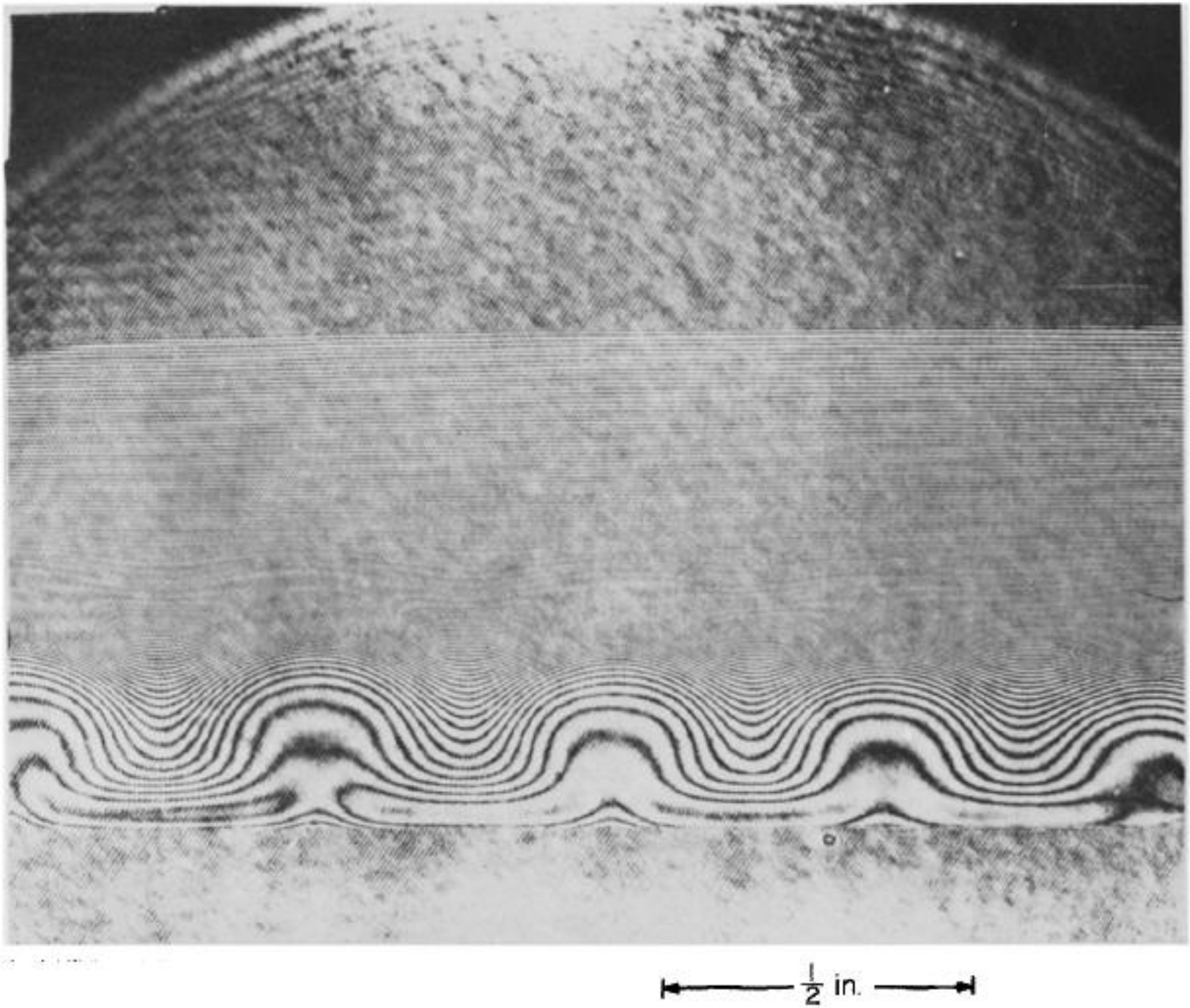
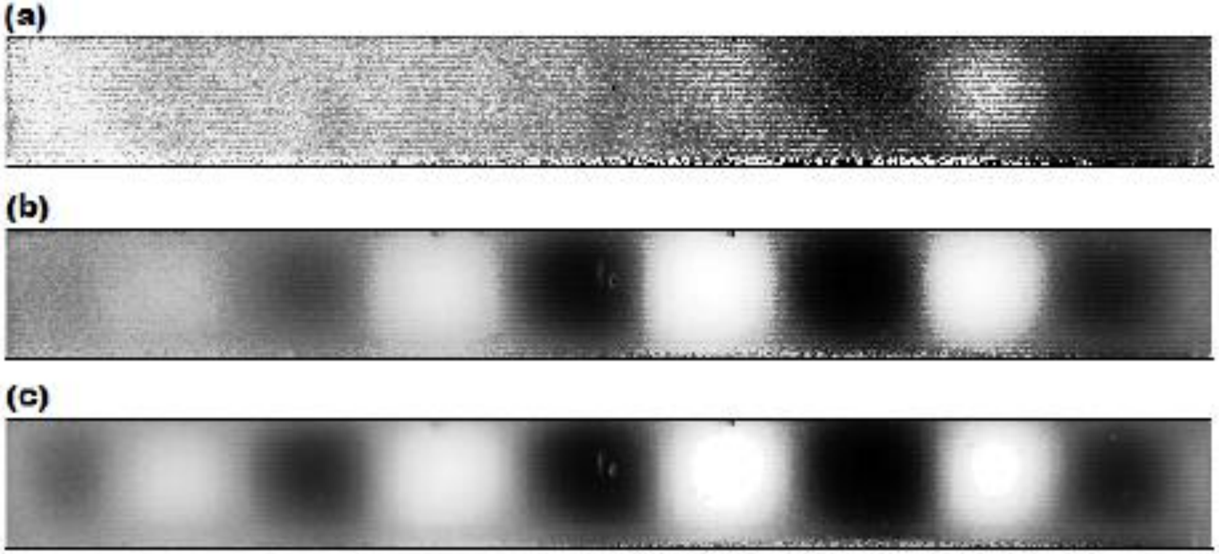


FIG. 7. Interferogram showing the cells that develop after the onset of instability.

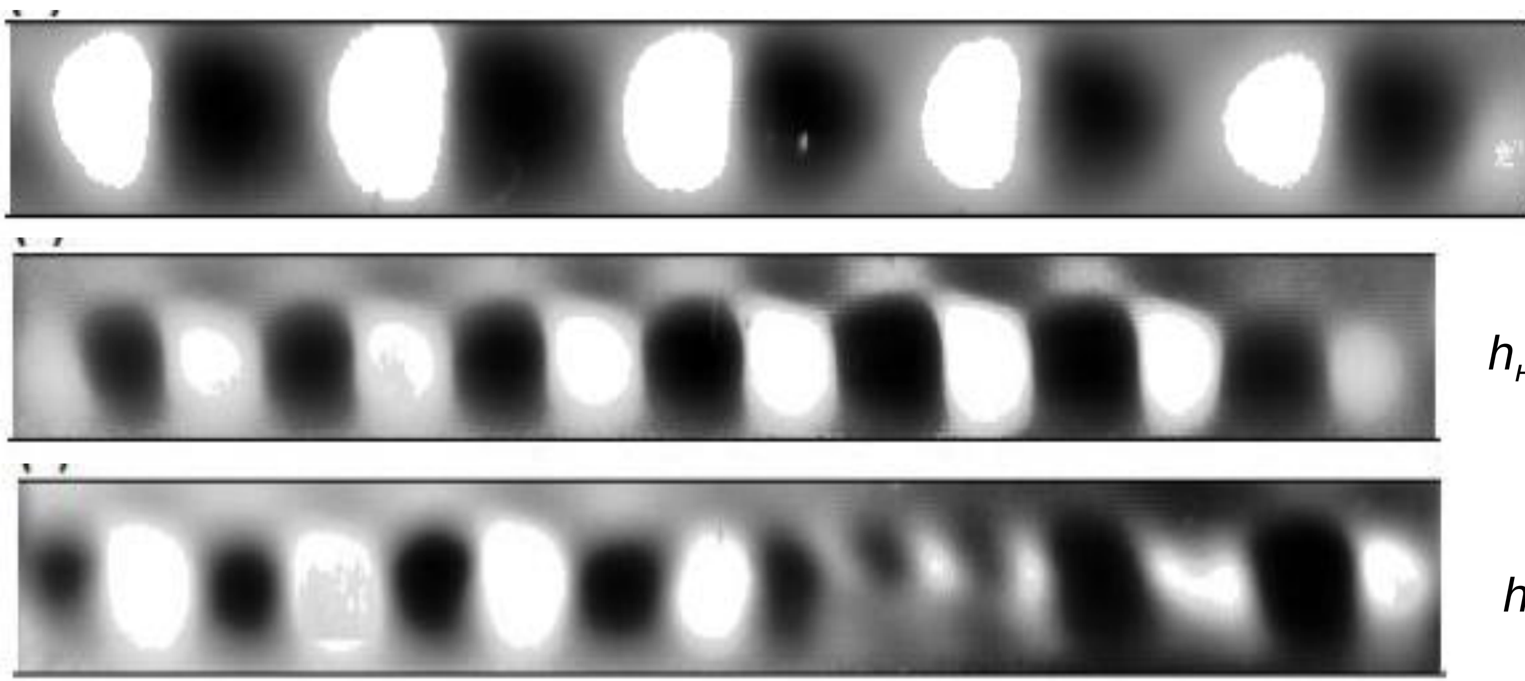
Tankin R., Farhadieh R. // Int.J. Heat Mass Transfer. 1971, V.14, P.953-960

Large E.D. **An Experimental Investigation of Penetrative Convection in Water Near 4°C. // Dissertation, The Ohio State University, 2010**



Symmetrical structures

Different ratios of stable and unstable sublayers



$h_{неуст}/h_{уст}=0.82$

$h_{неуст}/h_{уст}=0.49$

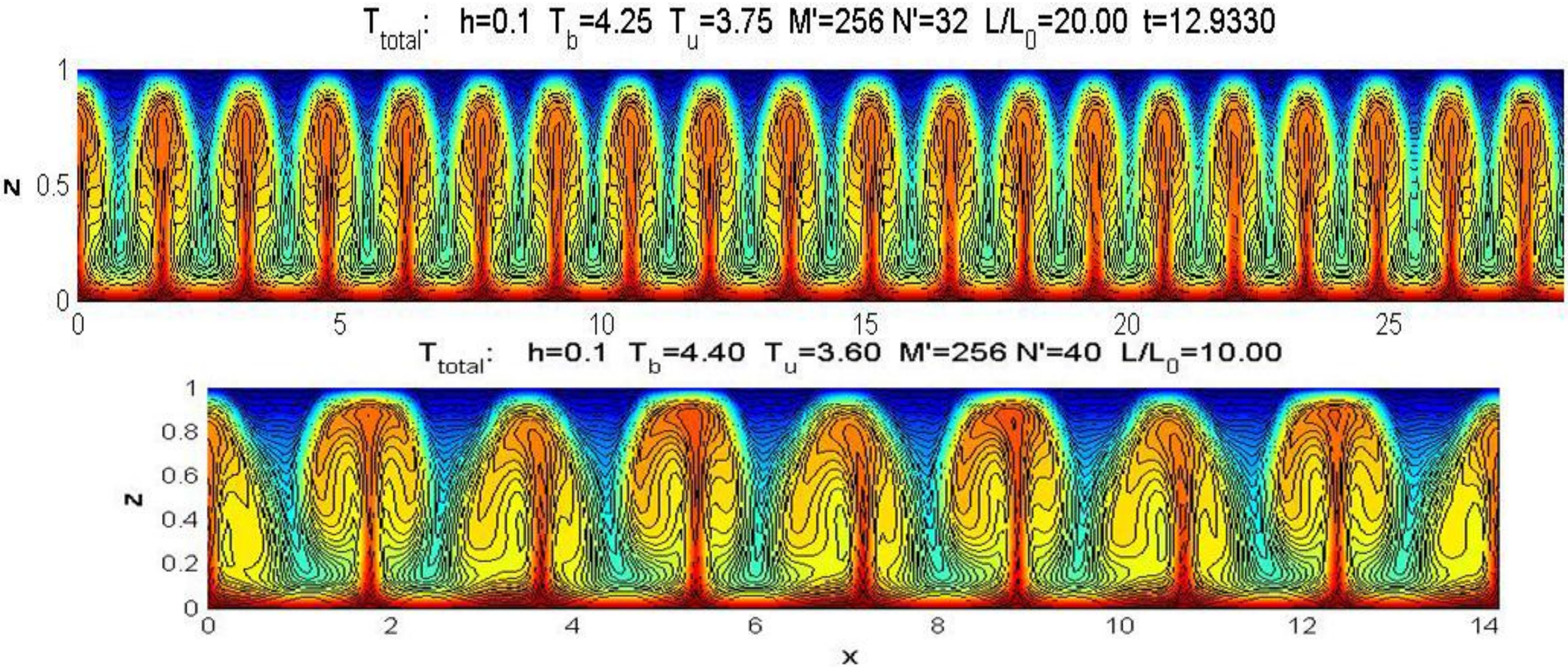
$h_{неуст}/h_{уст}=0.56$

Dependency on horizontal dimensions
(L=1/α – half a period)

To define dimensions for the study of transitions to chaos simulations were made for horizontal lengths up to 20L_0, L_0 – being taken from linear theory.

Conclusions:

- **for steady mode the period = L_0**
- **for periodic mode with 1 maximum period length = 2L_0**
- **for periodic mode with 2 maxima period length = 4L_0**



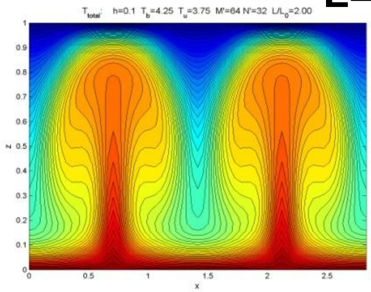
Dependence on aspect ratio (L=1/α – half a period)

Steady mode

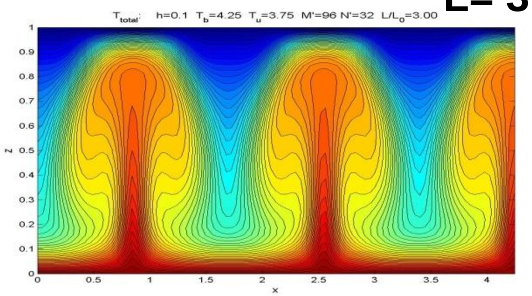
$$\alpha_0 = \frac{1}{\sqrt{2}}$$

$$L_0 = \sqrt{2}$$

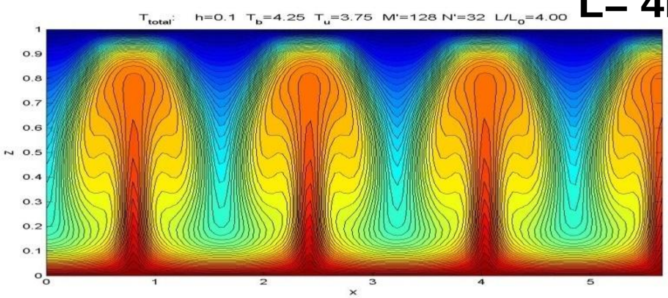
L= 2L_0



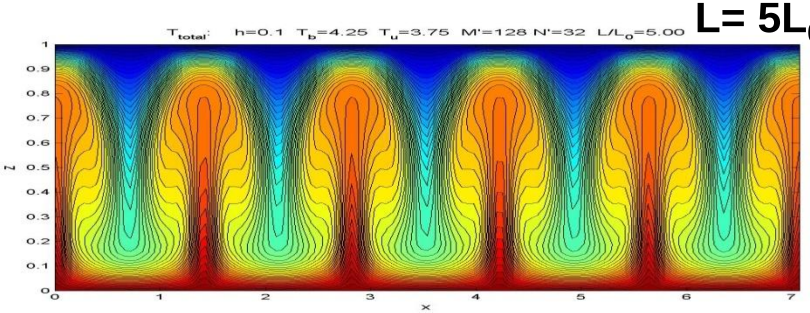
L= 3L_0



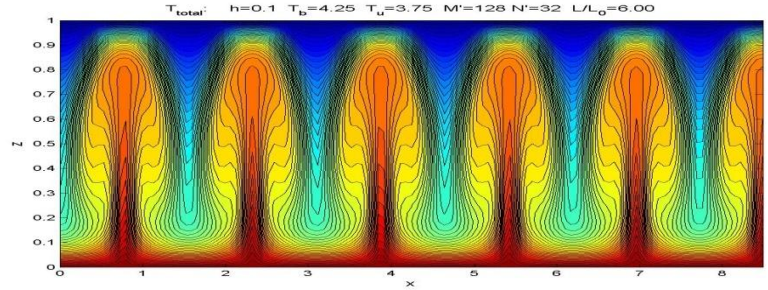
L= 4L_0



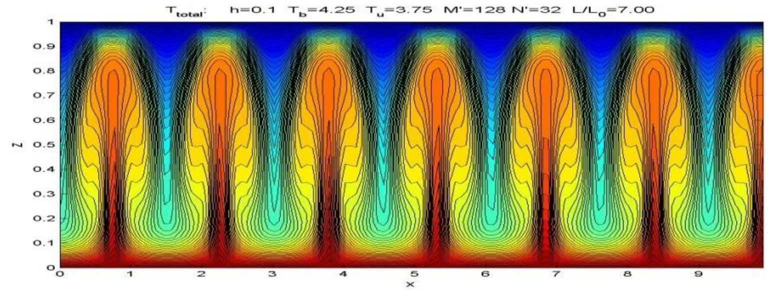
L= 5L_0



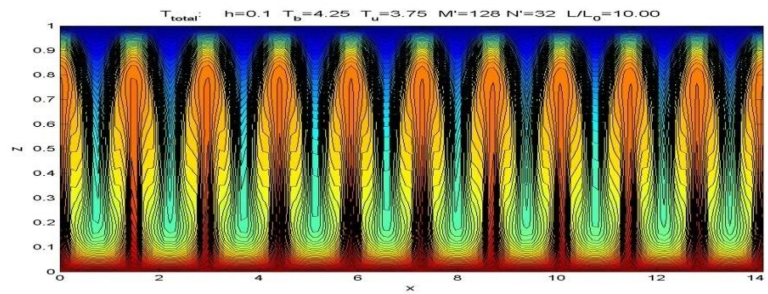
L= 6L_0



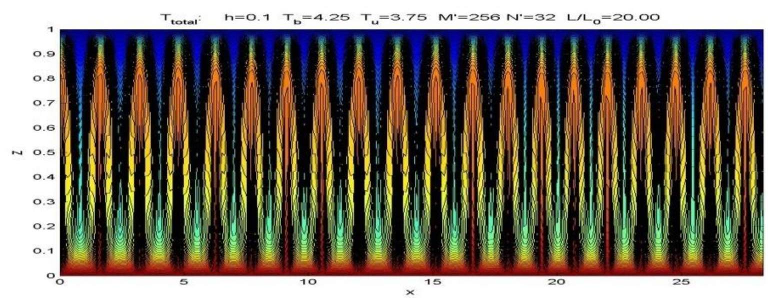
L= 7L_0

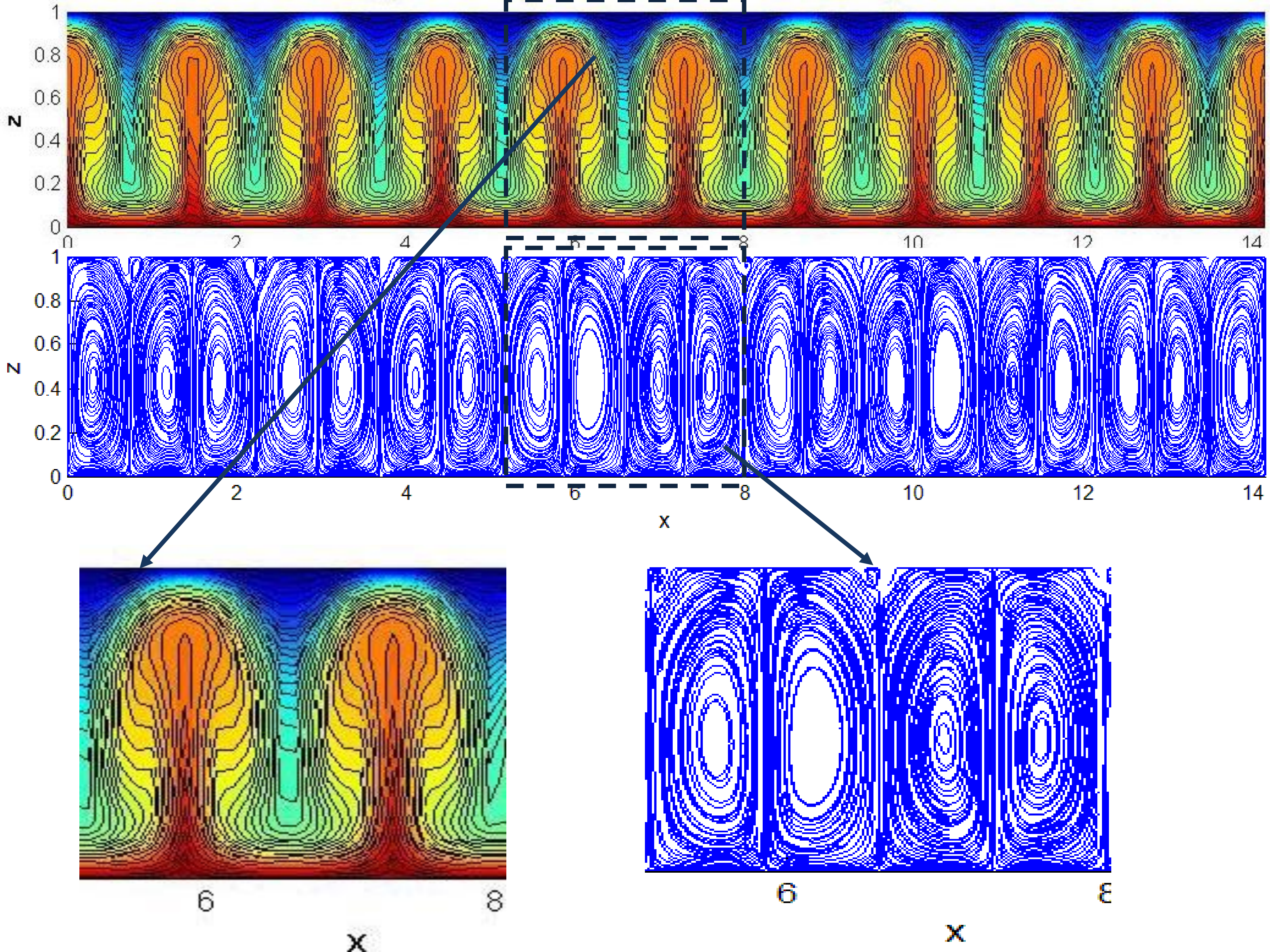


L= 10L_0



L= 20L_0

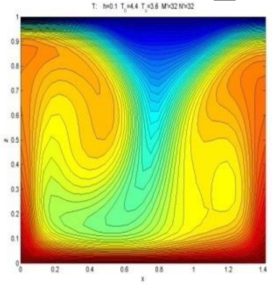




z
z
x
0
2
4
6
8
10
12
14
0
0.2
0.4
0.6
0.8
1
6
8
x
6
8
x

Dependence on aspect ratio (L=1/α – half a period)

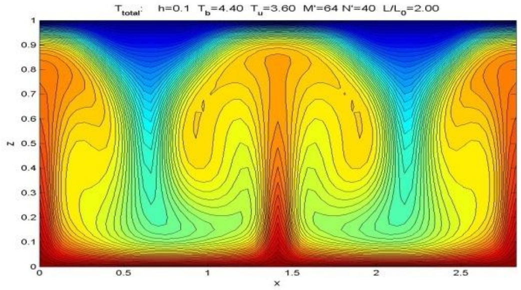
$L = L_0$



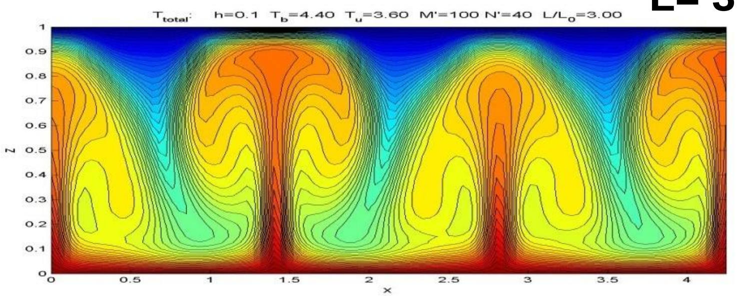
$$\alpha_0 = \frac{1}{\sqrt{2}}$$

$$L_0 = \sqrt{2}$$

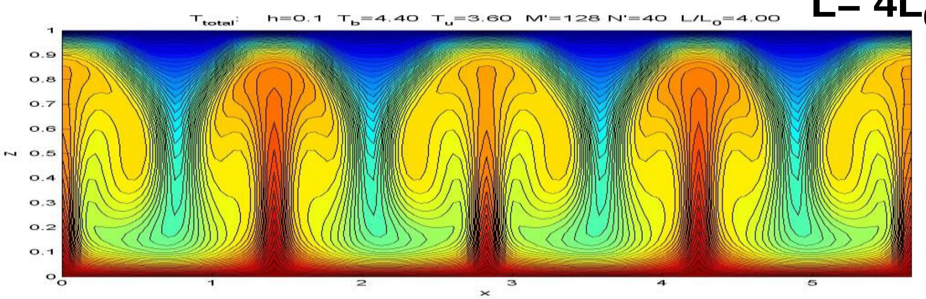
$L = 2L_0$



$L = 3L_0$

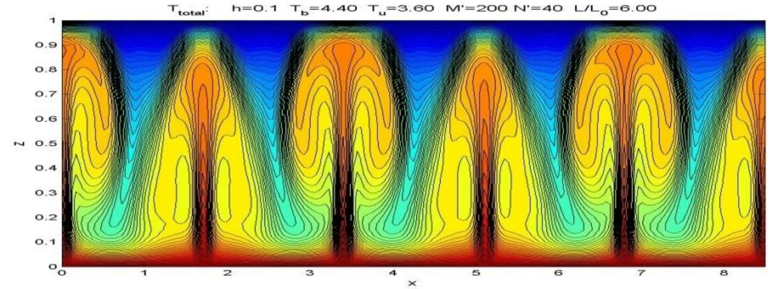


$L = 4L_0$

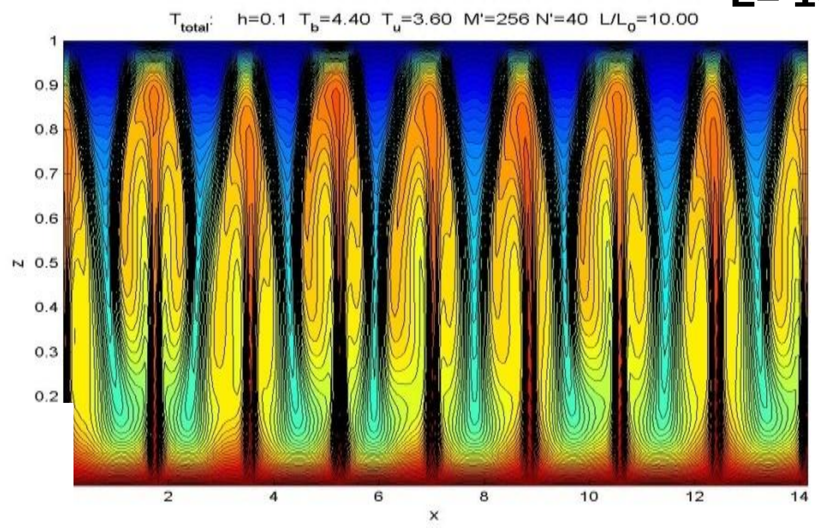


Periodical mode

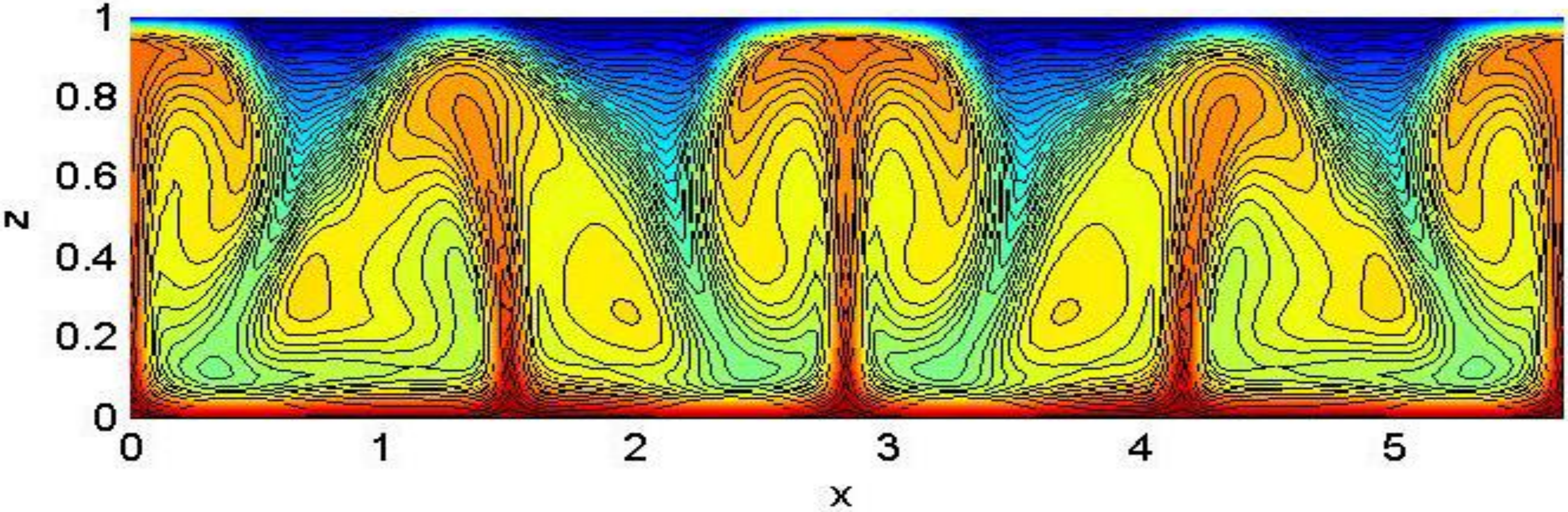
$L = 6L_0$



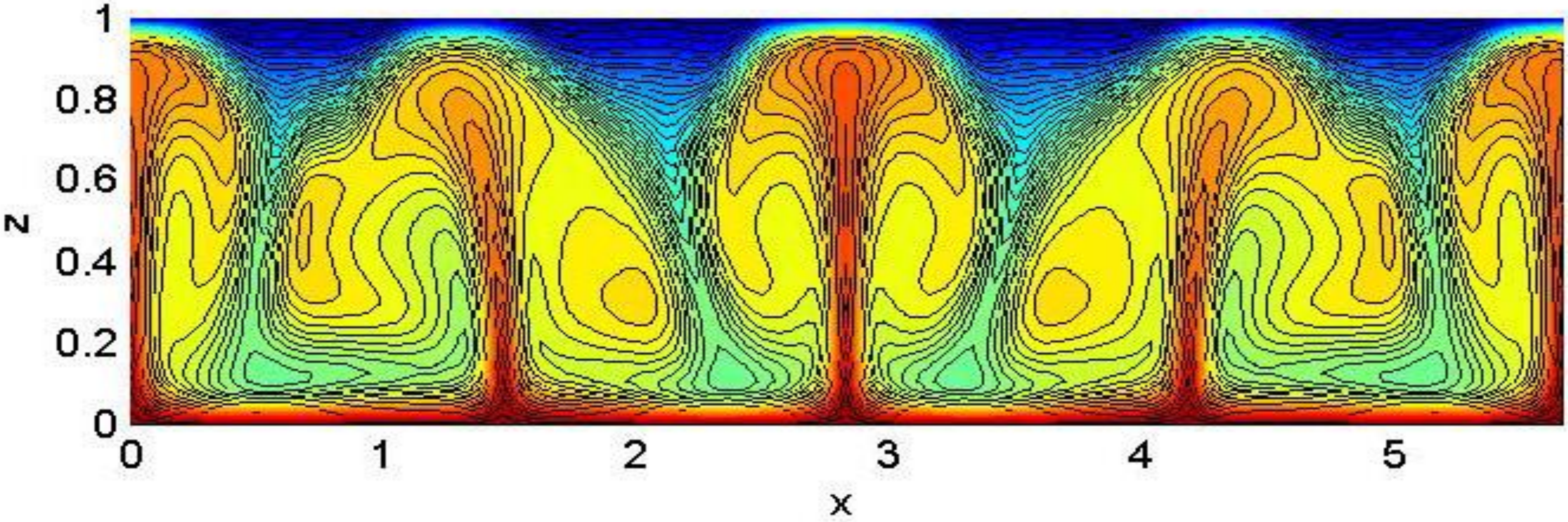
$L = 10L_0$



Period-2 solution



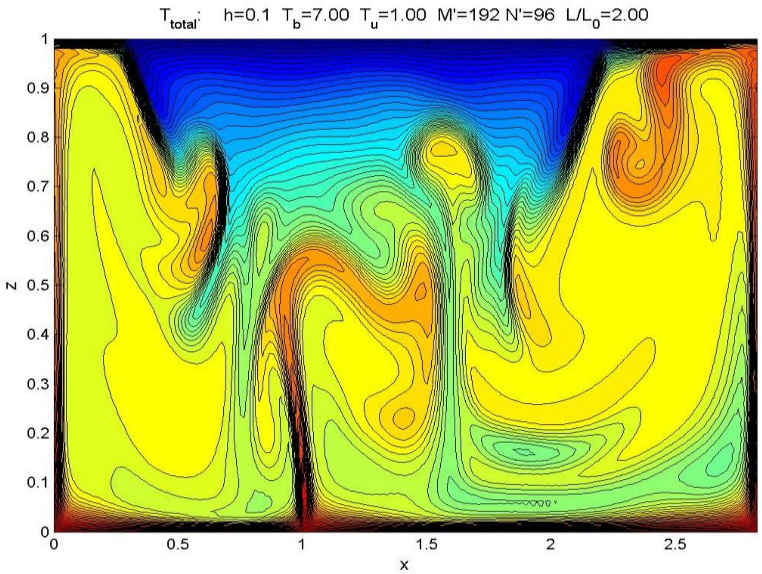
Quasiperiodic solution $4L_0$

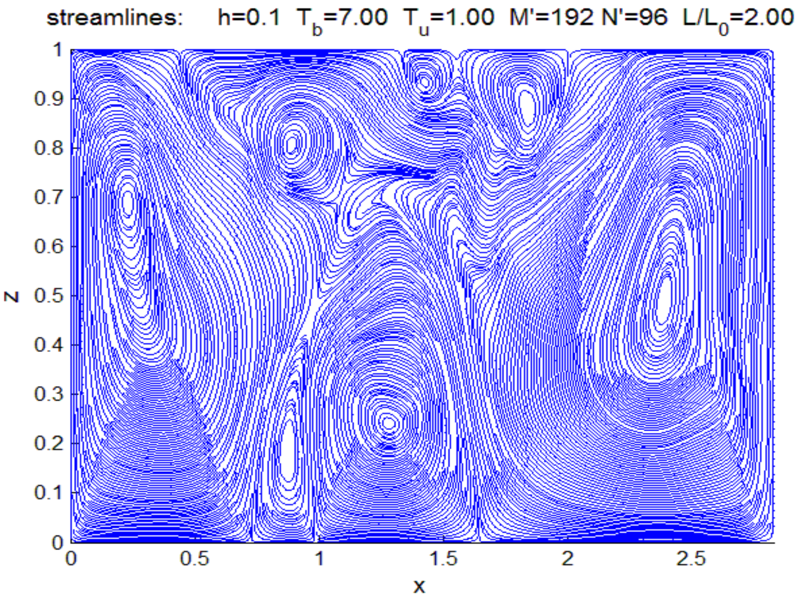


Dependence on aspect ratio ($L=1/\alpha$ – half a period)

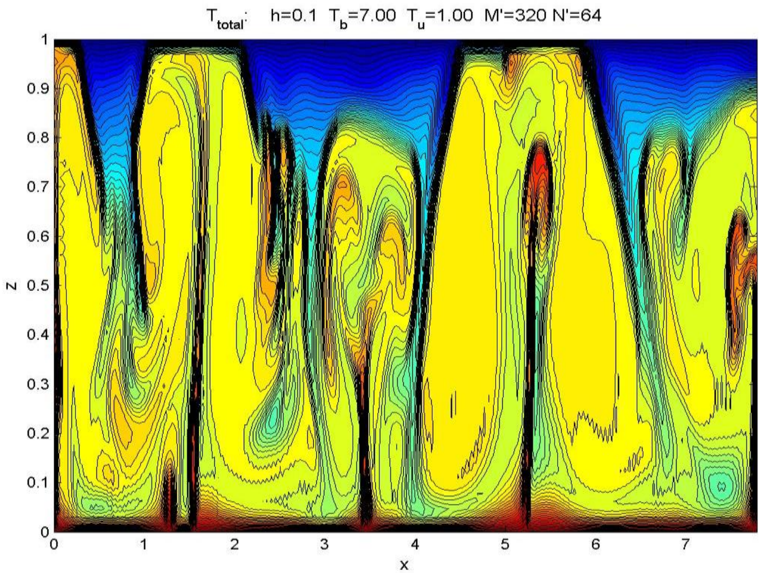
$L = 2L_0$

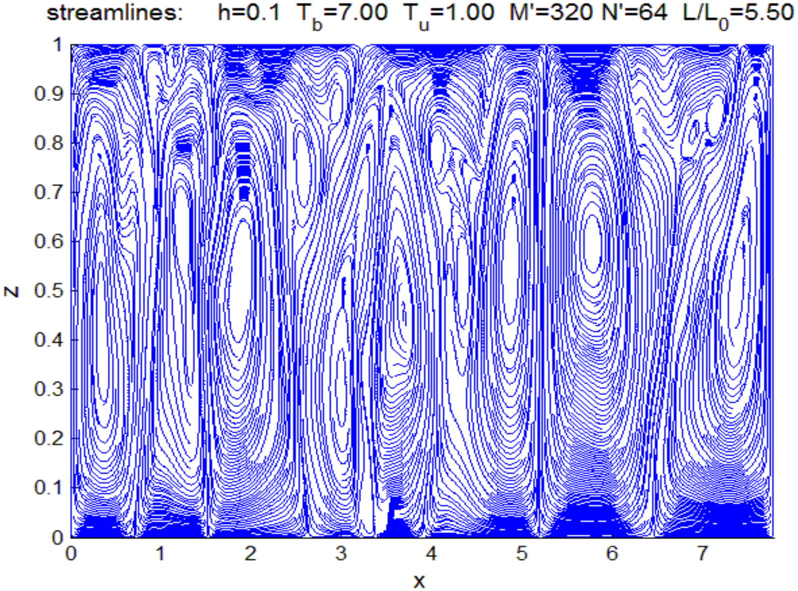
Fully chaotical mode

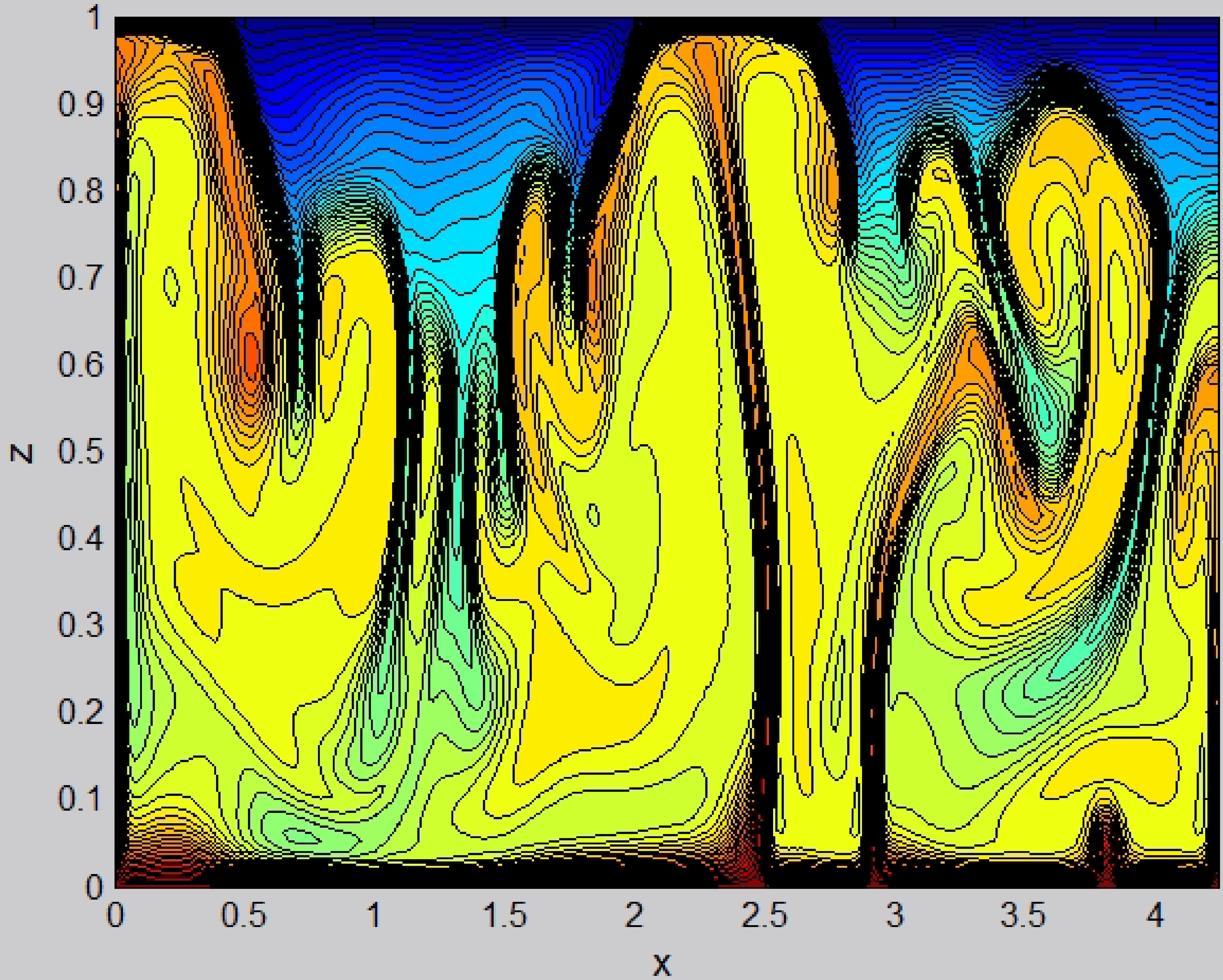




$L = 5.5L_0$



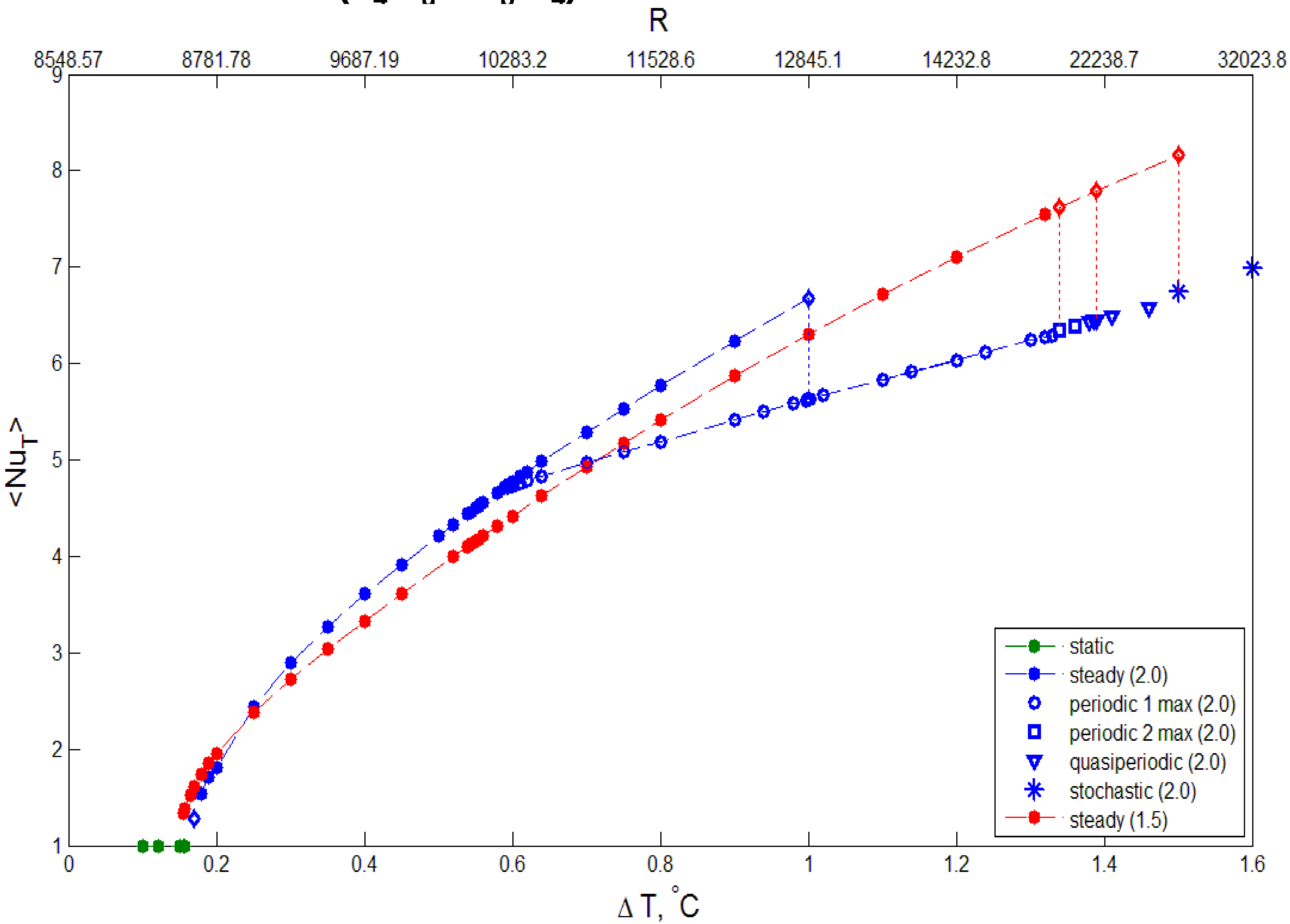


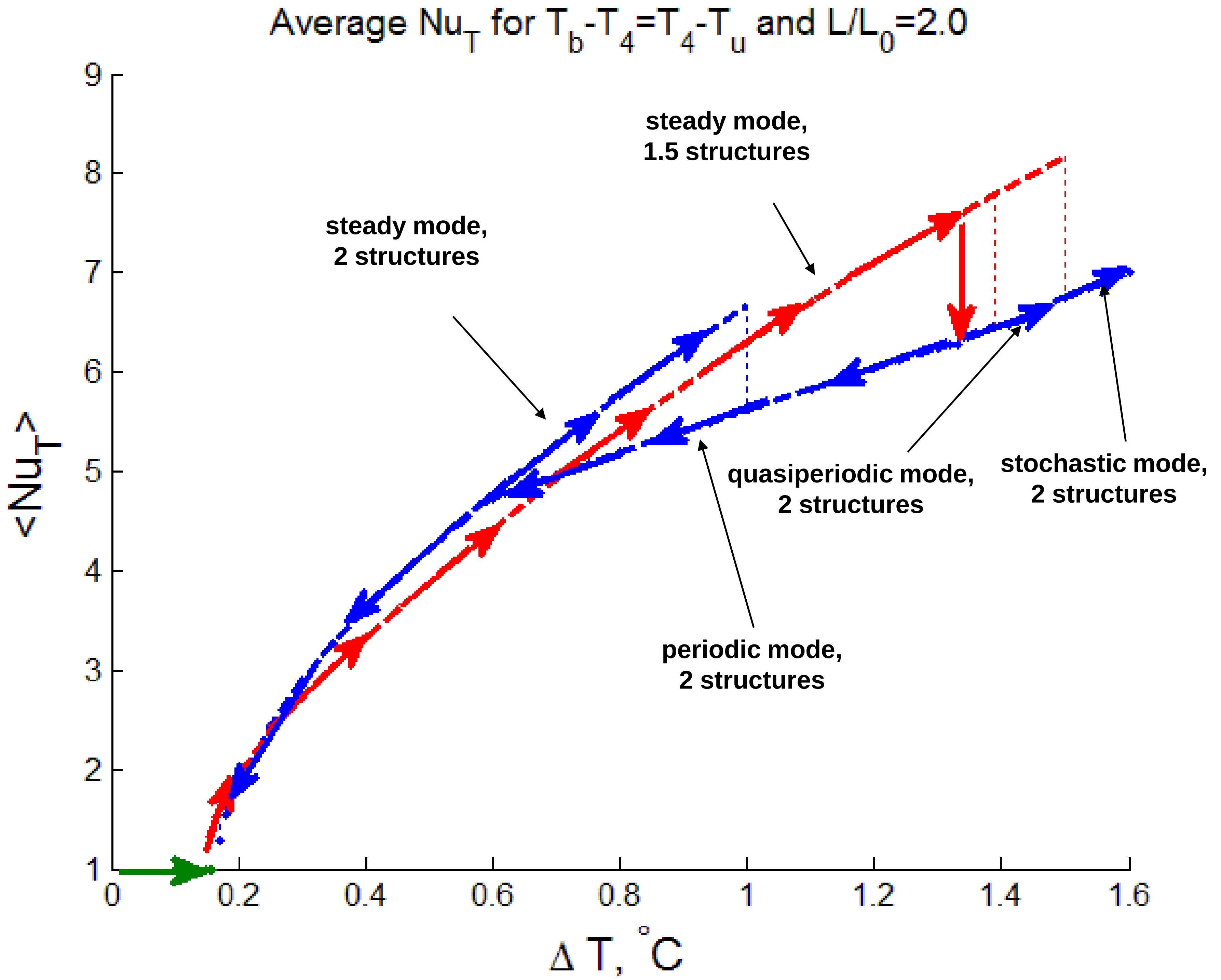


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
z
0
0.5
1
1.5
2
2.5
3
3.5
4
x

Transitional modes

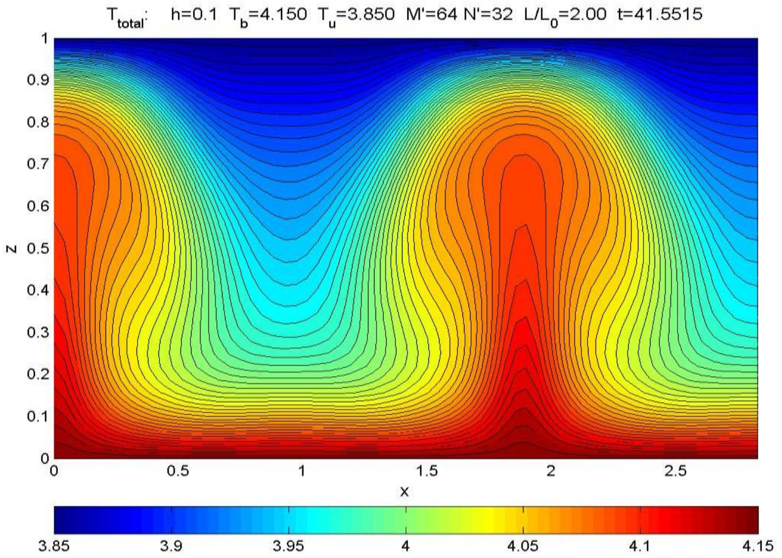
Mean Nusselt number on temperature on boundaries (and corresponding Rayleigh numbers) when density maximum is in the middle of conductive temperature distribution (T_4-T_u = T_b-T_4)

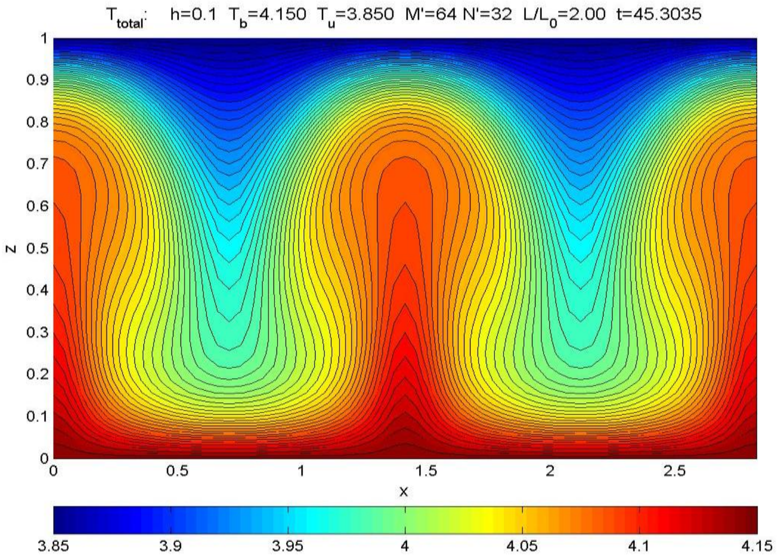



Average Nu_T for T_b-T_4=T_4-T_u and L/L_0=2.0
steady mode,
1.5 structures
steady mode,
2 structures
quasiperiodic mode,
2 structures
stochastic mode,
2 structures
periodic mode,
2 structures
<Nu_T>
Δ T, °C
9
8
7
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

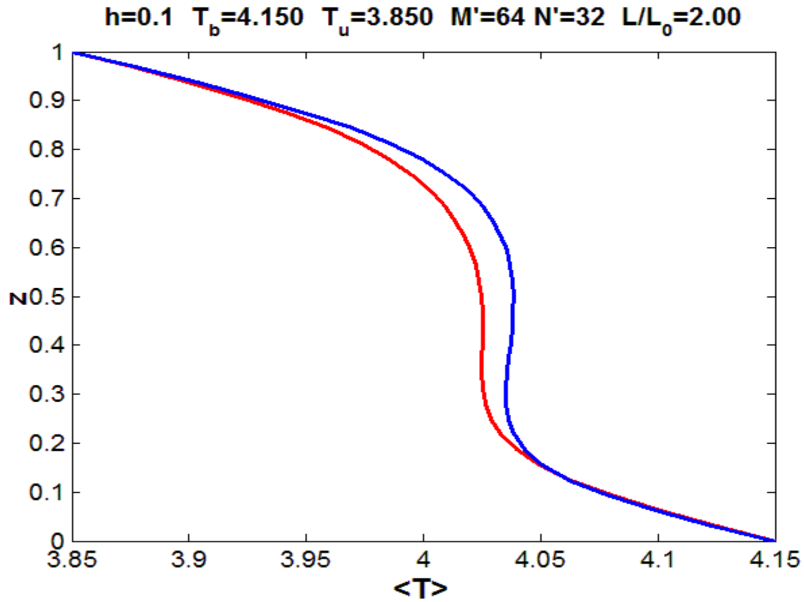
T_b=4.15 T_u=3.85, two steady solutions

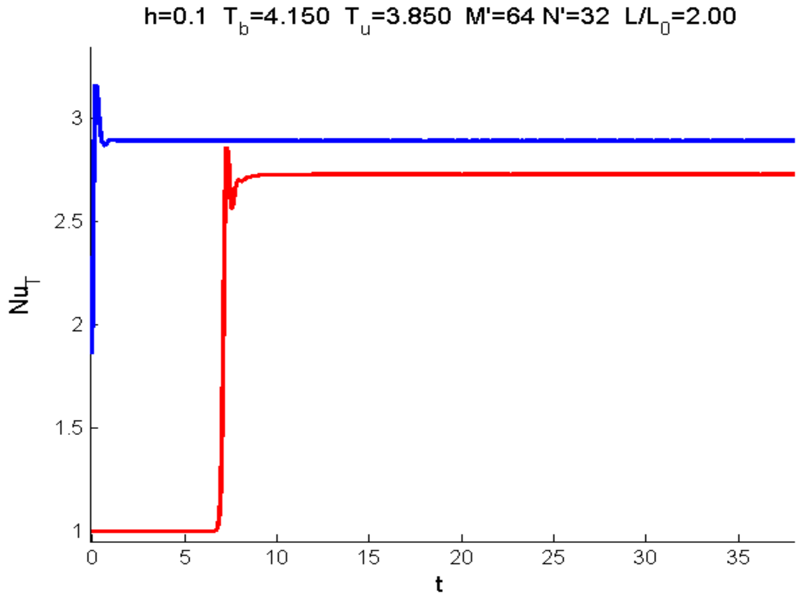
isotherms

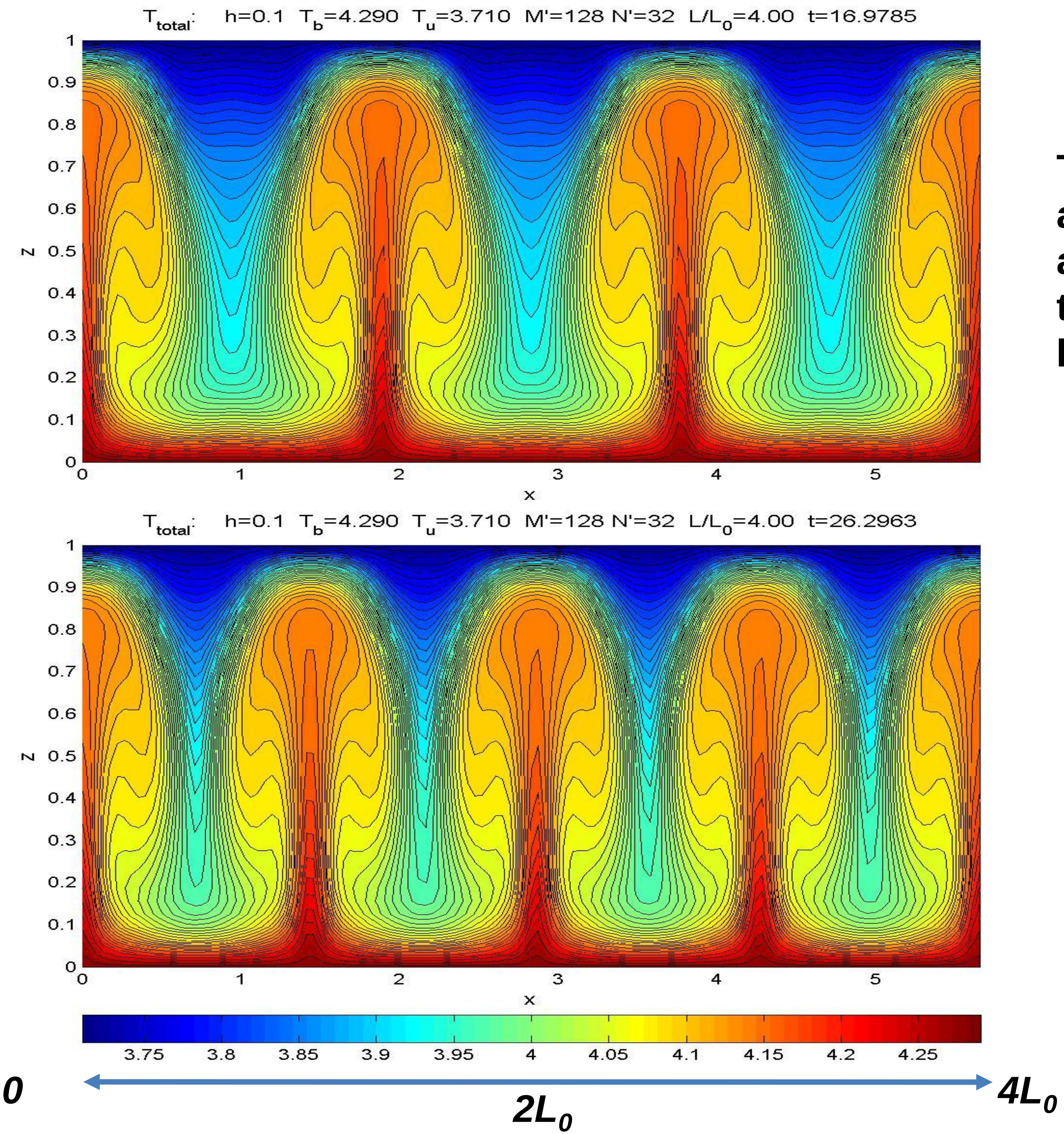




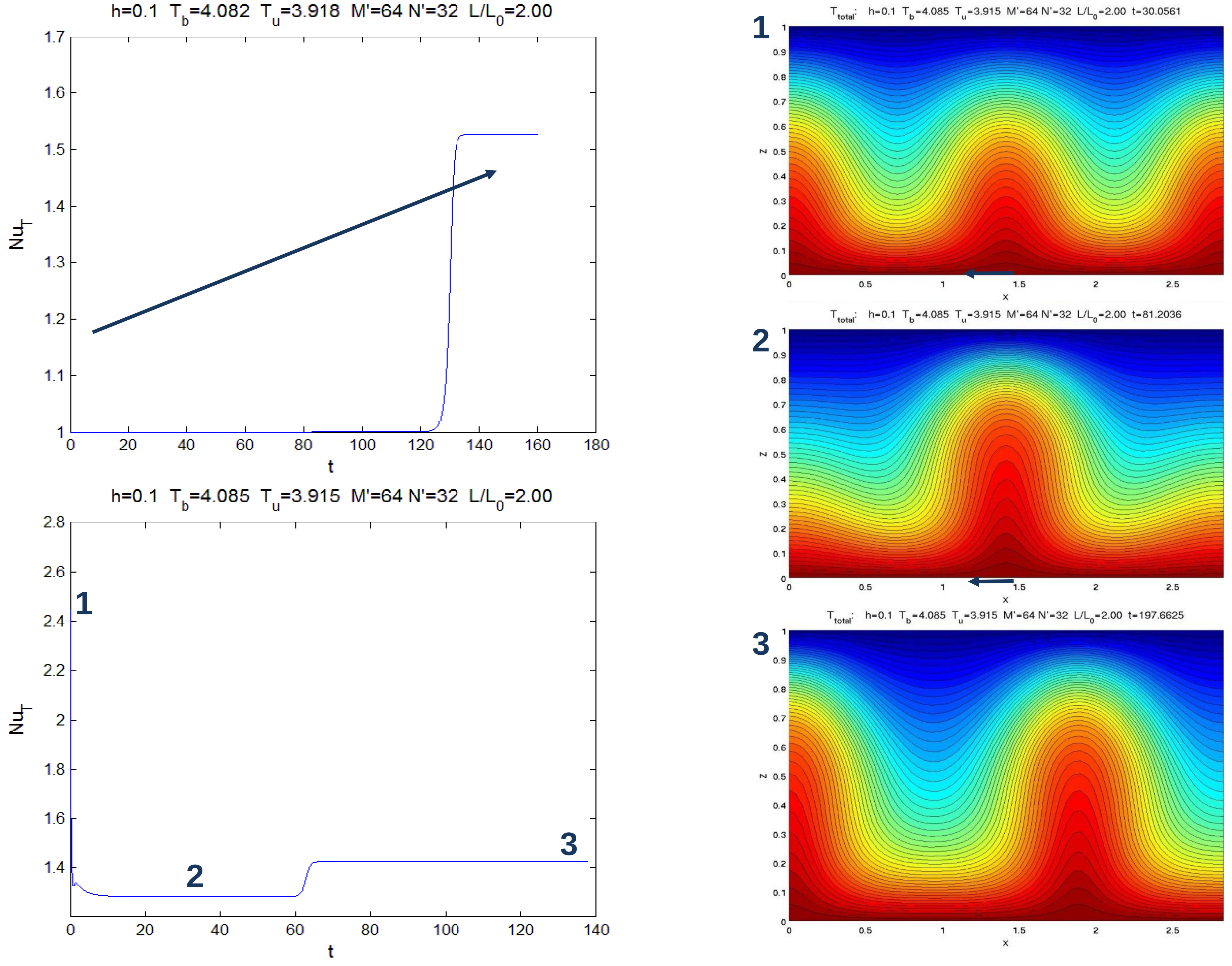
Mean temperature profile and Nusselt number





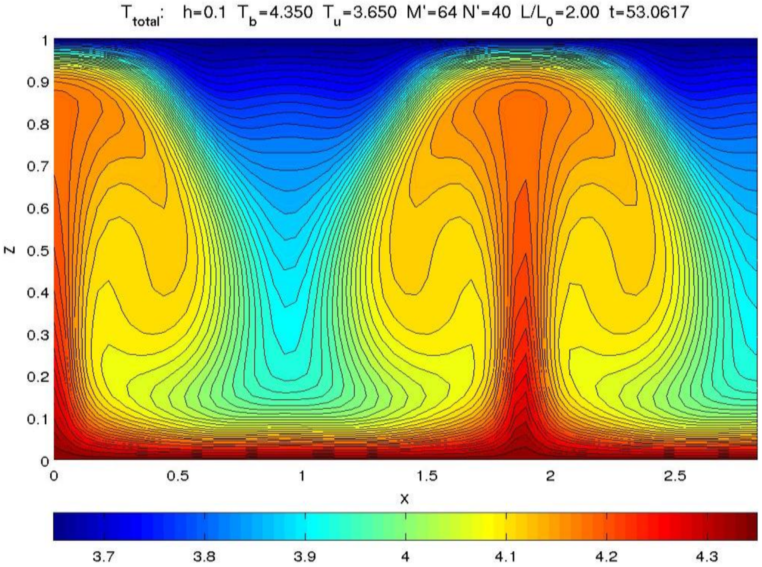


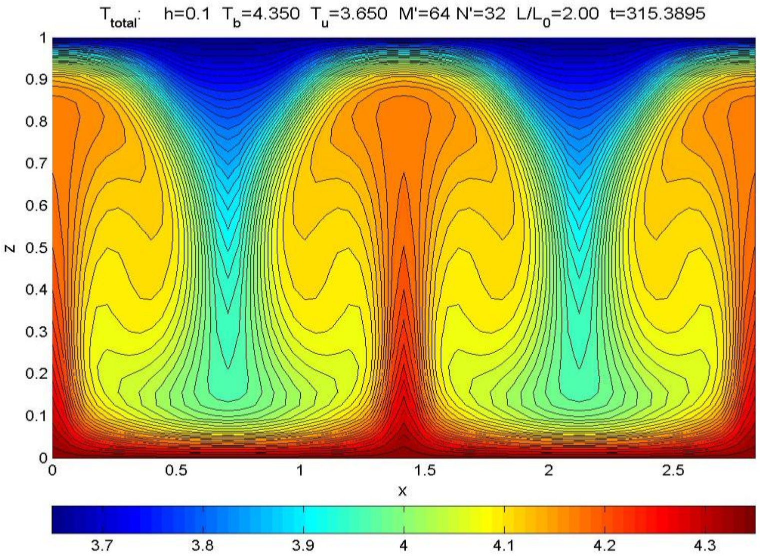
These steady modes are stable after doubling the horizontal length

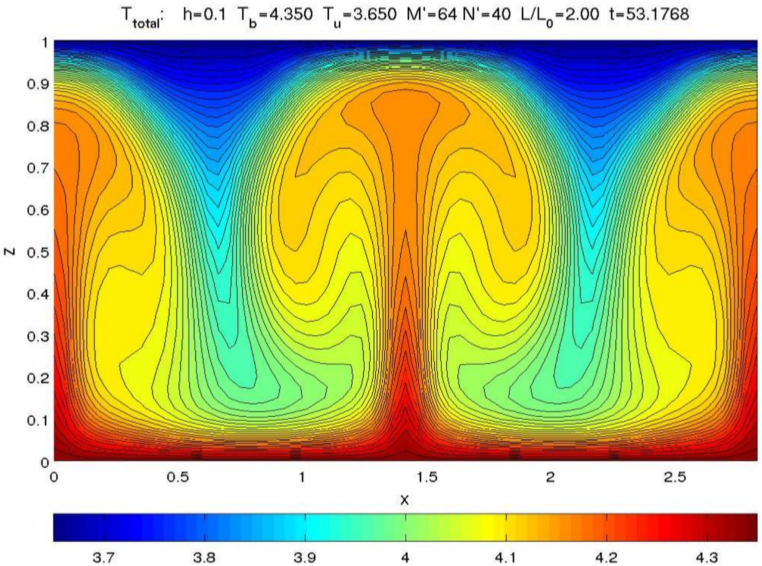

h=0.1 T_b=4.082 T_u=3.918 M'=64 N'=32 L/L_0=2.00
Nu_T
t
h=0.1 T_b=4.085 T_u=3.915 M'=64 N'=32 L/L_0=2.00
1
2
3
T_total: h=0.1 T_b=4.085 T_u=3.915 M'=64 N'=32 L/L_0=2.00 t=30.0561
T_total: h=0.1 T_b=4.085 T_u=3.915 M'=64 N'=32 L/L_0=2.00 t=81.2036
T_total: h=0.1 T_b=4.085 T_u=3.915 M'=64 N'=32 L/L_0=2.00 t=197.6625
z
x

T_b=4.35 T_u=3.65, steady and periodic modes

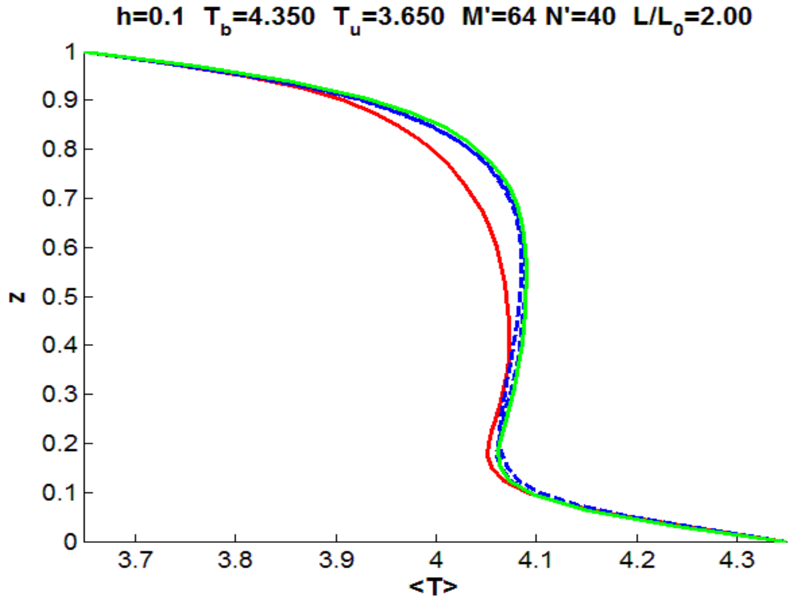
isotherms

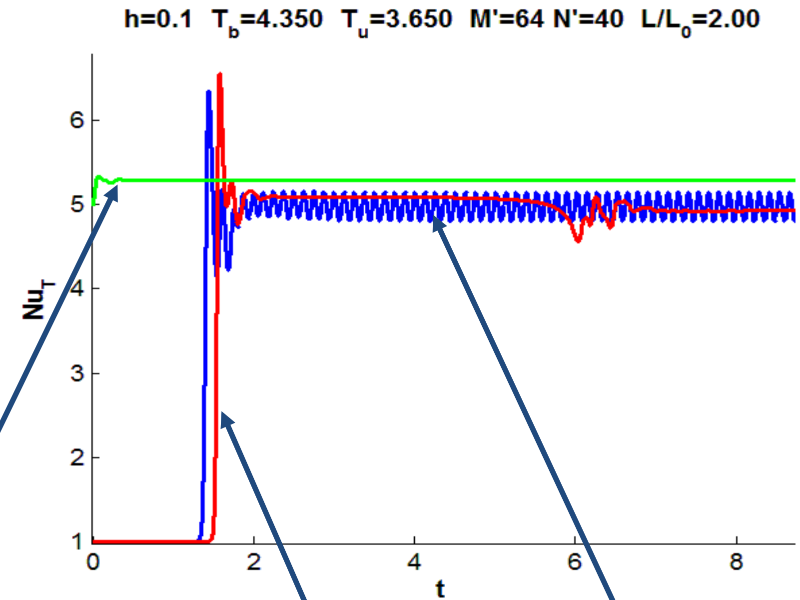






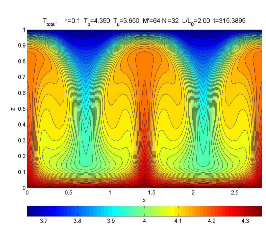
Mean temperature profile and Nusselt number



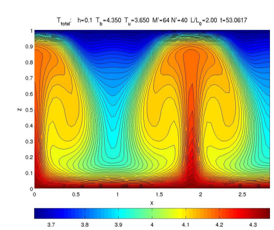


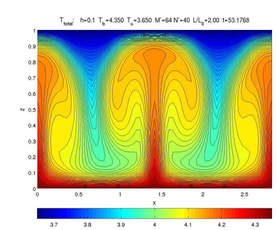
steady,
1.5 structures

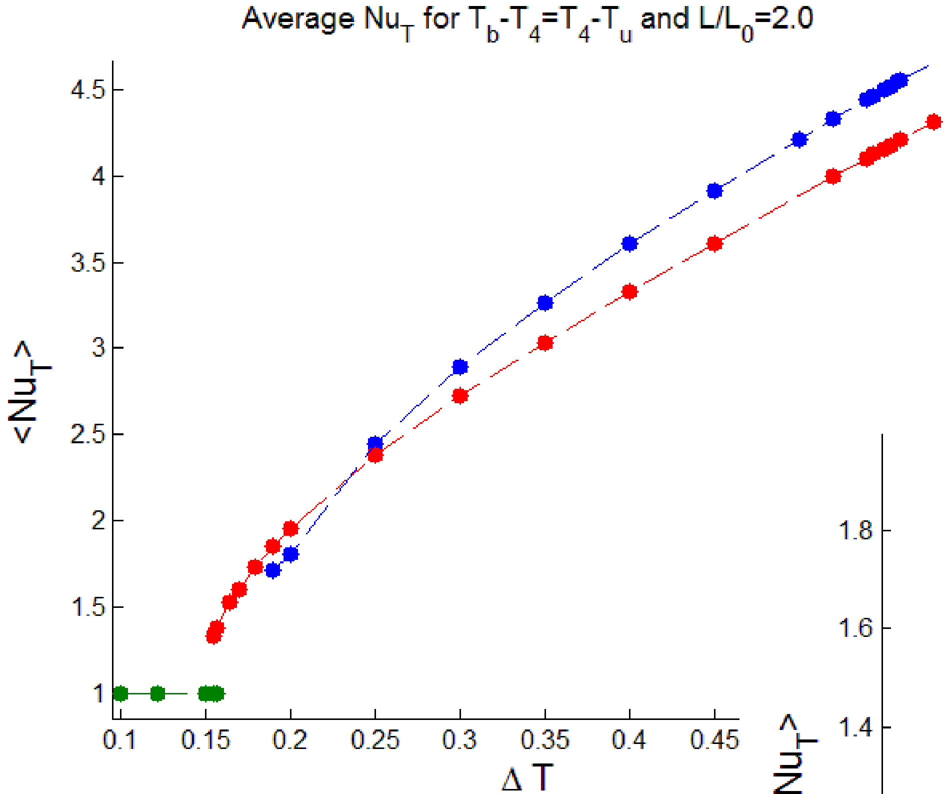
periodic,
2 structures

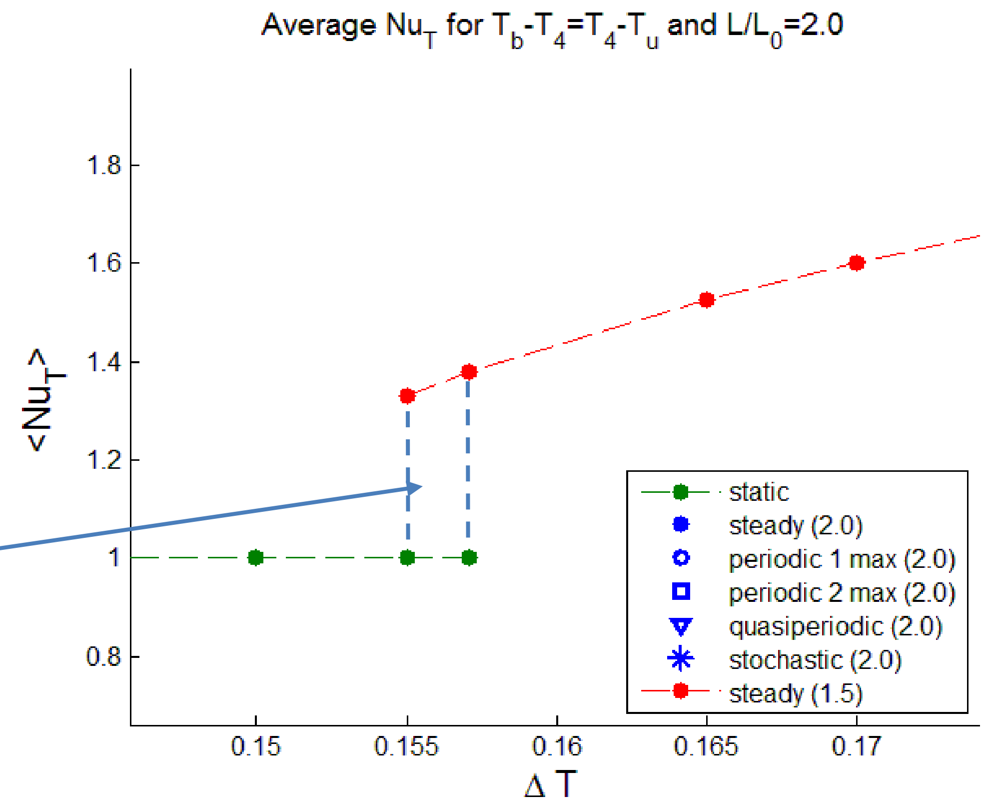


steady,
2 structures

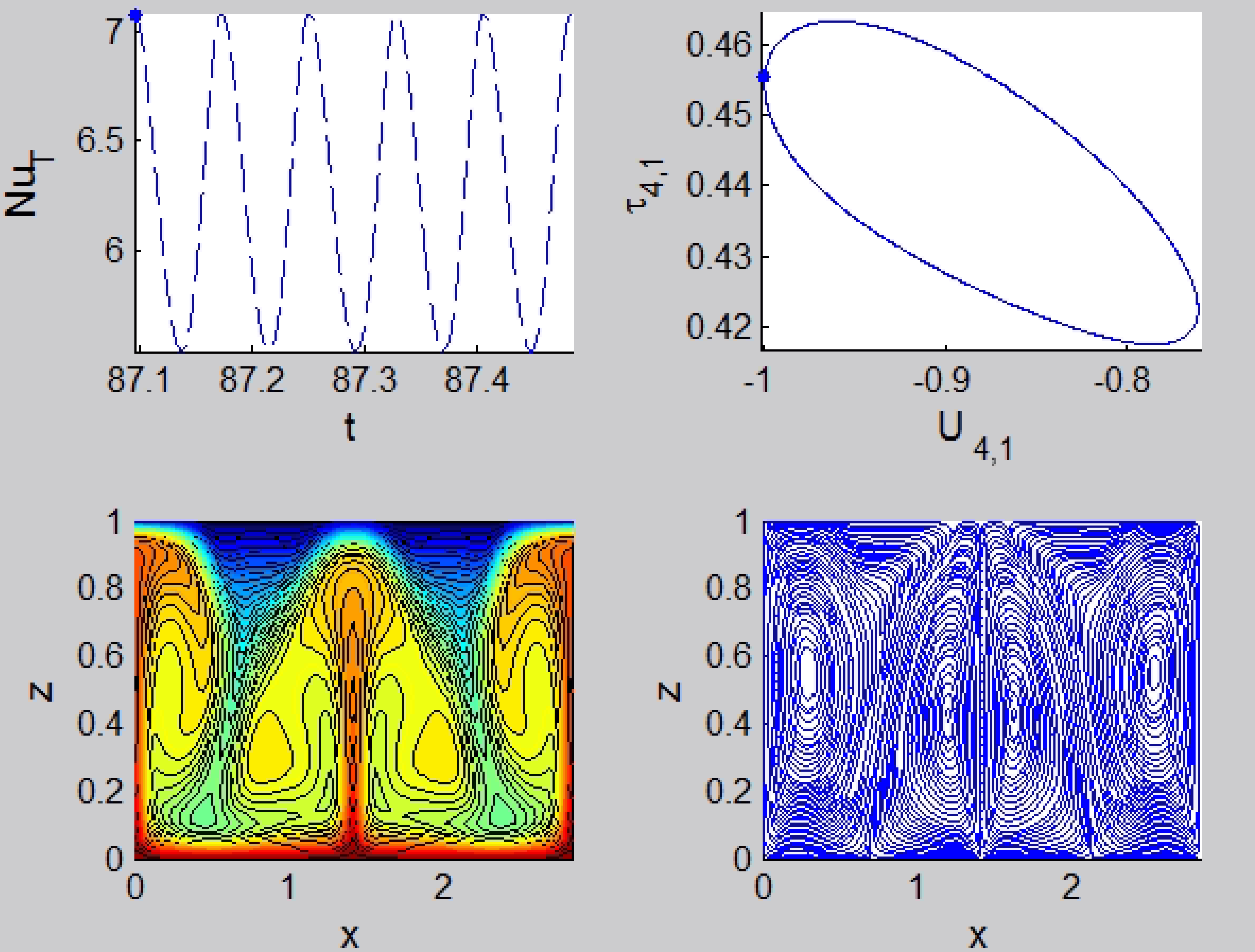




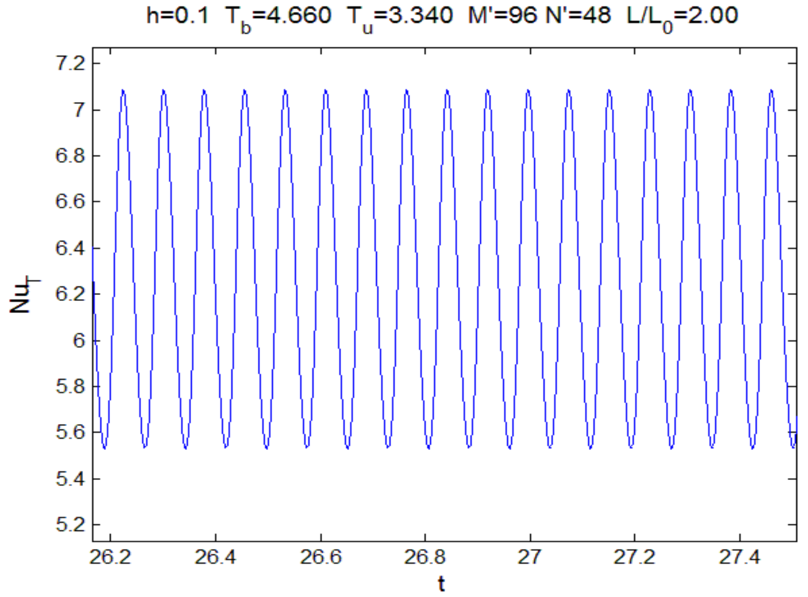


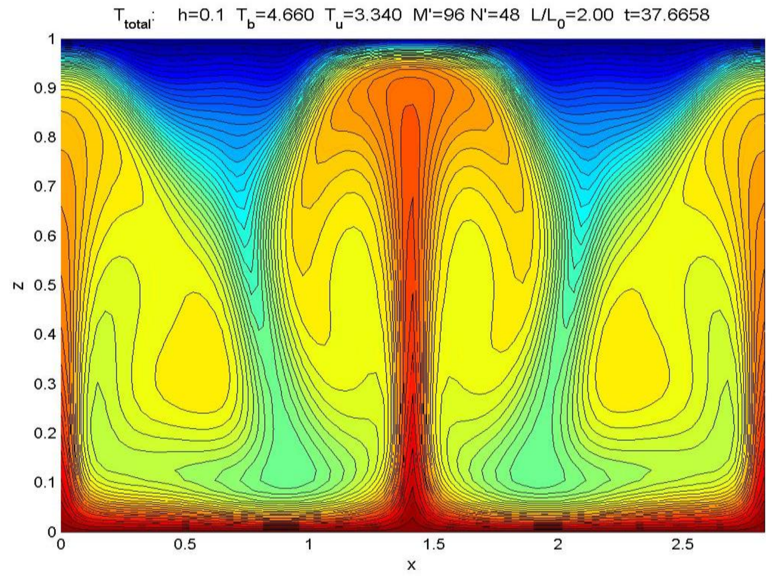


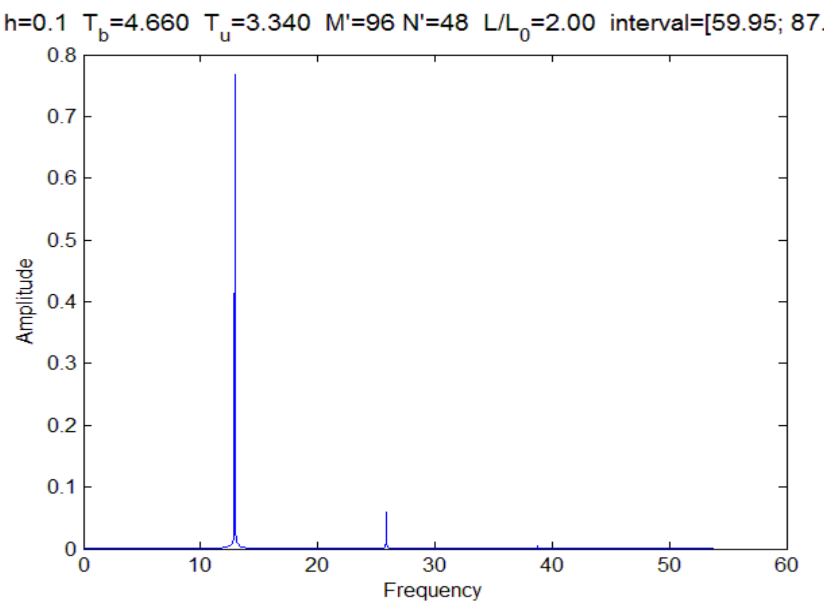
Domain of existence of two solutions: static and stationary

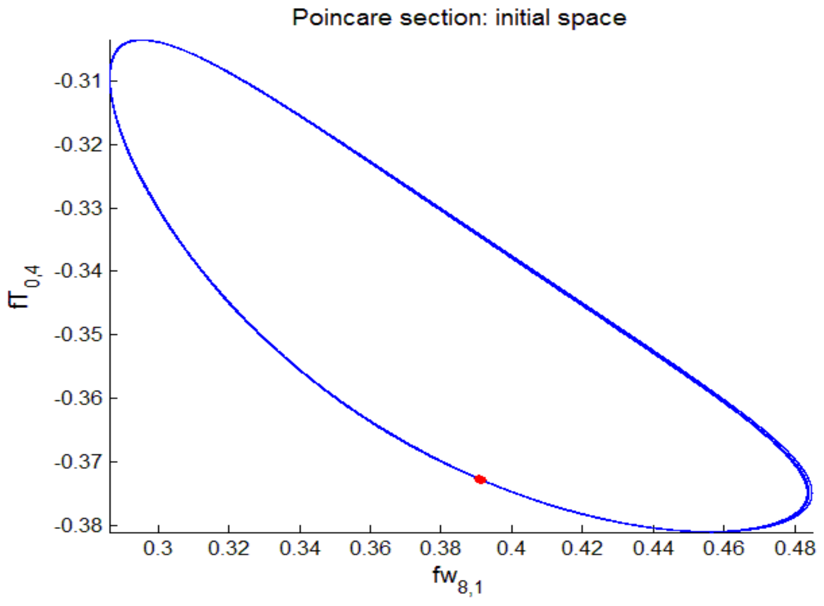

7
6.5
6
Nu_T
87.1
87.2
87.3
87.4
t
0.46
0.45
0.44
0.43
0.42
$\tau_{4,1}$
-1
-0.9
-0.8
$U_{4,1}$
1
0.8
0.6
0.4
0.2
0
z
0
1
2
x
1
0.8
0.6
0.4
0.2
0
z
0
1
2
x

T_b=4.66 T_u=3.34, periodic mode (1 maximum for a period)

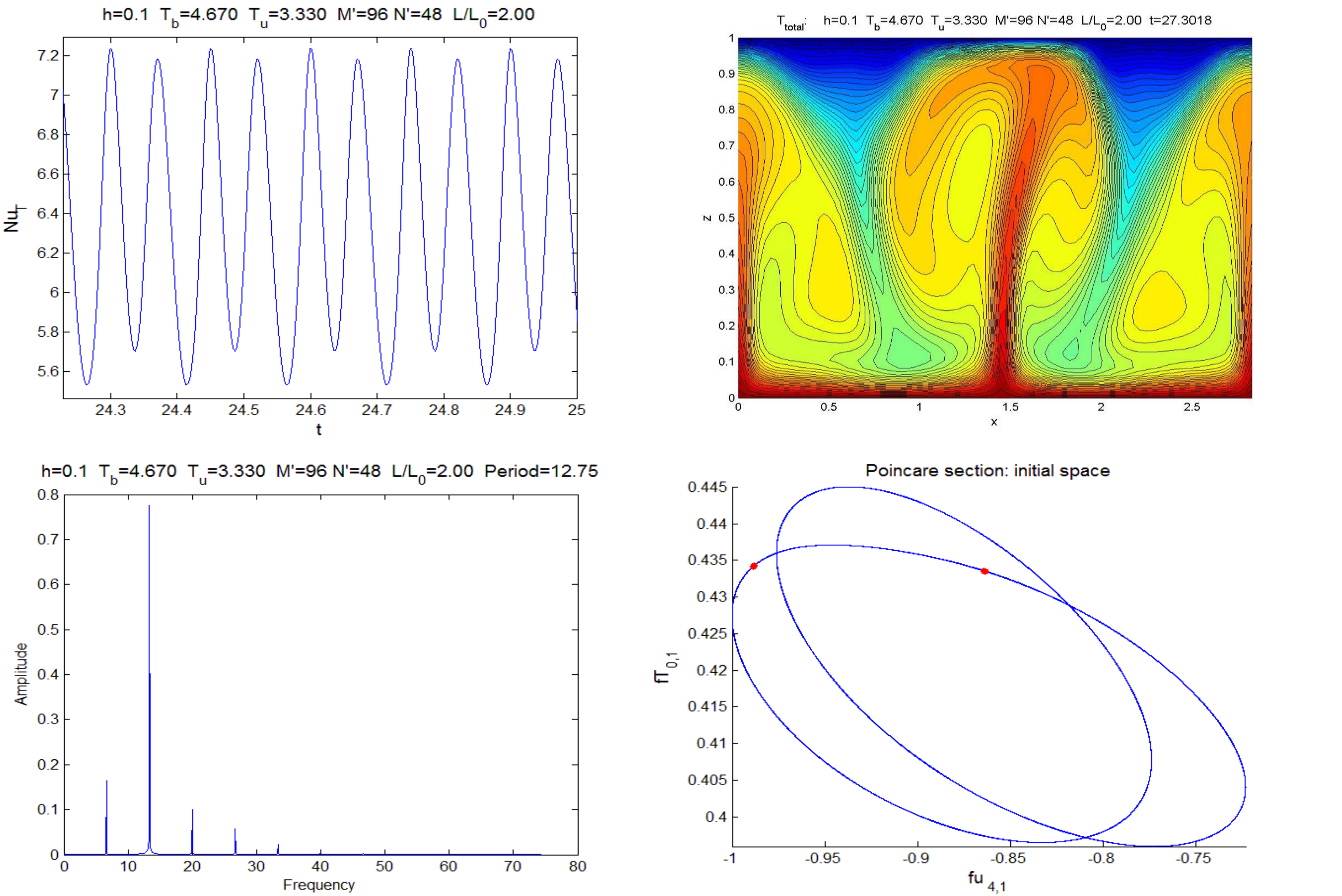




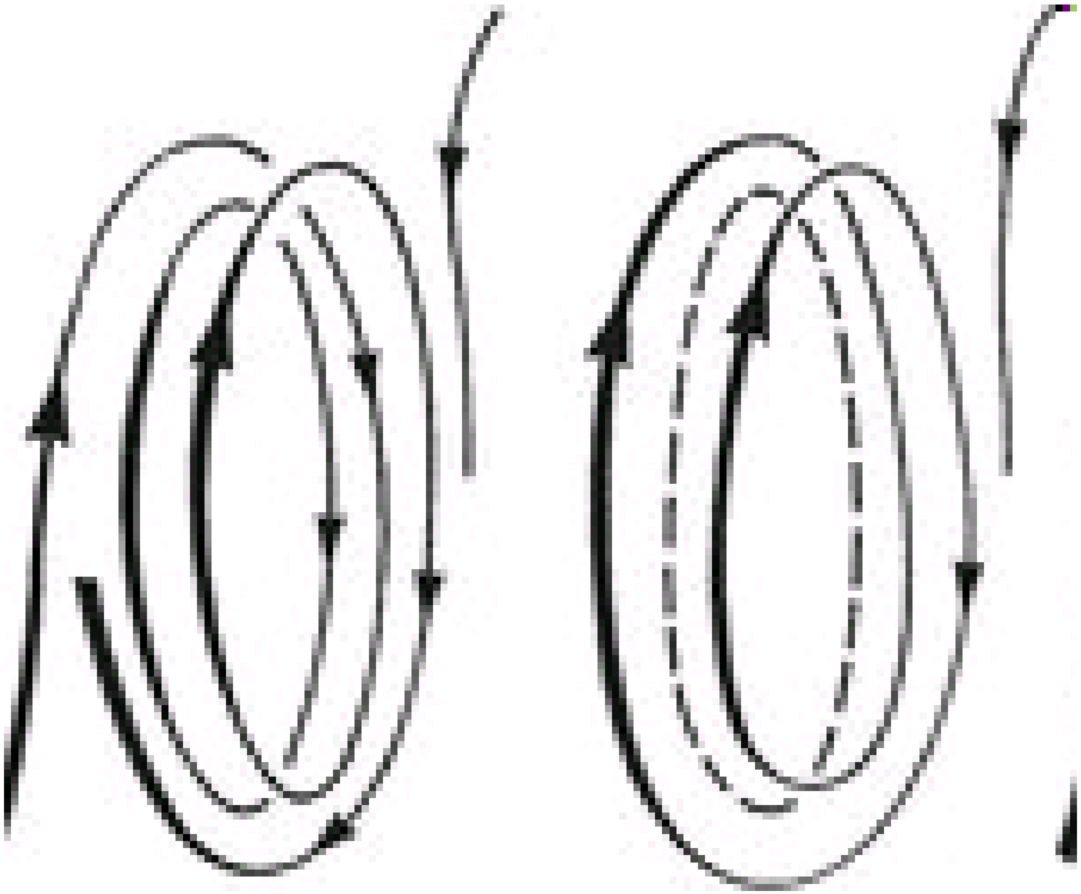




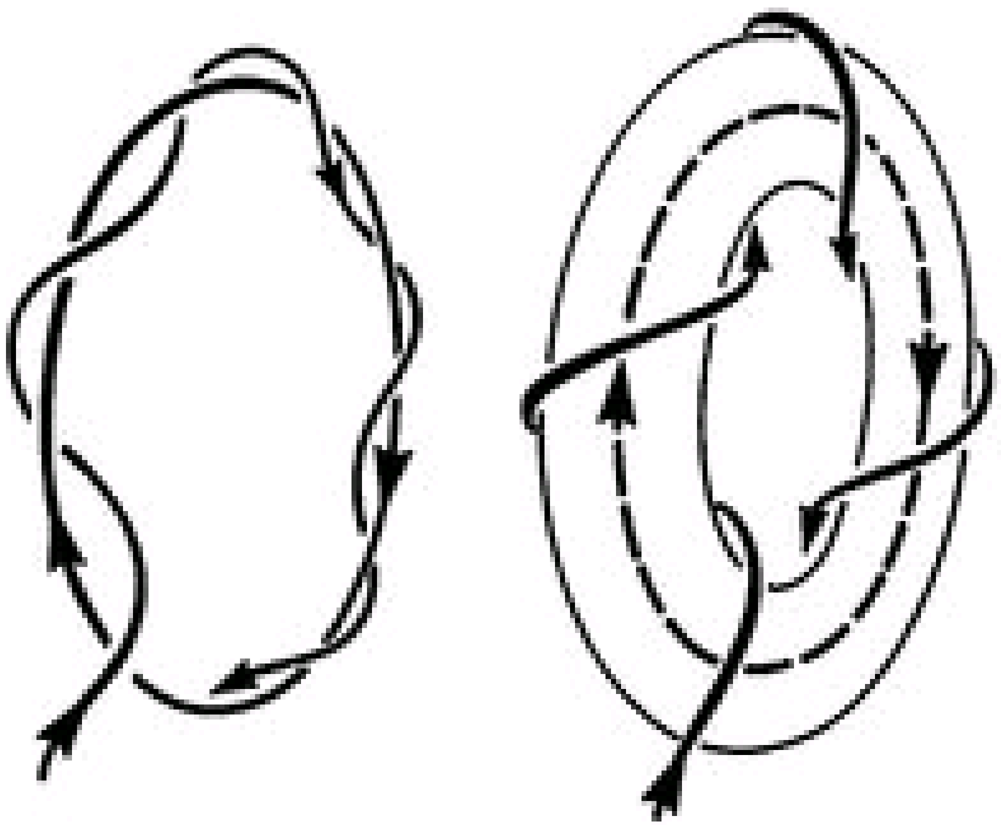
T_b=4.67 T_u=3.33, periodic mode (2 maxima for a period)



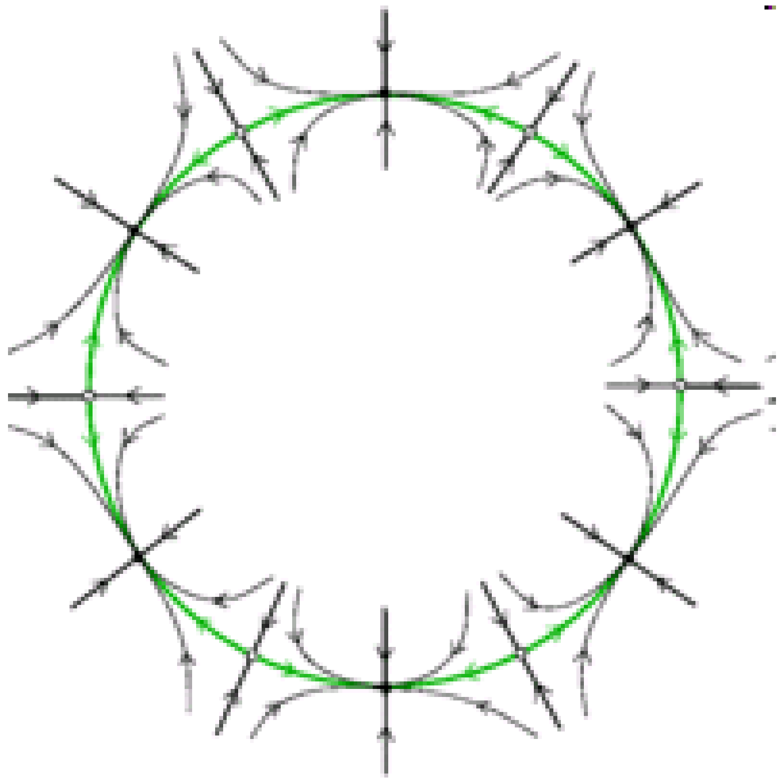
Period-doubling bifurcation



Supercritical torus bifurcation

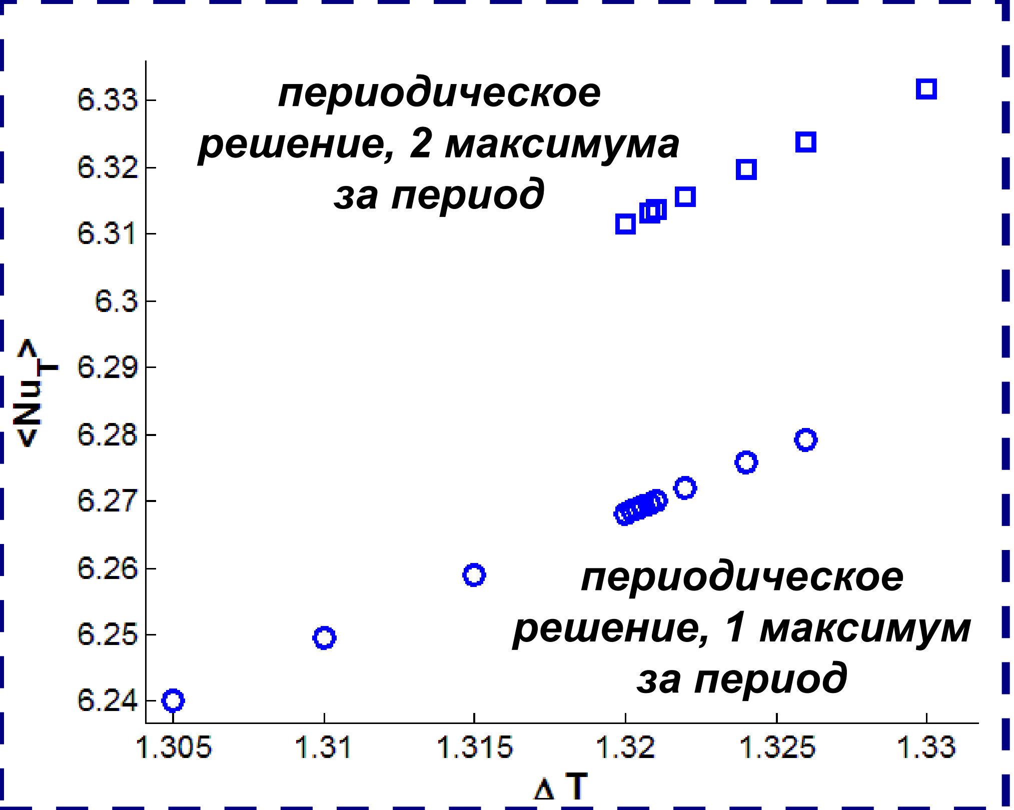


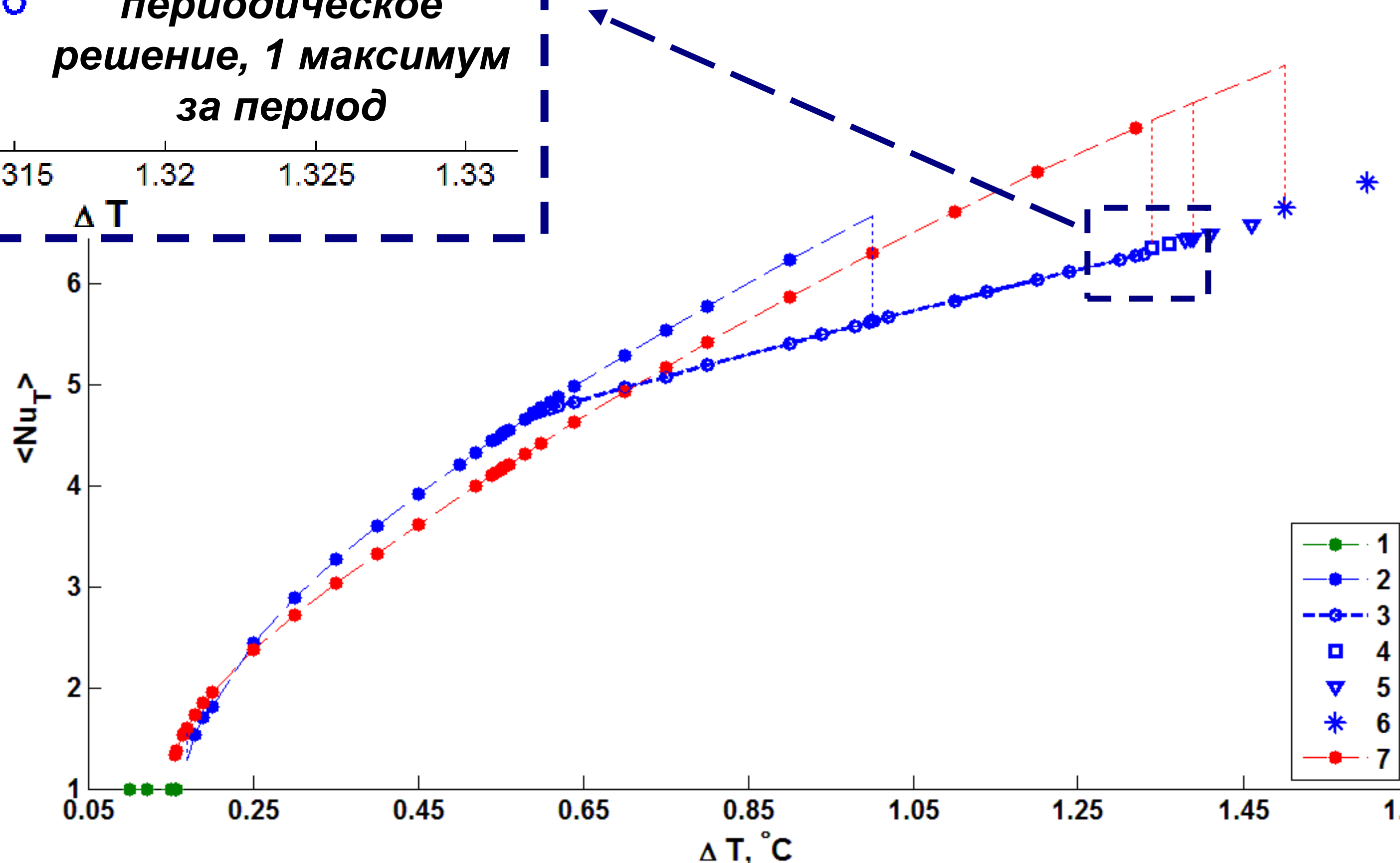
Period-6 cycle

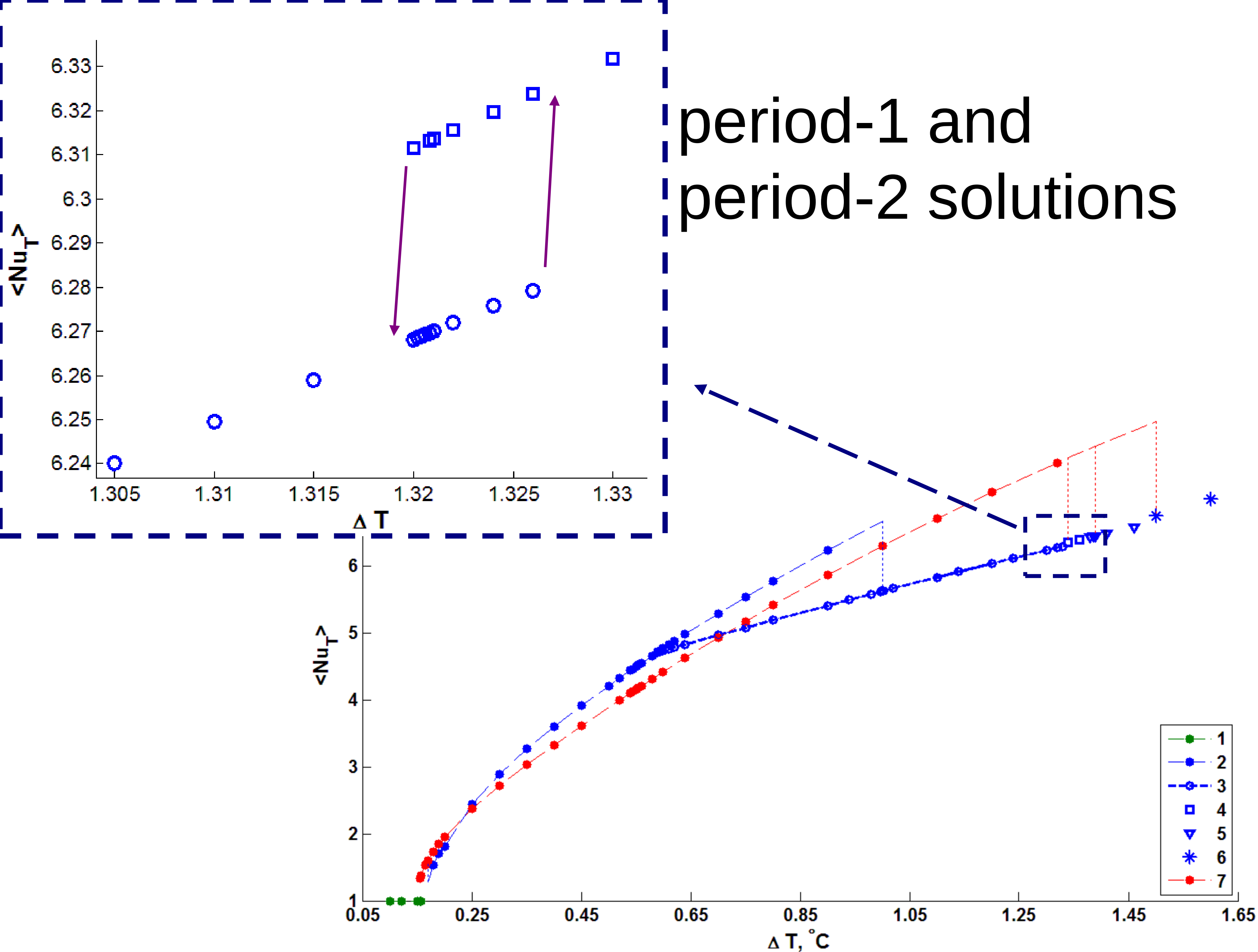


saddle-node bifurcation for maps

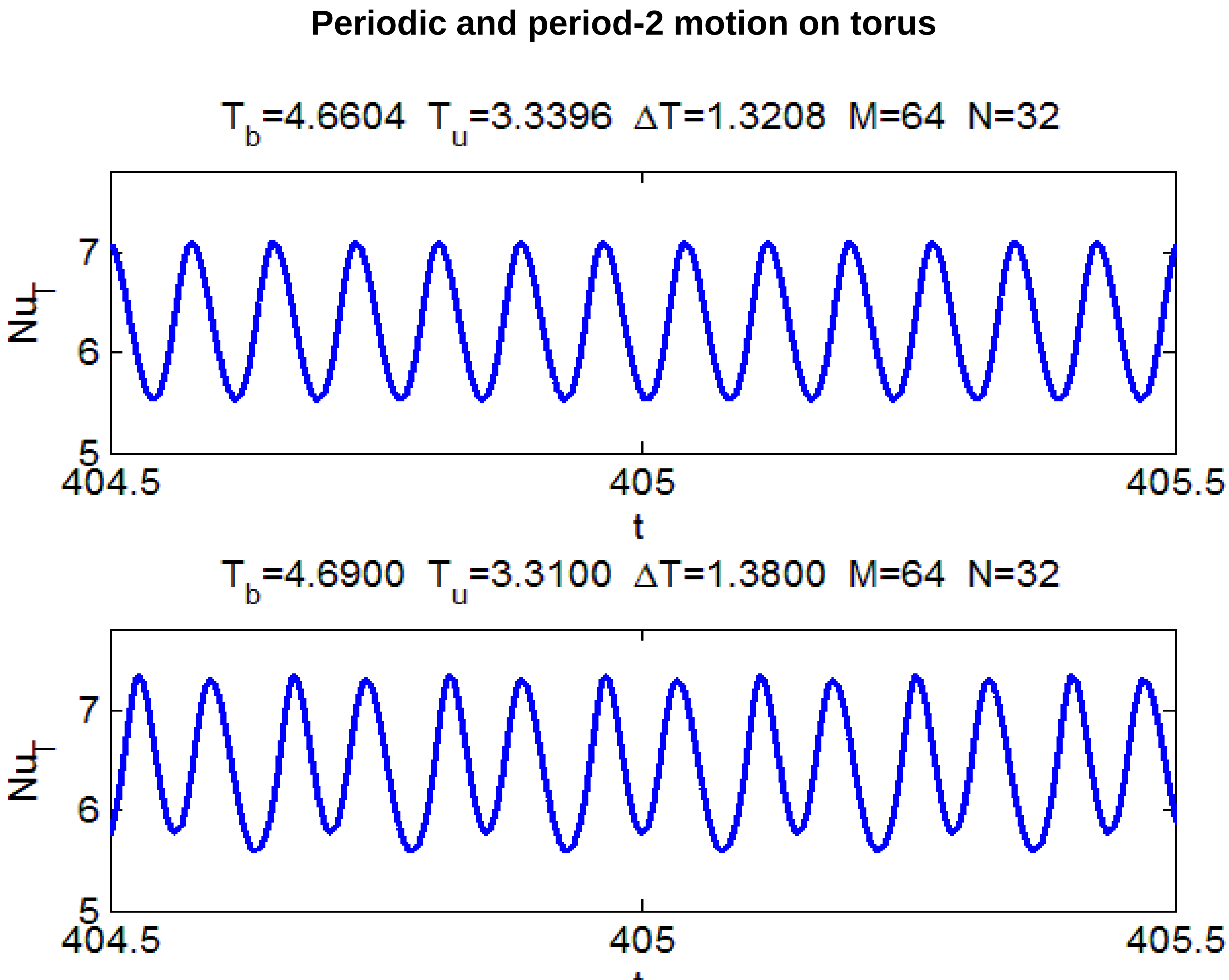
Существование гистерезиса: периодическое решение, 1 максимум – периодическое решение, 2 максимума за период

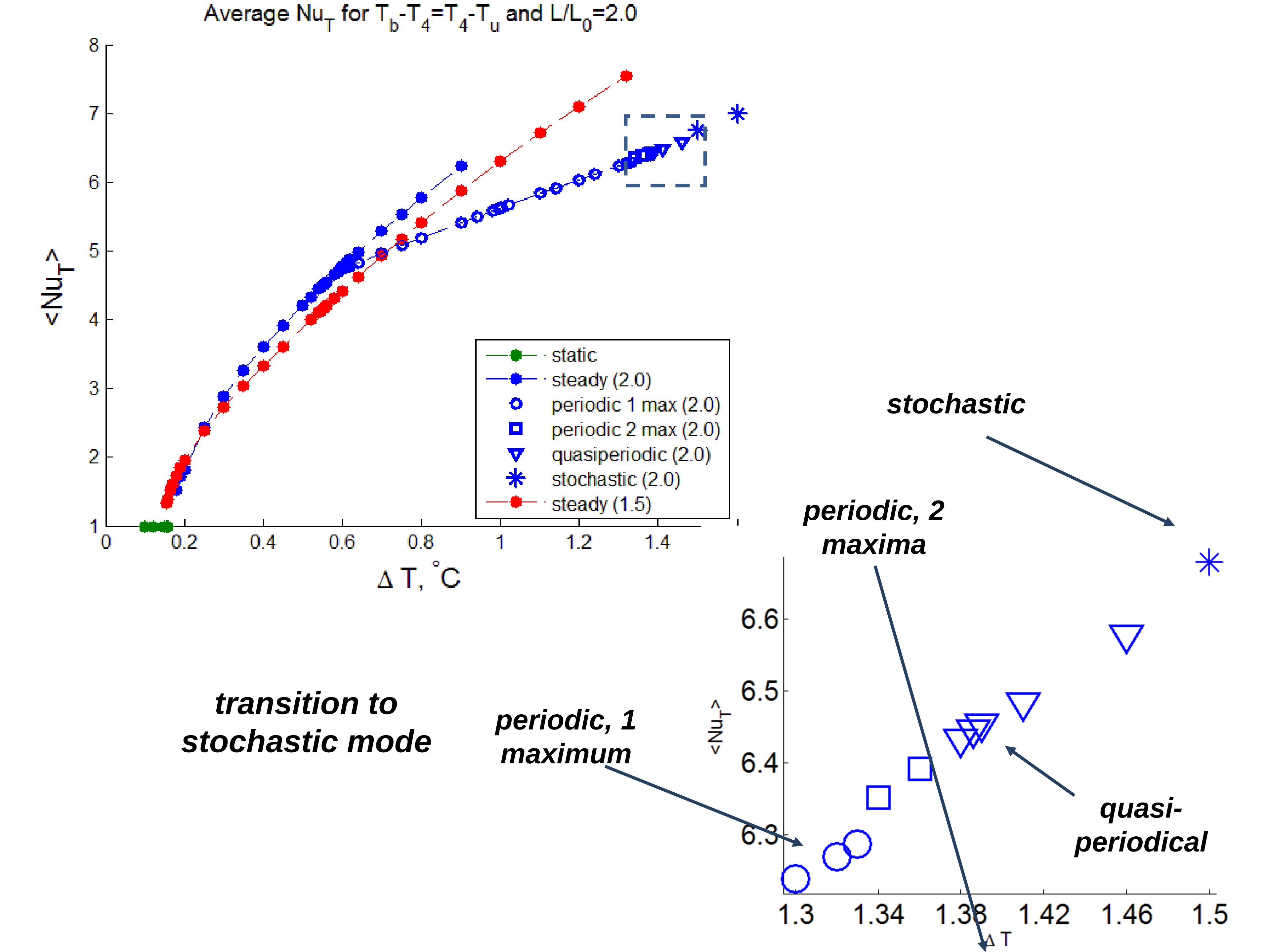


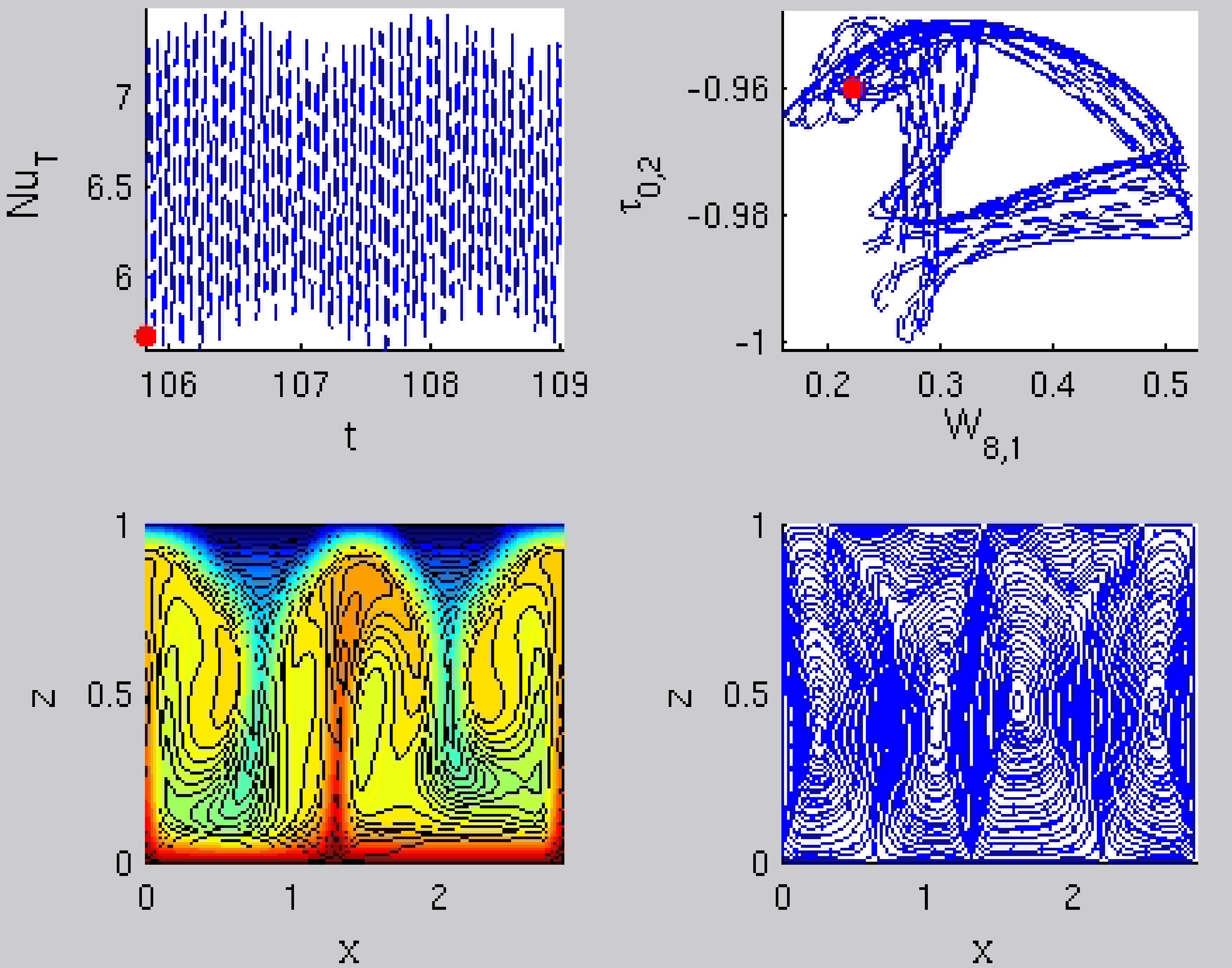




period-1 and
period-2 solutions

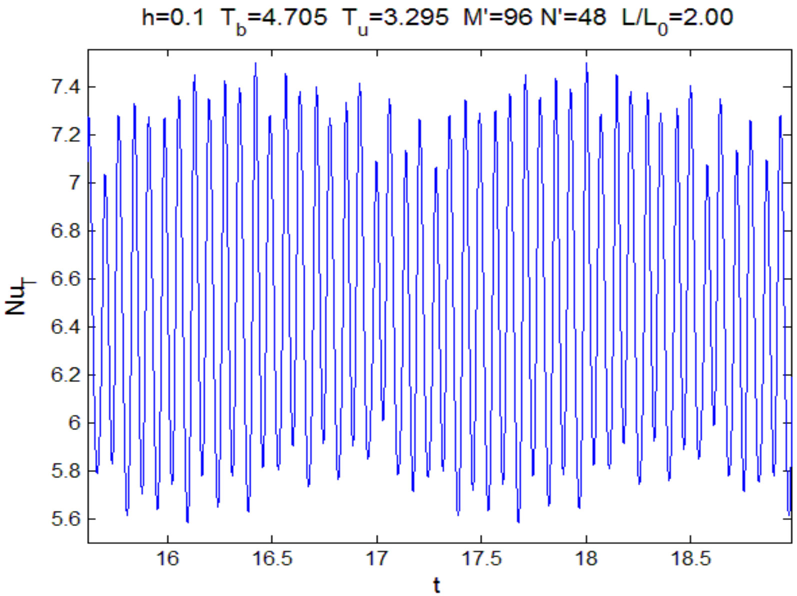

Periodic and period-2 motion on torus
T_b=4.6604 T_u=3.3396 ΔT=1.3208 M=64 N=32
Nu_T
7
6
5
404.5
405
405.5
t
T_b=4.6900 T_u=3.3100 ΔT=1.3800 M=64 N=32
Nu_T
7
6
5
404.5
405
405.5
t

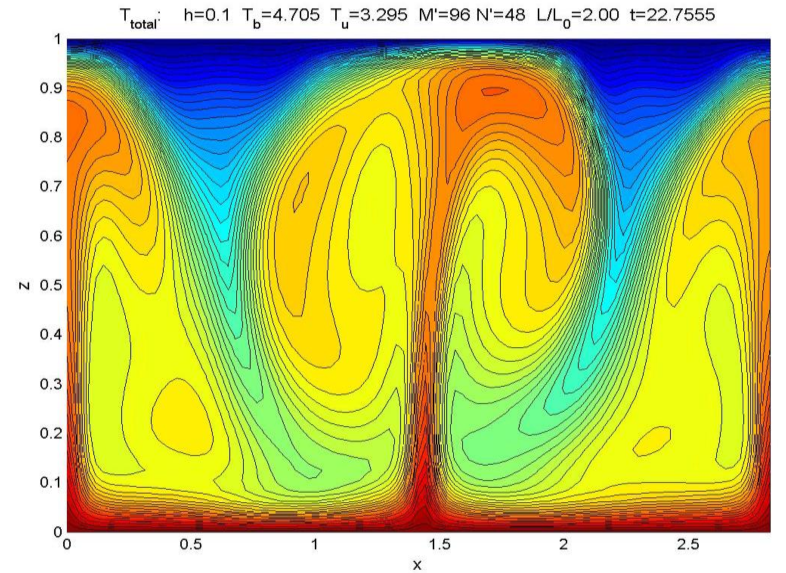

Average Nu_T for T_b-T_4=T_4-T_u and L/L_0=2.0
<Nu_T>
Δ T, °C
static
steady (2.0)
periodic 1 max (2.0)
periodic 2 max (2.0)
quasiperiodic (2.0)
stochastic (2.0)
steady (1.5)
stochastic
periodic, 2 maxima
transition to stochastic mode
periodic, 1 maximum
quasi-periodical
<Nu_T>
Δ T

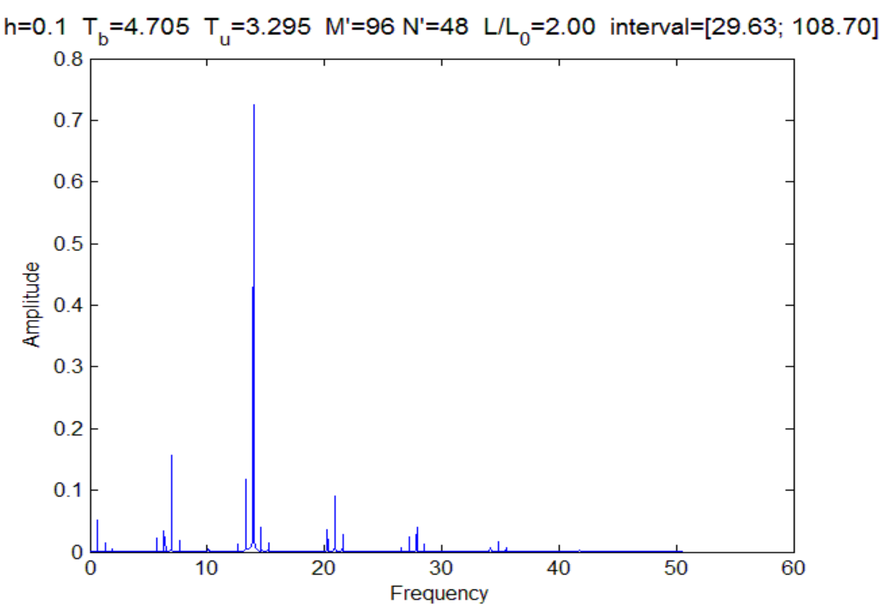


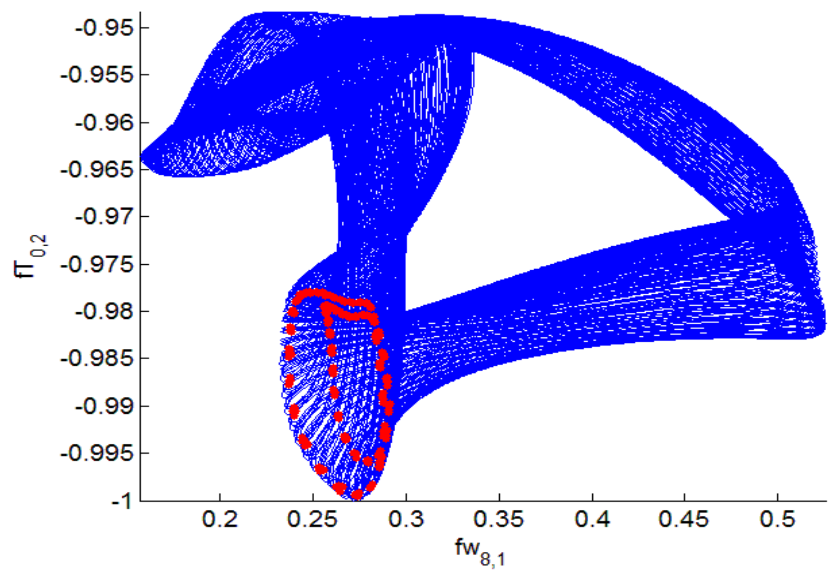
Nu_T
7
6.5
6
106
107
108
109
t
τ_{0,2}
-0.96
-0.98
-1
0.2
0.3
0.4
0.5
W_{8,1}
1
z
0.5
0
0
1
2
x
1
z
0.5
0
0
1
2
x

T_b=4.705 T_u=3.295, quasiperiodic mode

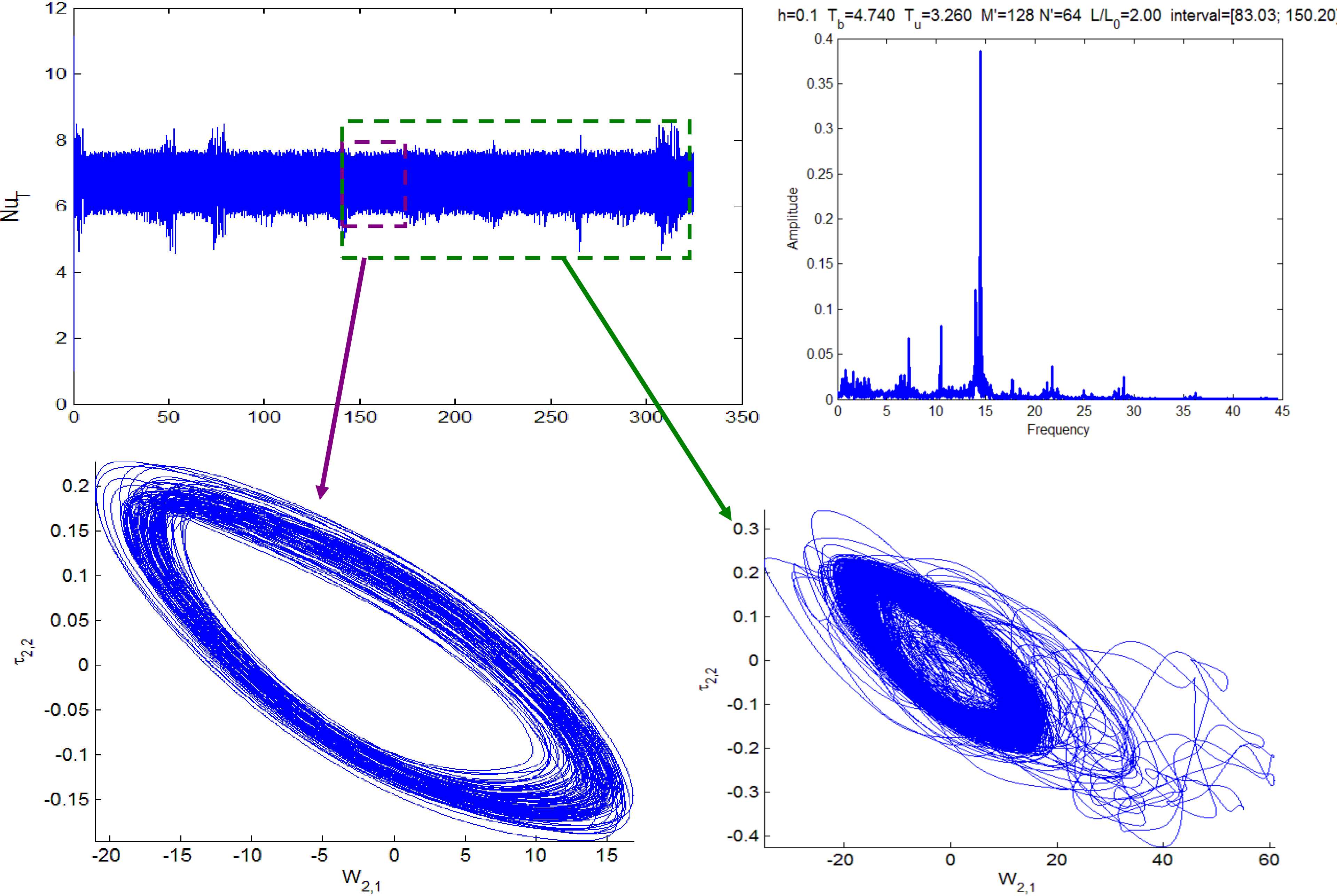




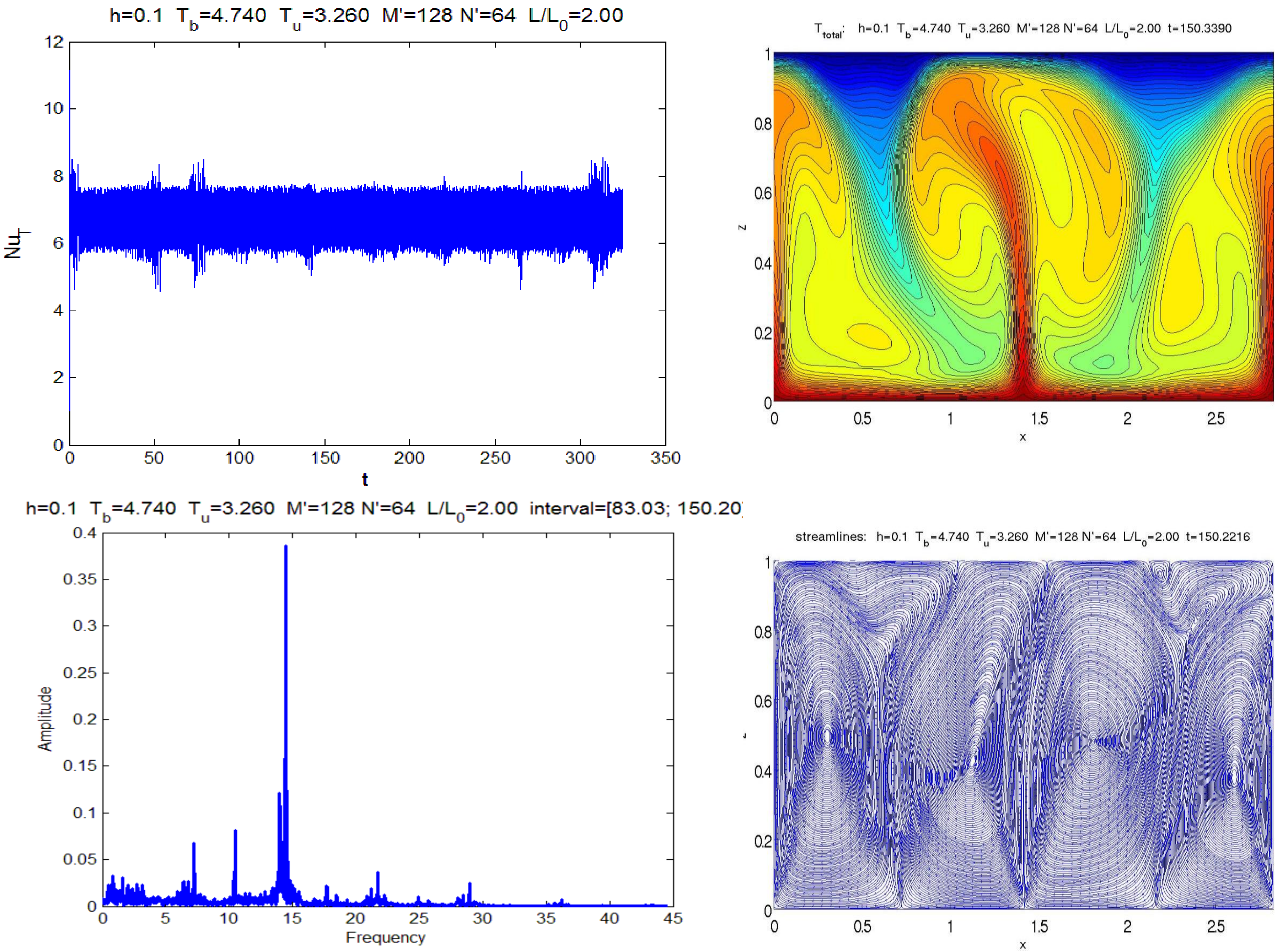




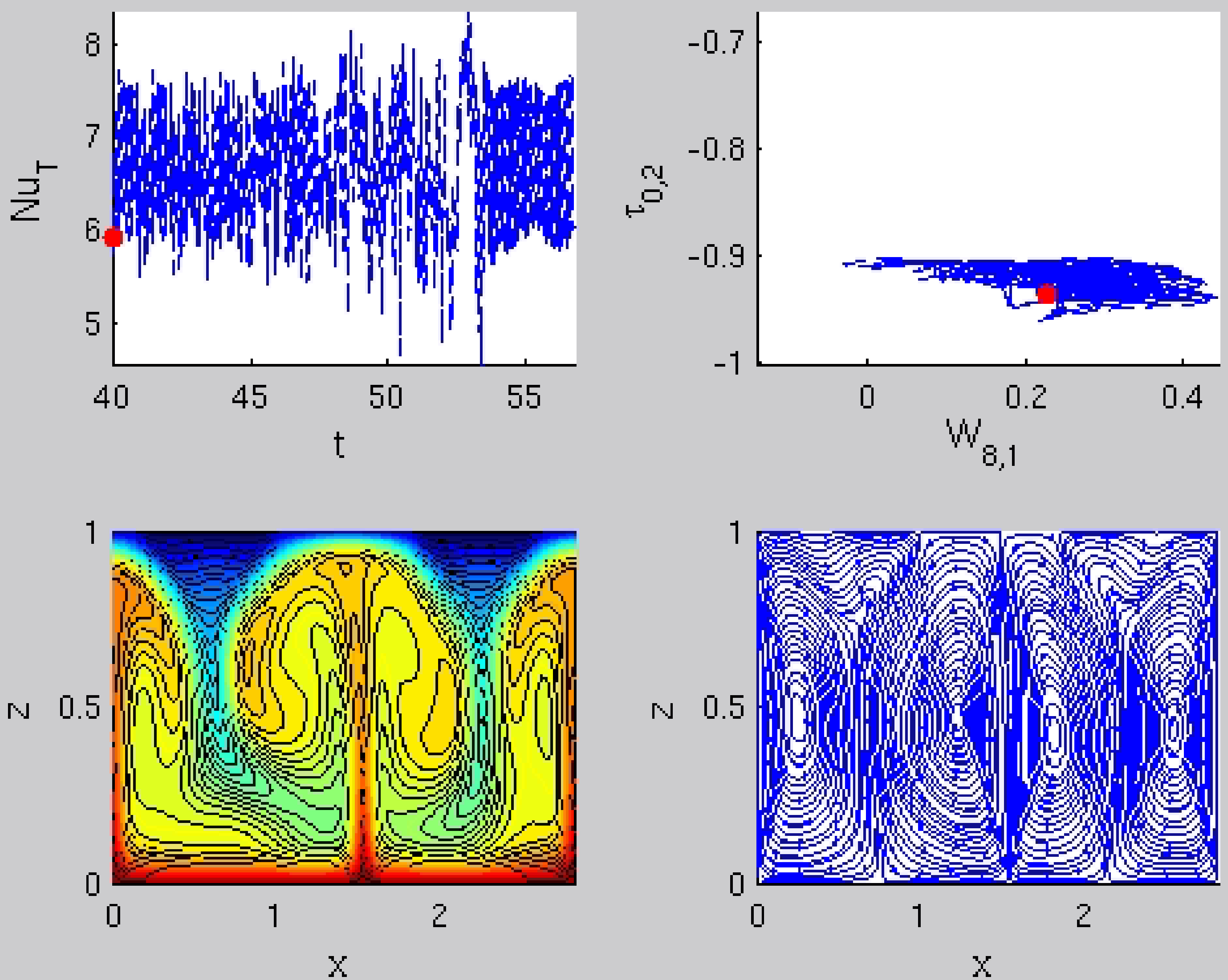
T_b=4.74 T_u=3.26
Intermittency and bursts of heat flux



T_b=4.74 T_u=3.26

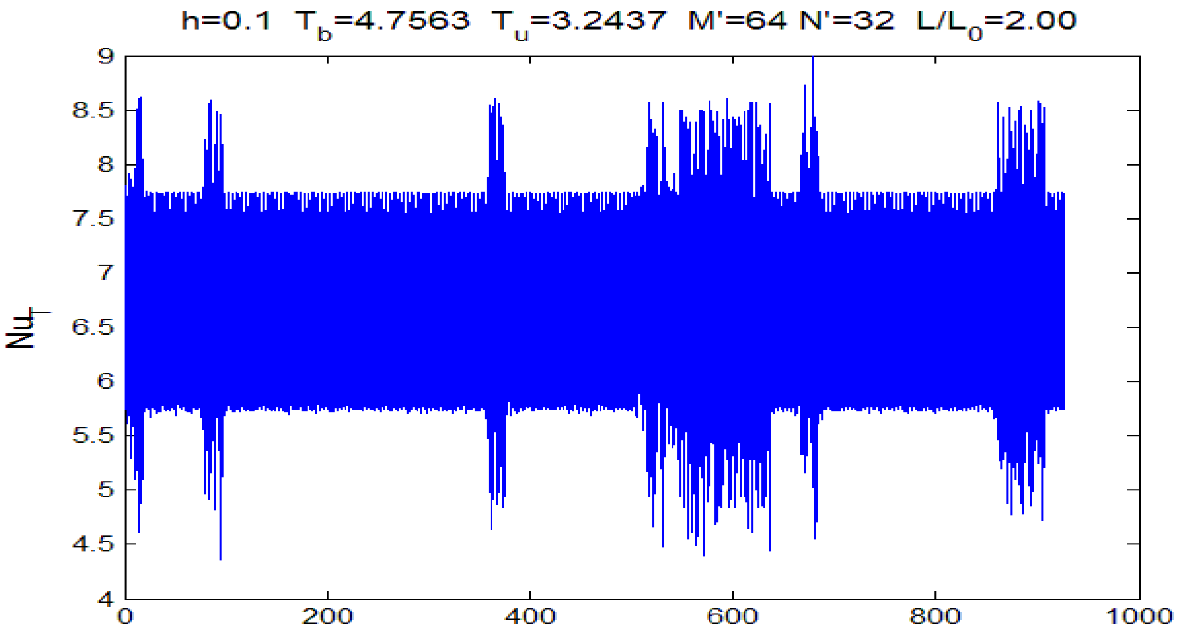


h=0.1 Tb=4.740 Tu=3.260 M'=128 N'=64 L/L0=2.00
NuT
t
Ttotal: h=0.1 Tb=4.740 Tu=3.260 M'=128 N'=64 L/L0=2.00 t=150.3390
z
x
h=0.1 Tb=4.740 Tu=3.260 M'=128 N'=64 L/L0=2.00 interval=[83.03; 150.20]
Amplitude
Frequency
streamlines: h=0.1 Tb=4.740 Tu=3.260 M'=128 N'=64 L/L0=2.00 t=150.2216
x

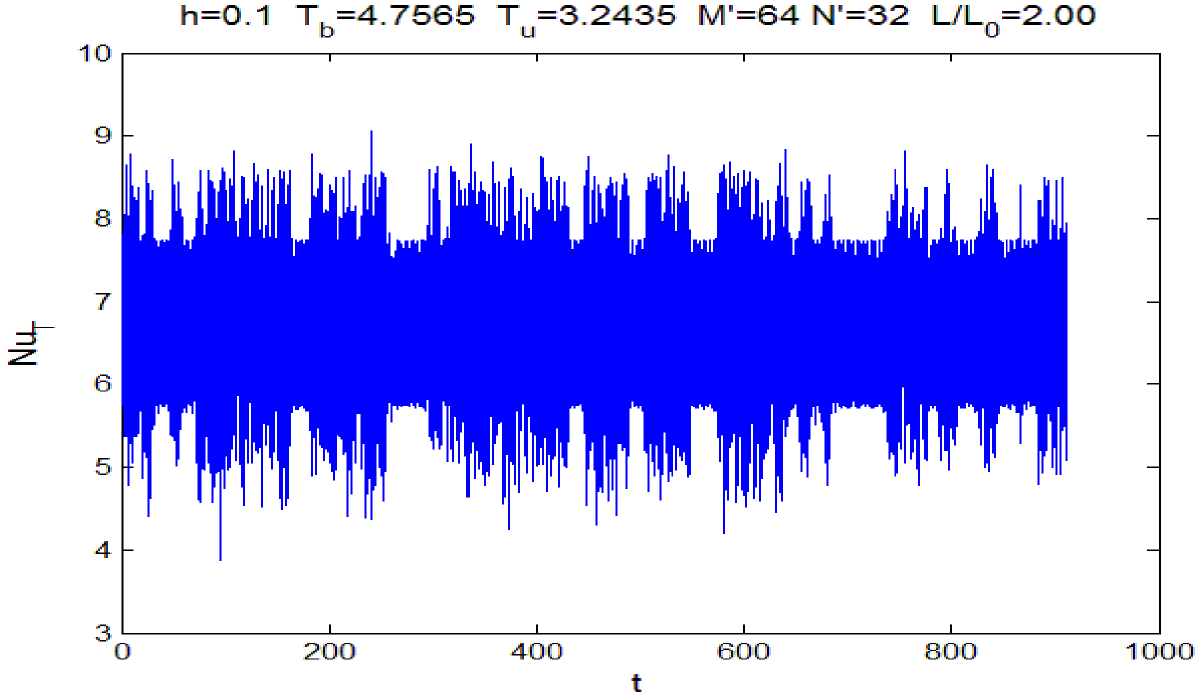


8
7
6
5
Nu_T
40
45
50
55
t
-0.7
-0.8
-0.9
-1
$\tau_{0,2}$
0
0.2
0.4
$W_{8,1}$
1
0.5
0
z
0
1
2
x
1
0.5
0
z
0
1
2
x

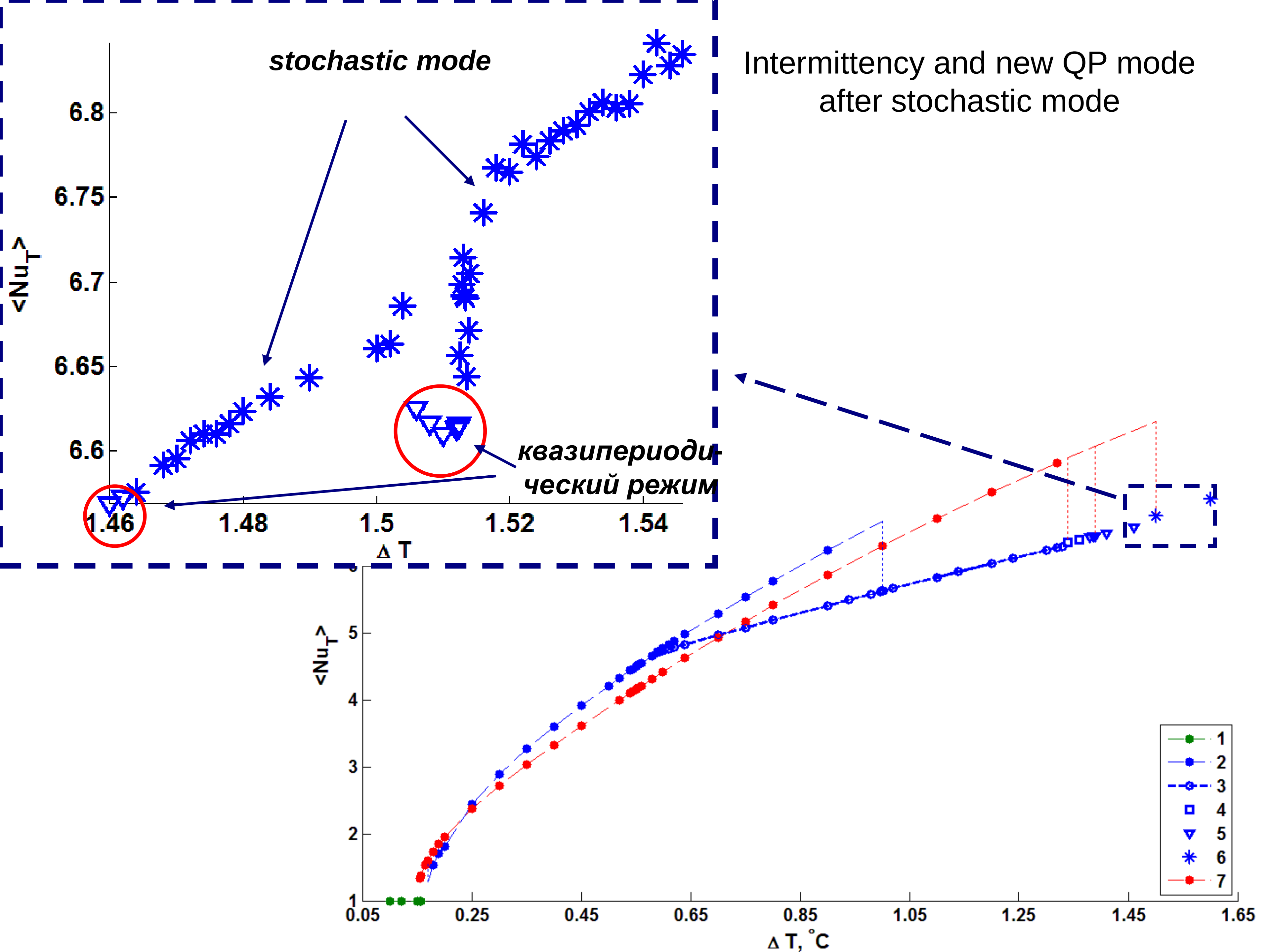
Intermittency and bursts of heat flux in stochastic modes



ΔT=1.5126

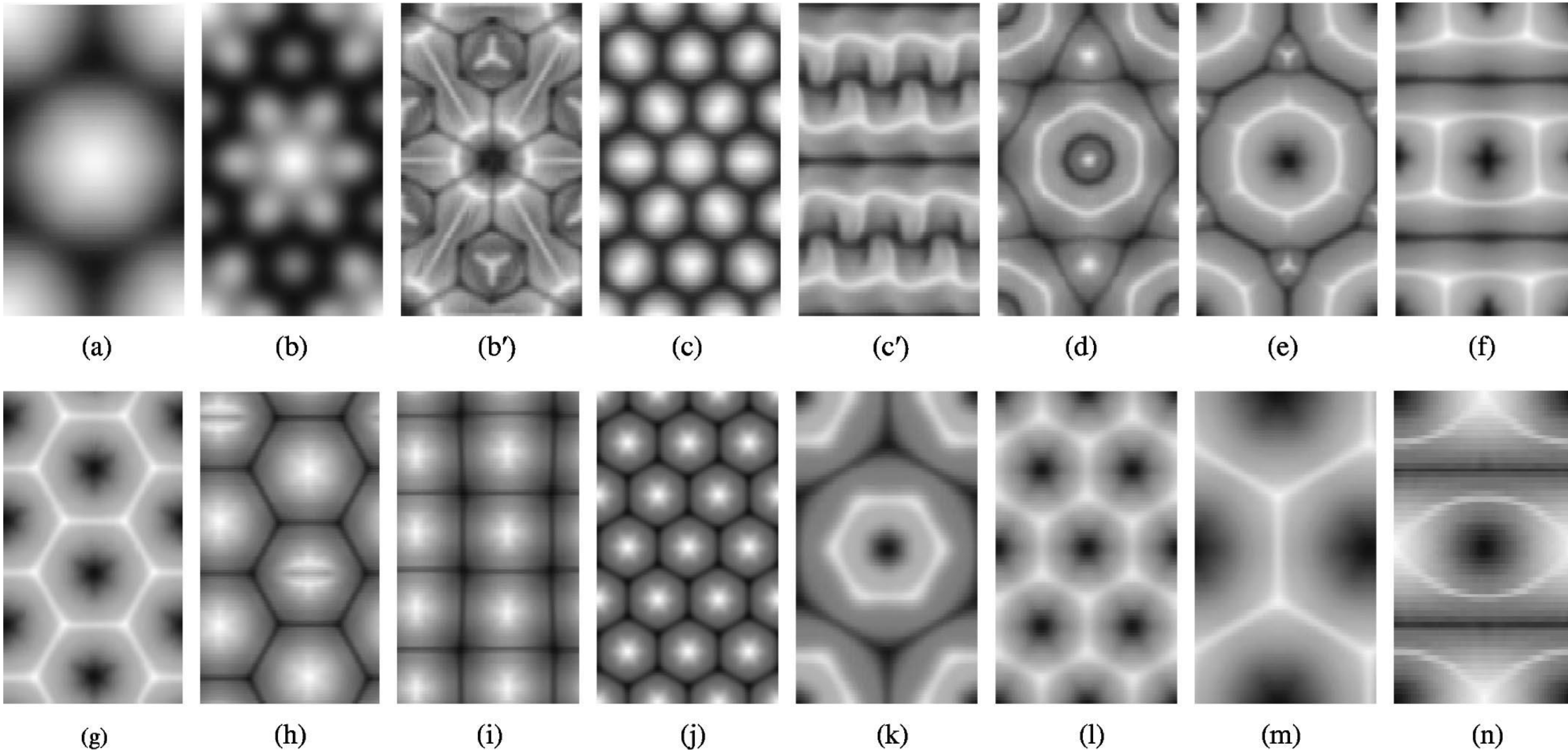


ΔT=1.5130



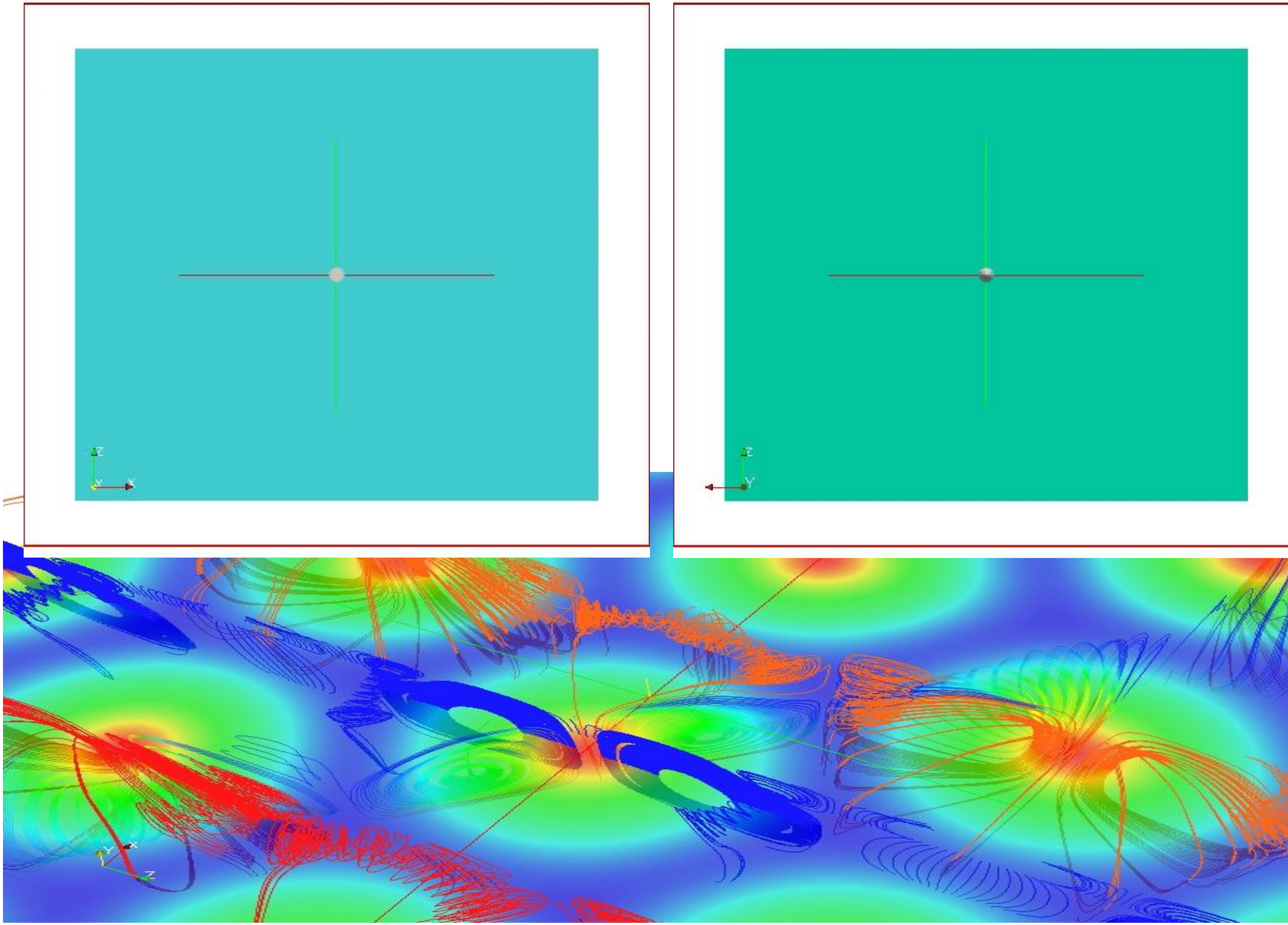
Intermittency and new QP mode
after stochastic mode
stochastic mode
квазипериоди-
ческий режим
<Nu_T>
Δ T
6.8
6.75
6.7
6.65
6.6
1.46
1.48
1.5
1.52
1.54
Δ T, °C
0.05
0.25
0.45
0.65
0.85
1.05
1.25
1.45
1.65
1
2
3
4
5
6
7

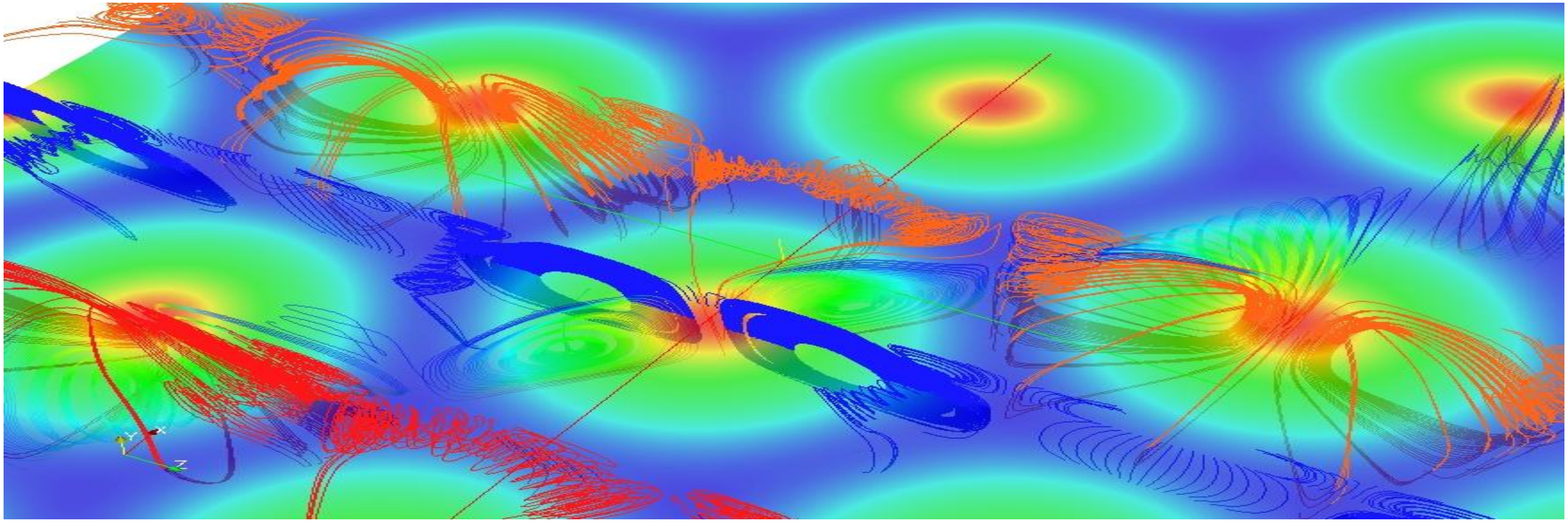
Three-dimensional patterns



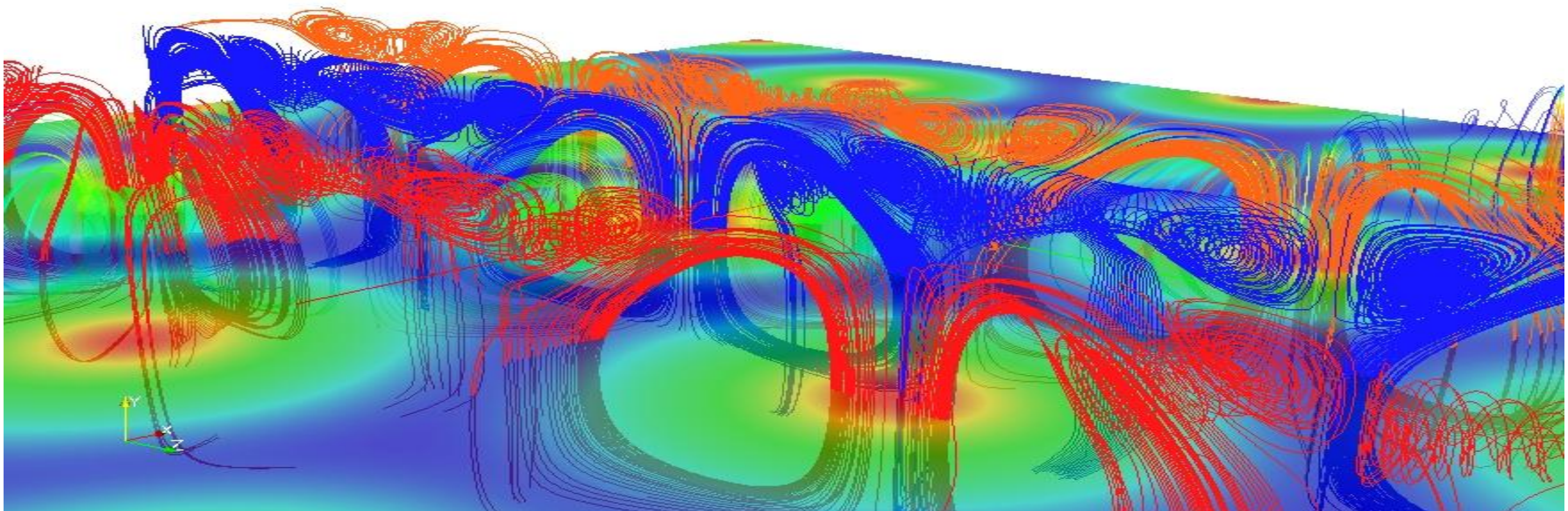
Alexander Getling 2003

Three-dimensional modes





Y
X
Z



Y
X
Z

Spectral methods

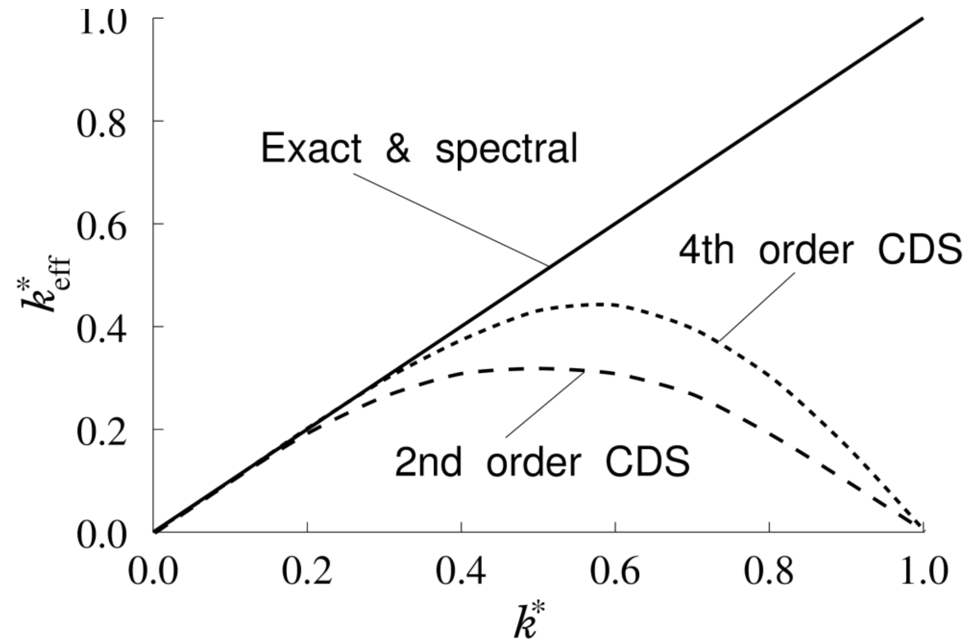
- Functions are represented with the aid of Fourier Series

$$f(x_i) = \sum_{q=-N/2}^{N/2-1} \hat{f}(k_q)\, \mathrm{e}^{\mathrm{i}k_q x_i} \ ,$$

- Spatial derivatives are evaluated exactly

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \sum_{q=-N/2}^{N/2-1} \mathrm{i}k_q \hat{f}(k_q)\, \mathrm{e}^{\mathrm{i}k_q x}$$

- Effective wavenumber, discretization error
- Joel H. Ferziger and Milovan Peric Computational Methods for Fluid Dynamics (to be published in Russian in MSU 2013)



Resolution up to 1000^3

Energy spectra and fluxes for turbulent convection in complex fluids

Energy transfer between various Fourier modes

Kolmogorov's energy spectrum

$$E^u(k) = K_{ko}(\epsilon^u)^{\frac{2}{3}} k^{-\frac{5}{3}} \qquad E^\theta(k) = K_\theta \epsilon^\theta (\epsilon^u)^{-\frac{1}{3}} k^{-\frac{5}{3}}$$

Bolgiano and Obukhov dual cascade for stratified fluids

$$E^u(k) = C_k(\epsilon^\theta)^{\frac{2}{5}} (\alpha g)^{\frac{4}{5}} k^{-\frac{11}{5}} \qquad E^\theta(k) = C_\theta(\epsilon^\theta)^{\frac{4}{5}} (\alpha g)^{-\frac{2}{5}} k^{-\frac{7}{5}}$$

M. Verma et al Mode-to-mode energy transfer, energy cascade in RBC

Further development

Classification of three-dimensional modes.

Comparative analysis with fully periodic boundary conditions.

Height of density maximum variation.

Slip boundary conditions.

Energy transfer between various Fourier modes and energy spectrum